Research Design - - Topic 5
Completely Randomized Factorial Designs

- Two Factor Designs (Kirk, Ch. 9)
- Experimental Design Approach (Kirk, pp. 364-413)
- Variance Accounted for and Power (Kirk, pp. 397-402)
- Tests of Means (Kirk, pp. 376-397)
- SPSS GLM Univariate Run
- Using G*Power for Specific Effects

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Two Factor Designs

- General Description. Two factor analysis of variance permits you to study the simultaneous effects of two factors. Consider the data for a 3X3 design, in which there are an equal number of observations in each cell.

	B1	B2	B3
A1	24	44	38
	33	36	29
	37	25	28
	29	27	47
	42	43	48
A2	30	35	26
	21	40	27
	39	27	36
	26	31	46
	34	22	45
A3	21	41	42
	18	39	52
	10	50	53
	31	36	49
	20	34	64

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- Table of means

	B1	B2	B3	A-means
A1	33	35	38	35.33
A2	30	31	36	32.33
A3	20	40	52	37.33
B-means	27.67	35.33	42	35.00

- Questions to ask of the Data

Main Effects of A
Do the A-means vary more than you would expect on the basis of chance?

Main Effects of B
Do the B-means vary more than you would expect on the basis of chance?

Interaction Effects of A and B
Do the AB means vary from what you would expect given the values of the A-means and the B-means?

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Experimental Design Approach

- **The model** $X_{abi} = \mu + \alpha_a + \beta_b + \alpha\beta_{ab} + \varepsilon_{abi}$

where:

$\mu =$ *population mean*

$\alpha_a = \mu_a - \mu$

$\beta_b = \mu_b - \mu$

$\alpha\beta_{ab} = \mu_{ab} - \mu_a - \mu_b + \mu$

$\varepsilon_{abi} = X_{abi} - \mu_{ab}$

This model results in different expected mean squares depending on whether each of the factors is fixed or random.

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Definitional Formulae for the Summary Table

Source	Sum of Squares	df
A	$nb\sum(\overline{X}_a - \overline{G})^2$	a-1
B	$na\sum(\overline{X}_b - \overline{G})^2$	b-1
AB	$n\sum\sum(\overline{X}_{ab} - \overline{X}_a - \overline{X}_b + \overline{G})^2$	(a-1)(b-1)
S/AB	$\sum\sum\sum(X_{abi} - \overline{X}_{ab})^2$	ab(n-1)
Total	$\sum\sum\sum(X_{abi} - \overline{G})^2$	abn - 1

Note. The experimental design approach requires equal sample sizes (n) in the ab cells. If n's are not equal, other computational procedures are followed. One is the unweighted means approach, where the harmonic mean of the n's is substituted for n, and all main effect means are the means of the corresponding cell means. Another approach uses the general linear model (GLM).

Expected Mean Squares

Expected mean squares are the parameters in the model estimated by the values of the mean squares computed in the experiment. These values will differ depending on whether the model is fixed (i.e., all factors except subjects is fixed) or random (i.e., at least one of the factors in addition to subjects is random). A general depiction is given on the next slide.

A **Fixed factor** is one in which all levels of the population (P) are represented in the experiment (p), or generalizations are to be made only to those levels in the population that are included in the experiment. Thus, (1-p/P) = 0

A **Random factor** is one in which only a random sample of the levels in the population (P) are included in the experiment (p). Thus, (1-p/P) = 1

Expected Mean Squares

Source	Cornfield Tukey
A	$nb\theta_A^2 + n(1-b/B)\theta_{AB}^2 + \sigma_\varepsilon^2$
B	$na\theta_B^2 + n(1-a/A)\theta_{AB}^2 + \sigma_\varepsilon^2$
AB	$n\theta_{AB}^2 + \sigma_\varepsilon^2$
S/AB	σ_ε^2

Where $\theta^2 = \Sigma\alpha^2/(a-1)$ or $\Sigma\beta^2/(b-1)$ or $\Sigma\alpha\beta^2/(a-1)(b-1)$ if the factor(s) are fixed.
Or $\theta^2 = \sigma_A^2$ or σ_B^2 or σ^2_{AB} if the factor(s) are random.
σ_ε^2 is expressed as a variance because error is random

Expected Mean Squares for the three models

Source	A fixed; B fixed	A fixed, B random	A random, B random
A	$\frac{nb\sum\alpha_a^2}{a-1} + \sigma_\varepsilon^2$	$\frac{nb\sum\alpha_a^2}{a-1} + n\sigma_{AB}^2 + \sigma_\varepsilon^2$	$nb\sigma_A^2 + n\sigma_{AB}^2 + \sigma_\varepsilon^2$
B	$\frac{na\sum\beta_b^2}{b-1} + \sigma_\varepsilon^2$	$na\sigma_B^2 + \sigma_\varepsilon^2$	$na\sigma_B^2 + n\sigma_{AB}^2 + \sigma_\varepsilon^2$
AB	$\frac{n\sum\sum\alpha\beta_{ab}^2}{(a-1)(b-1)} + \sigma_\varepsilon^2$	$n\sigma_{AB}^2 + \sigma_\varepsilon^2$	$n\sigma_{AB}^2 + \sigma_\varepsilon^2$
S/AB	σ_ε^2	σ_ε^2	σ_ε^2

Note this information can be used to identify the appropriate sources of variance to be used as error terms in the F-ratios or to estimate the proportion of the estimated population variance accounted for by the effect.

Assumptions

Independent Random Sampling. Ss are randomly and independently obtained from the AB populations.

Normality. The observations in the AB populations are normally distributed.

Homogeneity Of Variance. The variances in the AB populations are equal.

Null Hypotheses: There are three for the two factor design.

A $\mu_{a1} = \mu_{a2} = \mu_{a3} = \ldots$

B $\mu_{b1} = \mu_{b2} = \mu_{b3} = \ldots$

AB $\mu_{ab} - \mu_a - \mu_b + \mu = 0$ } for all AB

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Variance Accounted for and Power

- The formulae for the proportion of variance accounted for (i.e., ω^2, ρ_I, and η^2), and effect size (f) have the same form as those presented in Lecture 4, except now there are three possible sources of variance, A, B, and AB.
  - Note that the SPSS GLM output provides η^2 values and power in the analysis of variance summary table if requested.

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Tests of Means

- **Main Effect Means.** These tests evaluate differences between marginal (i.e., unweighted or weighted) means. The formulae are the same as those presented in Lecture 5 except that the value of the sample size would be the number of observations making up a main effect mean. Thus, the value for equal sample sizes would be *bn* for any A mean and *an* for any B mean.
- **Cell Means.** These tests evaluate differences between cell means. There are two types of such tests:
  - Tests of Simple Main Effects
  - Tests of Interaction Effects

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Tests of Simple Main Effects

- Omnibus Tests. These are comparable to single factor analyses of variance conducted for all means of one factor at each level of the other factor, using the Mean Square for error from the original analysis.

  Three Definitions of Type I error.
  1. Per F-ratio ($p<.05$)
  2. Per family in the set of interest ($p<.10/a$ or b)
  3. Per possible number of families ($p<.15/(a+b)$)

- Contrasts. These make use of the tests of significance discussed in Lecture 5 except that ***n*** is the number of observations associated with the cell in question.

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Tests of Interaction Effects

(Kirk, 1995, pp. 383-389)

Treatment/Contrast Interaction The interaction between one factor (eg., A) and a contrast for the other factor (eg., B1 – B3). This would yield an AX2 table.

Contrast/Contrast Interaction The interaction between a contrast on one factor (eg., A1-A3) and one for another factor (eg., B2-B3). This would yield a 2X2 table.

Tests of Significance. An F-ratio of the Mean Square for the interaction divided by the Mean Square for the error term for the Interaction from the larger analysis of variance. Kirk recommends using the Scheffé STP adjustment to evaluate significance.

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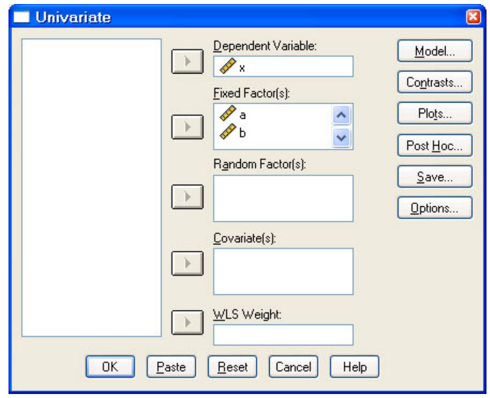
SPSS Window for GLM Univariate



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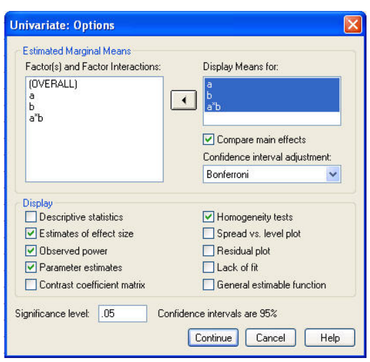
SPSS GLM Options Window



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SPSS GLM Univariate Output

Tests of Between-Subjects Effects

Dependent Variable: x

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Corrected Model	2970.000[b]	8	371.250	5.940	.000	.569	47.520	.999
Intercept	55125.000	1	55125.000	882.000	.000	.961	882.000	1.000
a	190.000	2	95.000	1.520	.232	.078	3.040	.302
b	1543.333	2	771.667	12.347	.000	.407	24.693	.993
a * b	1236.667	4	309.167	4.947	.003	.355	19.787	.934
Error	2250.000	36	62.500					
Total	60345.000	45						
Corrected Total	5220.000	44						

a. Computed using alpha = .05

b. R Squared = .569 (Adjusted R Squared = .473)

Levene's Test of Equality of Error Variances[a]

Dependent Variable: x

F	df1	df2	Sig.
.437	8	36	.891

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept+a+b+a * b

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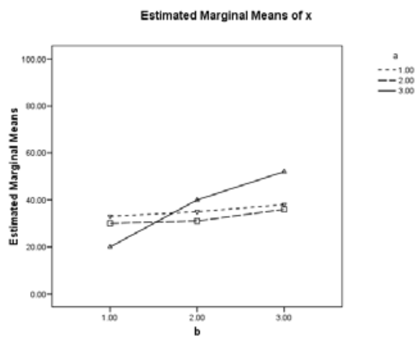
SPSS Plot of the Interaction



SPSS Tests of Main Effect Means

- SPSS uses two types of pairwise contrasts for main effects:

  1. Aposteriori Contrasts. There are 18 post hoc tests of means that can be used in SPSS GLM Univariate. They are accessed through the post hoc window. These contrasts involve the means presented in the Descriptive statistics (i.e., the Weighted means), and define the standard error of the difference between the means as follows:

$$SE = \sqrt{\frac{MS_{error}}{n_1} + \frac{MS_{error}}{n_2}}$$

where n_1 and n_2 equal the number of observations for each main effect mean

2. Tests of estimated means. There are three tests, LSD, Bonferroni, and Dunn Sidak that are accessed in the Options window. They make use of the estimated means (i.e., the unweighted means) and define the standard error of the difference as follows:

$$SE = \sqrt{\frac{MS_{error}}{an_1^{'}} + \frac{MS_{error}}{an_2^{'}}}$$

Where a = the number of cell means making up each mean and n' = the harmonic mean of the n's for those cells.

Tests of Simple Main Effect contrasts

- SPSS does not perform tests of simple main effects using the clope method, but you can obtain these tests by modifying the syntax file as shown below in lower case on the second EMMEANS line. You obtain the EMMEANS lines through Options

```
UNIANOVA
 x  BY a b
 /METHOD = SSTYPE(3)
 /INTERCEPT = INCLUDE
 /EMMEANS = TABLES(b) COMPARE ADJ(BONFERRONI)
 /EMMEANS = TABLES(a*b) compare (b) adj(bonferroni)
 /PRINT = DESCRIPTIVE ETASQ OPOWER PARAMETER HOMOGENEITY
 /CRITERIA = ALPHA(.05)
 /DESIGN = a b a*b .
```

SPSS GLM UnivariateTest of Cell Means

Estimates

Dependent Variable: x

a	b	Mean	Std. Error	95% Confidence Interval	
				Lower Bound	Upper Bound
1.00	1.00	33.000	3.536	25.830	40.170
	2.00	35.000	3.536	27.830	42.170
	3.00	38.000	3.536	30.830	45.170
2.00	1.00	30.000	3.536	22.830	37.170
	2.00	31.000	3.536	23.830	38.170
	3.00	36.000	3.536	28.830	43.170
3.00	1.00	20.000	3.536	12.830	27.170
	2.00	40.000	3.536	32.830	47.170
	3.00	52.000	3.536	44.830	59.170

Pairwise Comparisons

Dependent Variable: x

a	(I) b	(J) b	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a]	
						Lower Bound	Upper Bound
1.00	1.00	2.00	-2.000	5.000	1.000	-14.555	10.555
		3.00	-5.000	5.000	.972	-17.555	7.555
	2.00	1.00	2.000	5.000	1.000	-10.555	14.555
		3.00	-3.000	5.000	1.000	-15.555	9.555
	3.00	1.00	5.000	5.000	.972	-7.555	17.555
		2.00	3.000	5.000	1.000	-9.555	15.555
2.00	1.00	2.00	-1.000	5.000	1.000	-13.555	11.555
		3.00	-6.000	5.000	.714	-18.555	6.555
	2.00	1.00	1.000	5.000	1.000	-11.555	13.555
		3.00	-5.000	5.000	.972	-17.555	7.555
	3.00	1.00	6.000	5.000	.714	-6.555	18.555
		2.00	5.000	5.000	.972	-7.555	17.555
3.00	1.00	2.00	-20.000*	5.000	.001	-32.555	-7.445
		3.00	-32.000*	5.000	.000	-44.555	-19.445
	2.00	1.00	20.000*	5.000	.001	7.445	32.555
		3.00	-12.000	5.000	.065	-24.555	.555
	3.00	1.00	32.000*	5.000	.000	19.445	44.555
		2.00	12.000	5.000	.065	-.555	24.555

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

a. Adjustment for multiple comparisons: Bonferroni.

Tests of Interaction Effects

Consider the following table of means with a contrast for B shown at each level of A

B contrast coefficients →

A		(1)	(-1)	(0)	Ψ
	1	33	35	38	-2
A	2	30	31	36	-1
	3	20	40	52	-20
		B1	B2	B3	-23

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The Treatment/Contrast Interaction (Kirk, p. 386)is:

$$SSA\psi_{(B)} = \frac{n\left[\sum_a \psi^2_{B@A_a} - \frac{\left(\sum \psi_{B@A_a}\right)^2}{a}\right]}{\sum_b c_b^2}$$

$$SSA\psi_{(B)} = \frac{5\left[(-2)^2 + (-1)^2 + (-20)^2 - \frac{(-23)^2}{3}\right]}{2} = 571.67$$

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B contrast coefficients

A contrast coefficients ↓	(1)	(0)	(-1)	Ψ
(1)	33	35	38	-5
(-1)	30	31	36	-6
(0)	20	40	52	-32

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The Contrast/Contrast Interaction (Kirk, p. 387) is:

Given c_a = 1 -1 0 and c_b = 1 0 -1

$$SS\psi_{(A)}\psi_{(B)} = \frac{n\left[\sum c_a \psi_{(B)@A_a}\right]^2}{\sum_a c_a^2 \sum_b c_b^2}$$

$$SS\psi_{(A)}\psi_{(B)} = \frac{5\left[(1)(-5)+(-1)(-6)+(0)(-32)\right]^2}{(2)(2)} = 1.25$$

STP $$F^* = \frac{v_1 F^T(v_1, v_2, \alpha)}{v_3}$$ Applies to both types

where $v_1 = df_{AB}$ $v_2 = df_{error}$ $v_3 = df_{effect}$

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Using G*Power for Special Effects

By clicking on F-ratios and selecting: **ANOVA Fixed Effects, Special, main effects and interactions**, you can compute estimates for each main effect and interaction effect for power, effect size, total sample size, required alpha level, or compromise power.

Necessary parameters as required are effect size, power, alpha level, total sample size, degrees of freedom for the numerator of the F-ratio, and/or total number of groups in the analysis

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