Research Design - - Topic 4
Multiple Comparison Tests (Kirk, Ch. 4)

Tests of Means following an analysis of variance

(a) Types of Contrasts

(b) Conceptual Unit for Type I Error Rate

(c) Basic Test Statistics: t, q, F

(d) An Example Using SPSS

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Types of Contrasts

A Contrast *is a comparison between two means. It is formally defined as a linear combination of weighted means thus they need not be pairwise comparisons. The general form Is:*

$$\psi_i = c_1\overline{X}_1 + c_2\overline{X}_2 + \ldots + c_a\overline{X}_a$$

•*An* a priori contrast *is one that is planned (i.e., a planned comparison) to test a specific hypothesis.*

•*An* aposteriori contrast *is unplanned. It is a post hoc comparison concerned with identifying differences that exist in the data.*

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- Orthogonal contrasts. *Two contrasts are orthogonal if they are independent of each other. There is no overlap in the data represented in each contrast.*
- Non-Orthogonal contrasts. *Two contrasts are non-orthogonal if they share similar information.*

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A set of orthogonal contrasts with 4 means.

$$\psi_i = c_1\overline{X}_1 + c_2\overline{X}_2 + c_3\overline{X}_3 + c_4\overline{X}_4$$

$$\psi_1 = (1)\overline{X}_1 + (-1)\overline{X}_2 + (0)\overline{X}_3 + (0)\overline{X}_4 \ = \ \overline{X}_1 - \overline{X}_2$$

$$\psi_2 = (0)\ \overline{X}_1 + (0)\overline{X}_2 + (1)\overline{X}_3 + (-1)\overline{X}_4 \ = \ \overline{X}_3 - \overline{X}_4$$

$$\psi_3 = (.5)\overline{X}_1 + (.5)\overline{X}_2 + (-.5)\overline{X}_3 + (-.5)\overline{X}_4 = \frac{\left(\overline{X}_1 + \overline{X}_2\right)}{2} - \frac{\left(\overline{X}_3 + \overline{X}_4\right)}{2}$$

These contrasts are orthogonal because the sum of the cross product of the coefficients is 0 for each pair of contrasts.

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Conceptual Unit for Type I Error Rate

- Per Contrast error rate (α_{pc}). The probability that any one contrast will be incorrectly declared significant.

$$\alpha_{PC} = \frac{\text{No. of contrasts falsely declared significant}}{\text{Total No. of contrasts}}$$

- Familywise error rate (α_{FW}) The probability of incorrectly declaring one or more contrasts significant per family

$$\alpha_{FW} = \frac{\text{No. of families with at least one contrast falsely declared significant}}{\text{Total No. of families}}$$

- Per Family Error Rate (α_{PF}) The average number of contrasts incorrectly declared significant per family (not a true probability).

$$\alpha_{PF} = \frac{\text{No. of contrasts falsely declared significant}}{\text{Total No. of families}}$$

$$\alpha_{PC} < \alpha_{FW} < \alpha_{PF}$$

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Definitions of Type I error rates

$\alpha_{PC} =$ *per contrast* Defined by the LSD t-test

$\alpha_{FW} = 1 - (1 - \alpha_{PC})^{C}$ Used in the Dunn Šidák test

$\alpha_{PF} = \sum_{j}^{C} \alpha_{PCj} = C\alpha_{PC}$ Used in the Dunn (Bonferroni) test

Where the C in the equations for α_{FW} and α_{PF} refers to the number of contrasts.

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The three test Statistics

The t-test

$$t = \frac{c_1\bar{X}_1 + c_2\bar{X}_2 + \ldots + c_a\bar{X}_a}{\sqrt{MS_{error}\left[\frac{c_1^2}{n_1} + \frac{c_2^2}{n_2} + \ldots\right]}} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{MS_{error}\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}$$

The q-test

$$q = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{MS_{error}}{n}}}$$

The F-test

$$F = \frac{(c_1\bar{X}_1 + c_2\bar{X}_2 + \ldots + c_a\bar{X}_a)^2}{MS_{error}\left(\frac{c_1^2}{n_1} + \frac{c_2^2}{n_2} + \ldots +\right)} = \frac{\left(\bar{X}_1 - \bar{X}_2\right)^2}{MS_{error}\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}$$

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Comparing Mean 1 with Mean 4

The t-test

$$t = \frac{\bar{X}_1 + 0 + 0 - \bar{X}_4}{\sqrt{MS_{error}\left[\frac{1}{n_1} + \frac{0}{n_2} + \frac{0}{n_3} + \frac{1}{n_4}\right]}} = \frac{\bar{X}_1 - \bar{X}_4}{\sqrt{MS_{error}\left[\frac{1}{n_1} + \frac{1}{n_4}\right]}} = \frac{3.0 - 6.25}{\sqrt{2.179\left[\frac{1}{8} + \frac{1}{8}\right]}} = \frac{-3.25}{.738} = -4.40$$

The q-test

$$q = \frac{\bar{X}_1 - \bar{X}_4}{\sqrt{\frac{MS_{error}}{n}}} = \frac{3.00 - 6.25}{\sqrt{\frac{2.179}{8}}} = \frac{-3.25}{.522} = -6.23$$

The F-test

$$F = \frac{(\bar{X}_1 + 0 + 0 - \bar{X}_4)^2}{MS_{error}\left[\frac{1}{n_1} + 0 + 0 + \frac{1}{n_4}\right]} = \frac{(\bar{X}_1 - \bar{X}_4)^2}{MS_{error}\left[\frac{1}{n_1} + \frac{1}{n_4}\right]} = \frac{(3.0 - 6.25)^2}{2.179\left[\frac{1}{8} + \frac{1}{8}\right]} = \frac{10.56}{.545} = 19.37$$

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Basic Test Statistics (Kirk,1995)

Tests Using the t-statistic:

Student's Multiple t (LSD) (Table E.3)
Dunnett's Multiple Comparison Test (Table E.7)
Hochberg & Tamhane Tables for unequal n's
Dunn (Bonferroni) (Table E.14)
Dunn Šidák (Table E.15)
Holm (Table E.14 or E.15)

Tests using the q-statistic (pairwise contrasts only):

Tukey HSD (Table E.6)
Tukey-Kramer (Table E.6)
Fisher-Hayter (Table E.6)
REGW Q (Table E.6)
Peritz Q (Table E.6)

Not recommended by Kirk

Newman Keuls (Table E.6)
Duncan's New Multiple Range Test

Tests Using the F-statistic:

REGW F (Table E.4)
Scheffé S (Table E.4)
Peritz F (Table E.4)

Other Tests:

Variants of t
- Dunnett's T3 (Table E.14 (?))

Variants of q
- Dunnett's C (Table E.6)
- Games Howell (Table E.6)

Variants of F
- Brown-Forsythe (Table E.4)

An Example Using SPSS

Descriptive Statistics

Dependent Variable: x

b	Mean	Std. Deviation	N
1.00	3.0000	1.51186	8
2.00	3.5000	.92582	8
3.00	4.2500	1.03510	8
4.00	6.2500	2.12132	8
Total	4.2500	1.88372	32

Univariate Tests

Dependent Variable: x

	Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Contrast	49.000	3	16.333	7.497	.001	.445	22.492	.972
Error	61.000	28	2.179					

The F tests the effect of b. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

a. Computed using alpha = .05

Multiple Comparisons

Dependent Variable: x
Bonferroni

(I) b	(J) b	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1.00	2.00	-.5000	.73800	1.000	-2.5951	1.5951
	3.00	-1.2500	.73800	.608	-3.3451	.8451
	4.00	-3.2500*	.73800	.001	-5.3451	-1.1549
2.00	1.00	.5000	.73800	1.000	-1.5951	2.5951
	3.00	-.7500	.73800	1.000	-2.8451	1.3451
	4.00	-2.7500*	.73800	.005	-4.8451	-.6549
3.00	1.00	1.2500	.73800	.608	-.8451	3.3451
	2.00	.7500	.73800	1.000	-1.3451	2.8451
	4.00	-2.0000	.73800	.068	-4.0951	.0951
4.00	1.00	3.2500*	.73800	.001	1.1549	5.3451
	2.00	2.7500*	.73800	.005	.6549	4.8451
	3.00	2.0000	.73800	.068	-.0951	4.0951

Based on observed means.

*. The mean difference is significant at the .05 level.

Note

$$\bar{X}_1 - \bar{X}_2 = 3.00 - 3.50 = -.50 \qquad \sqrt{\frac{2MS_{error}}{n}} = \sqrt{\frac{2(2.179)}{8}} = .738$$

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If it is desired to compute the t-statistics, this can be done by dividing the contrast by the standard error. The degrees of freedom for the t-tests are the degrees of freedom for the Mean Square Error, which is 28 for the example. Thus, to compare means 1 and 4:

$$t(28) = \frac{-3.25}{.738} = -4.404 \qquad p < .001$$

This could be tested for significance using the LSD approach, the Bonferroni approach, or any other using the appropriate tables. The alpha levels shown in slide 12 are based on the Bonferroni approach, assuming 6 contrasts.

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