Research Design: Topic 18
Hierarchical Linear Modeling (Measures within Persons)

General Rationale, Purpose, and Applications

Linear Growth Models

1. Random Coefficients Model

   Level 1: DV = f(Time); Level 2; Individual

2. Intercepts and Slopes-as-Outcomes Model

   Level 1; DV = f(Time): Level 2; Sex of Individual

3. Random Coefficients Model

   Level 1; DV = f(Cov): Level 2; Individual

1

HLM can also be used with repeated measures designs. In this case, it is typically referred to as Measures within Persons or Linear Growth Models. In this case, the individual is Level 2 and Time (or the repeated measures factor) is Level 1. This is depicted on the figure presented on slide 4.

In essence, the focus here is to calculate a regression for each individual using the repeated observations as the replication. The model can be bivariate with respect to Level 1 or multivariate, but the number of repeated observations will determine how many variables can be handled at level 1. With 3 repeated measures, for example, you are limited to one predictor. This predictor can be trials or any other variable repeated over trials and can be both (or many) assuming there are enough repeated measures.

2

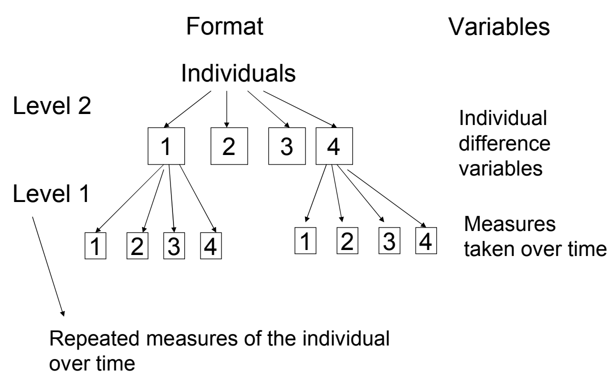
In this form, HLM can be applied to:
"(a) estimate a mean growth curve and the extent of individual variation around it
(b) Assess the reliability of measures for studying both status and change
(c) estimate the correlation between initial status and rate of change, and
(d) model relations of person-level predictors to both status and change" (Raudenbush & Bryk, 2002, p. 163).

We will consider only (a) and (d) in this Topic.

The simplest form of the random coefficients model is concerned with the regression of the individual's score on each trial against the trial number (i.e., (a) above). This can be expressed in equation form in a manner very similar to that used for persons within groups. The equations are shown in Slides 7 and 8. The examples make use of the Raudenbush & Bryk HLM program.

3

HLM Measures within Persons



4

Sample Data to be used in Topic 20
(Note: This is the Level 1 data file)

Subject	Time	DV	Cov	Sex
1	1	3	7	1
1	2	2	5	1
1	3	5	9	1
2	1	2	6	1
2	2	3	8	1
2	3	4	8	1
3	1	1	5	1
3	2	1	4	1
3	3	1	5	1
4	1	3	6	2
4	2	3	6	2
4	3	2	5	2
5	1	2	5	2
5	2	2	6	2
5	3	4	7	2
6	1	2	5	2
6	2	1	3	2
6	3	3	6	2
7	1	2	4	2
7	2	1	3	2
7	3	2	4	2

5

If your data are not sorted, you will also need a Level 2 file presenting the data common to an individual. In this case, it is Sex (which wouldn't have to be shown in the Level 1 file).

Level 2 file

Subject	Sex
1	1
2	1
3	1
4	2
5	2
6	2
7	2

6

Random Coefficients Model

Level 1 model $y_{ij} = \beta_{0j} + \beta_{1j}C_{ij} + \gamma_{ij}$

where: β_{0j} is the intercept for individual j
β_{1j} is the unstandardized regression coefficient (slope) for individual j
C_{ij} is a code for individual j (i.e., Trial i)
γ_{ij} is the error in prediction.

Level 2 models $(1)\ \beta_{0j} = \gamma_{00} + \mu_{0j}$

where:

$\gamma_{00} =$ mean of the intercepts

$\mu_{0j} = \beta_{0j} - \gamma_{00} =$ deviation of each intercept from the mean intercept

$(2)\ \beta_{1j} = \gamma_{10} + \mu_{1j}$

where:

$\gamma_{10} =$ mean of the slopes

$\mu_{1j} = \beta_{1j} - \gamma_{10} =$ deviation of each slope from the mean slope

7

Substituting the two Level 2 values into the Level 1 equation yields the full equation:

$$y_{ij} = \gamma_{00} + \gamma_{10}C_{ij} + \mu_{0j} + \mu_{1j}C_{ij} + \gamma_{ij}$$

Given this equation we can solve simultaneously (i.e., the unique solution) for the following parameters and their standard errors of estimate:

Mean Intercept $\gamma_{00}\ (i.e., \beta_{00})$

Mean Slope $\gamma_{10}\ (i.e., \beta_{10})$

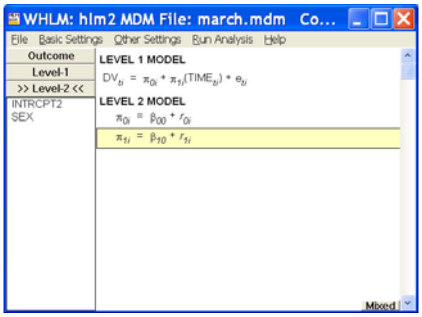
Variance of the intercepts - - based on μ_{0j}

Variance of the slopes - - based on μ_{1j}

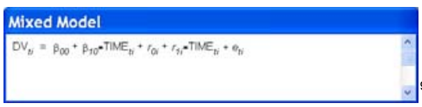
Error - - based on γ_{ij}

8

Basic set-up of Level 1 and Level 2 models. Note that Time is not centered, though it could be if desired.



Merged Level 1 and Level 2 model



9

Following are the OLS and ML estimates for each person (i.e., Level 2). They can be output if desired but they are not part of the default. The OLS values can be requested in Other Settings but the ML values can be output in the Resfil2 output under Basic Settings. Note that the values differ even though the means are the same.

	Ordinary Least Squares		Maximum Likelihood (REML)	
Person	Intercepts	Slopes	Intercepts	Slopes
1	1.3333	1.0000	1.426	.819
2	1.0000	1.0000	1.439	.711
3	1.0000	0.0000	1.538	-.053
4	3.6667	-0.5000	1.478	.428
5	0.6667	1.0000	1.453	.603
6	1.0000	0.5000	1.489	.329
7	1.6667	0.0000	1.511	.163
Means	1.4762	.4286	1.4763	.4286

10

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx. d.f.	P-value
For INTRCPT1, P0					
INTRCPT2, B00	1.476190	0.462242	3.194	6	0.021
For TIME slope, P1					
INTRCPT2, B10	0.428571	0.254278	1.685	6	0.142

Final estimation of variance components:

Random Effect		Standard Deviation	Variance Component	df	Chi-square	P-value
INTRCPT1,	R0	0.06177	0.00382	6	4.14951	>.500
TIME slope,	R1	0.36458	0.13292	6	6.92648	0.327
level-1,	E	0.79960	0.63937			

11

Interpretation of Results

The tests of fixed effects indicate that:

a. The mean intercept (1.4762) differs significantly from 0, $t(6) = 3.194$, $p < .021$. Note that Time was not centered so that this value refers to the mean intercept when Time = 0.

b. The mean slope (.4286) does not differ significantly from 0, $t(6) = 1.685$.

The tests of the random effects indicate that:

a. The variance of the intercepts for the 7 subjects (.00382) is not significantly greater than 0, $\chi^2(6) = 4.14591$, ns. That is, there is no evidence of individual differences among the subjects.

b. The variance of the slopes for the 7 subjects (.13292) does not differ significantly from 0 $\chi^2(6) = 6.92648$, ns. That is, there is no evidence of individual differences in the slopes.

c. The variance due to error in Level 1 is .63937.

12

Intercepts and Slopes-as-Outcome Variables

This model is comparable to the Random Coefficients model except that in addition to solving for the slopes and intercepts for each subject, we can also determine whether the slopes and intercepts vary as a function of a Level 2 variable (in this case Sex). This is an example of point (d) in slide 3.

13

Intercepts and Slopes-as-Outcome Variables

Level 1 model $y_{ij} = \beta_{0j} + \beta_{1j}C_{ij} + \gamma_{ij}$

where: β_{0j} is the intercept for individual j
β_{1j} is the unstandardized regression coefficient (slope) for individual j
C_{ij} is a code for individual j
γ_{ij} is the error in prediction.

Level 2 models (1) $\beta_{0j} = \gamma_{00} + \gamma_{01}W_j + \mu_{0j}$

(2) $\beta_{1j} = \gamma_{10} + \gamma_{10}W_j + \mu_{1j}$

where: $\gamma_{00} =$ mean of the intercepts

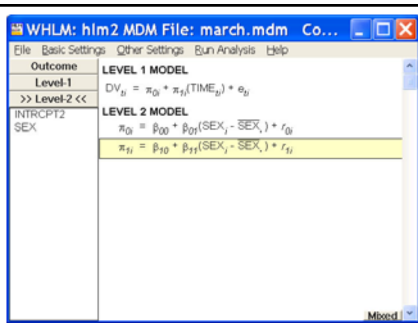
$\gamma_{10} =$ mean of the slopes

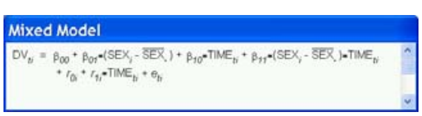
$W_j =$ Level 2 variable

$\mu_{0j} = \beta_{0j} - \gamma_{00} =$ deviation of each intercept from the mean intercept

$\mu_{1j} = \beta_{1j} - \gamma_{10} =$ deviation of each slope from the mean slope

14





15

Following are the OLS and ML estimates that can be obtained for this run. Often individual researchers do not output this information. It is done here to show you precisely what HLM does. Note that the values differ, but the means are the same (this isn't always necessarily the case).

	Ordinary Least Squares		Maximum Likelihood (REML)	
Person	Intercepts	Slopes	Intercepts	Slopes
1	1.3333	1.0000	1.131	.979
2	1.0000	1.0000	1.124	.875
3	1.0000	0.0000	1.078	.146
4	3.6667	-0.5000	1.756	.304
5	0.6667	1.0000	1.761	.457
6	1.0000	0.5000	1.746	.197
7	1.6667	0.0000	1.737	.042
Means	1.4762	.4286	1.4777	.4286

16

```
Final estimation of fixed effects:
----------------------------------------------------------------------------
                                      Standard             Approx.
   Fixed Effect         Coefficient   Error      T-ratio   d.f.     P-value
----------------------------------------------------------------------------
For       INTRCPT1, P0
   INTRCPT2, B00          1.476190   0.472291     3.126       5      0.030
        SEX, B01          0.638889   0.954371     0.669       5      0.533
For      TIME slope, P1
   INTRCPT2, B10          0.428571   0.258889     1.655       5      0.158
        SEX, B11         -0.416667   0.523144    -0.796       5      0.462
----------------------------------------------------------------------------

Final estimation of variance components:
-----------------------------------------------------------------------------
Random Effect           Standard      Variance     df    Chi-square  P-value
                        Deviation     Component
-----------------------------------------------------------------------------
INTRCPT1,       R0       0.05089       0.00259      5      3.52237    >.500
   TIME slope, R1        0.36760       0.13513      5      5.73795    0.332
 level-1,       E        0.81735       0.66807
-----------------------------------------------------------------------------
```

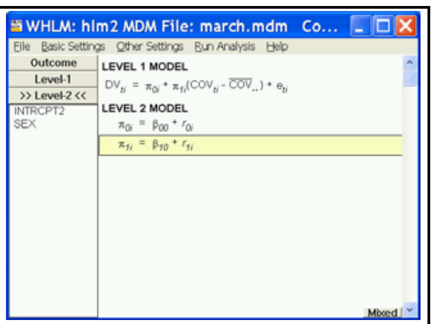
17

The next example is concerned with whether the Outcome variable at each trial covaries as a function of the other variable (COV), but does not include trial number (i.e., Time) as part of the model. If it were desired to include Time along with COV you would need at least one more trial because with only three data points, you cannot have more than one predictor.

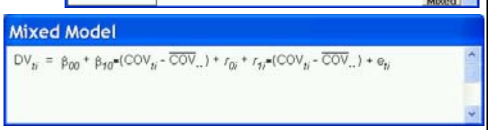
Note that this analysis is identical to what would be done for Persons within Groups, and in fact we could run the example through this option and obtain identical results.

18

Basic Level 1 and Level 2 models



Merged (mixed) model



19

	Ordinary Least Squares		Maximum Likelihood (REML)	
Person	Intercepts	Slopes	Intercepts	Slopes
1	2.26190	0.75000	2.349	.683
2	1.67857	0.75000	1.911	.689
3	1.00000	0.00000	1.785	.691
4	2.57143	1.00000	2.539	.680
5	2.23810	1.00000	2.362	.683
6	2.58163	0.64286	2.552	.680
7	3.57143	1.00000	2.815	.676
Means	2.27187	0.73469	2.3304	.6831

Note, in this case the mean OLS and ML values do not agree.

20

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx. d.f.	P-value
For INTRCPT1, P0					
INTRCPT2, B00	2.330653	0.182884	12.744	6	0.000
For COV slope, P1					
INTRCPT2, B10	0.683154	0.077186	8.851	6	0.000

Final estimation of variance components:

Random Effect	Standard Deviation	Variance Component	df	Chi-square	P-value
INTRCPT1, R0	0.42537	0.18094	6	12.57221	0.050
COV slope, R1	0.00772	0.00006	6	4.39839	>.500
level-1, E	0.39922	0.15938			

21

References

Gardner, R. C. (2007) Hierarchical Linear Modeling: Measures within Persons. Unpublished manuscript. Department of Psychology, University of Western Ontario.
http://publish.uwo.ca/~gardner/DataAnalysisDotCalm

Raudenbush, S., Bryk, A., Cheong, Y. F., & du Toit, M. (2004). *HLM6:Hierarchical and NonLinear Modeling*. Lincolnwood, NJ: SSI Scientific Softward International.

22