Research Design - - Topic 6
Single Factor Repeated Measure Designs

- General Description, Example, Purpose
- The Univariate Approach
  - The Experimental Design Model (Kirk, pp. 251-289)
- The Multivariate Approach (Kirk, pp. 281-282; O'Brien & Kaiser, 1985, pp. 316-333.)
- Running SPSS GLM – Repeated Measures
- Running G*Power

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General Description

- This analysis is comparable to the single factor analysis of variance except that in this case the treatment conditions are not independent. That is, there is a link between the scores in each treatment condition.
- This occurs if the same individual is tested under each condition (repeated measures), individuals are matched on some other variable and then assigned to a different treatment level or are tested in sets with a different individual in each treatment (randomized blocks).

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An Example Using the Data from Kirk, p. 270

	A_1	A_2	A_3	A_4	$\overline{P}_i$
	3	4	4	3	3.50
	2	4	4	5	3.75
	2	3	3	6	3.50
	3	3	3	5	3.50
	1	2	4	7	3.50
	3	3	6	6	4.50
	4	4	5	10	5.75
	6	5	5	8	6.00
Means	3.00	3.50	4.25	6.25	$\overline{G} = 4.25$
Variances	2.29	0.86	1.07	4.50	$\overline{S^2} = 2.18$

Major Question to ask of the data:

Do the A-means vary more than can be reasonably attributed to chance? 3

In this data set, each line represents data for the different treatments all obtained from the same subject or from the same random block. Thus, the data are not independent, and it is possible to compute correlations between the pairs of treatments. The associated covariance matrix is presented below.

Covariance Matrix

	A_1	A_2	A_3	A_4
A_1	2.29	1.14	0.71	1.29
A_2	1.14	0.86	0.29	0.29
A_3	0.71	0.29	1.07	0.93
A_4	1.29	0.29	0.93	4.50

$\overline{S^2} = 2.18$

$\overline{\text{cov}} = 0.78$

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Following is the summary table for the analysis giving the sources of variance, the definitional formulae and the numerical values for this data set.

Source	Sum Of Squares	df	SS	df	MS	F
Between Ss	$a\sum^{n}\left(\overline{P}_i - \overline{G}\right)^2$	$n-1$	31.50	7	4.50	
Within Ss	$\sum^{a}\sum^{n}\left(X_{ai} - \overline{P}_i\right)^2$	$n(a-1)$	78.50	24		
A	$n\sum^{a}\left(\overline{X}_a - \overline{G}\right)^2$	$a-1$	49.00	3	16.33	11.63
AS	$\sum^{a}\sum^{n}\left(X_{ai} - \overline{P}_i - \overline{X}_a + \overline{G}\right)^2$	$(a-1)(n-1)$	29.50	21	1.40	
Total	$\sum^{a}\sum^{n}\left(X_{ai} - \overline{G}\right)^2$	$an-1$	110.00	31		

Note. $MS_{AS} = \overline{S}^2 - \overline{\text{cov}} = 2.18 - .78 = 1.40$

Mean Squares and F-ratio[1]

$MS_A \longrightarrow \dfrac{n\sum^{a}\left(\overline{X}_a - \overline{G}\right)^2}{a-1}$

$MS_{AS} \longrightarrow \dfrac{\sum^{a}\sum^{n}\left(X_{ai} - \overline{P}_i - \overline{X}_a + \overline{G}\right)^2}{(a-1)(n-1)}$

$$F_A = \frac{MS_A}{MS_{AS}}$$

1 There are only two Mean Squares of interest in this analysis, that assessing variation in the A means, and that assessing the interaction between A and Subjects (or Blocks). This latter term is sometimes referred to as the Residual Mean Square, and it is the appropriate error term for A for all models where Subjects (and Blocks) are random (see the table of expected mean squares).

Two possible models

The non-additive model. $X_{ai} = \mu + \alpha_a + \pi_i + \alpha\pi_{ai} + \varepsilon_{ai}$

Assumes there is an interaction between Treatments and Subjects

where:

$\mu = population\ mean$

$\alpha_a = \mu_a - \mu$

$\pi_i = \mu_i - \mu$

$\alpha\pi_{ai} = \mu_{ai} - \mu_a - \mu_i + \mu$

$\varepsilon_{ai} = X_{ai} - \mu_{ai}$

The additive model. $X_{ai} = \mu + \alpha_a + \pi_i + \varepsilon_{ai}$

Assumes there is no interaction between Treatments and Subjects

where:

$\mu = population\ \ mean$

$\alpha_a = \mu_a - \mu$

$\pi_i = \mu_i - \mu$

$\varepsilon_{ai} = X_{ai} - \mu_a - \mu_i + \mu$

Expected Mean Squares

General Form Based on the Cornfield-Tukey Algorithm

Source	Non-Additive Model	Additive Model
A	$n\theta_A^2 + (1 - n/N)\sigma_{\pi A}^2 + \sigma_\varepsilon^2$	$n\theta_A^2 + \sigma_\varepsilon^2$
A x S	$\sigma_{\pi A}^2 + \sigma_\varepsilon^2$	σ_ε^2
Between Ss	$a\sigma_\pi^2 + (1 - a/A)\sigma_{\pi A}^2 + \sigma_\varepsilon^2$	$a\sigma_\pi^2 + \sigma_\varepsilon^2$

$\Theta_A^2 = \Sigma\alpha_A{}^2/(a-1)$ if A is fixed or σ_A^2 if A is random. All others considered random.

The final form of the Expected Mean Squares can be constructed from this table. The A factor can be fixed or random, but S is typically random, and thus (1-n/N) is generally equal to 1(but see Kirk (pp. 265-271) when S is defined as a block and considered fixed).

Assumptions

Independent Random Sampling. The replications (i.e., Subjects or Blocks) are randomly obtained from the population.

Normality. The treatment populations are normally distributed.

Circularity of the Covariance Matrix. The covariance matrix satisfies the assumption of circularity when the variance of the differences for any two populations is the same as that for any other two populations. **Note, it is sometimes stated that a necessary assumption is that of compound symmetry but this is incorrect; it is a sufficient but not necessary condition (cf., Kirk, 1995, pp 274-282).**

Null Hypothesis. The population means are equal.

$$H_0: \quad \mu_1 = \mu_2 = \mu_3 = \mu_4$$

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Conditions of Matrices

A matrix satisfies the assumption of **compound symmetry** if the variances are equal and the covariances are equal among themselves.

2	1.4	1.4	1.4
1.4	2	1.4	1.4
1.4	1.4	2	1.4
1.4	1.4	1.4	2

A matrix satisfies the assumption of **sphericity** if the variances are equal and the covariances are 0.

2	0	0	0
0	2	0	0
0	0	2	0
0	0	0	2

A matrix satisfies the assumption of **circularity** if the variances of the differences are equal. That is $\sigma^2_{1\text{-}2} = \sigma^2_{1\text{-}3} = \sigma^2_{1\text{-}4} = \ldots = \sigma^2_{3\text{-}4}$.

1	.25	.75	1.25
.25	2	1.25	1.75
.75	1.25	3	2.25
1.25	1.75	2.25	4

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Mauchly's Test of Sphericity. If this test is significant, it indicates that the covariance matrix violates the assumption of Circularity.

Epsilon Multipliers. These are values used to reduce the degrees of freedom for each source of variance depending on the extent to which the assumption of circularity is violated. This is done by multiplying the degrees of freedom by the epsilon multiplier. There are three different values:

- Greenhouse-Geisser (Recommended by Kirk, 1995)
- Huynh-Feldt (The most liberal)
- Lower Bound (The most conservative)

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The Multivariate Approach

Repeated measure designs can also be investigated from a multivariate perspective, where the data are considered to be a set of **a** variables administered to one group of subjects. The multivariate test under consideration is whether the means of these **a** variables are equivalent in the population. This can be tested using Hotelling's T^2 statistic. In SPSS GLM Repeated Measures, 4 statistics are given, Pillais Trace, Wilks' Lambda, Hotelling's Trace, and Roy's Largest Root, but they all yield the same F-ratio (which is Hotelling's T^2).

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The multivariate approach assumes that given a variables all measured on the same scale, one can form (a-1) new variables comprised of difference scores (contrasts) between pairs of variables. Given 4 levels of the factor, one example might be:

$$(\bar{X}_1 - \bar{X}_4),\quad (\bar{X}_2 - \bar{X}_4),\quad (\bar{X}_3 - \bar{X}_4)$$

Note. Any set of contrasts involving all the means would produce the same estimate of the multivariate effect. Also, **note** that it is not necessary to identify these contrasts when running SPSS GLM Repeated Measures.

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Assumptions

Independent Random Sampling. The replications (i.e., Subjects or Blocks) are randomly obtained from the population.

Multivariate Normality. Each variable is normally distributed in the population, any linear combination of the variables is normally distributed, and all pairs of variables are bivariate normal, etc. Generally, the analysis is robust with respect to violation of this assumption.

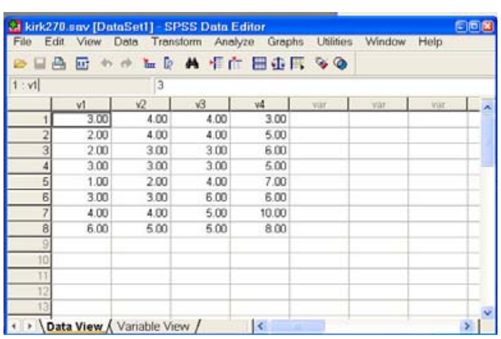
Null Hypothesis. The population means are equal. This is expressed in matrix format as follows:

$$\begin{bmatrix} \mu_1 - \mu_4 \\ \mu_2 - \mu_4 \\ \mu_3 - \mu_4 \end{bmatrix} = 0 \quad \text{or} \quad \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} = \begin{bmatrix} \mu_4 \\ \mu_4 \\ \mu_4 \end{bmatrix}$$

Note. This is identical to the univariate null hypothesis: $\mu_1 = \mu_2 = \mu_3 = \mu_4$. 14

Running SPSS GLM Repeated Measures

The data are presented in the Data Editor as follows:



	v1	v2	v3	v4
1	3.00	4.00	4.00	3.00
2	2.00	4.00	4.00	5.00
3	2.00	3.00	3.00	6.00
4	3.00	3.00	3.00	5.00
5	1.00	2.00	4.00	7.00
6	3.00	3.00	6.00	6.00
7	4.00	4.00	5.00	10.00
8	6.00	5.00	5.00	8.00

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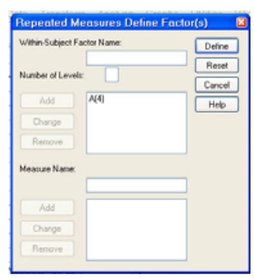
From the tool bar, click Analyze → GLM → Repeated Measures. This yields the following window. Enter any variable name (eg., **A**) and the number of levels, and click Add. This will yield the following with **A** as the factor name. Then click Define.



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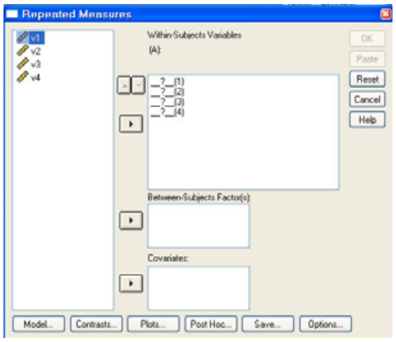
This yields the following window. Move the variables (here labelled v1, v2, …) to define the 4 levels of A. Then choose the options, etc., desired.



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```
GLM A1 A2 A3 A4
  /WSFACTOR=A 4 Polynomial
  /METHOD=SSTYPE(3)
  /PLOT=PROFILE(A)
  /EMMEANS=TABLES(A) COMPARE ADJ(LSD)
  /PRINT=ETASQ OPOWER
  /CRITERIA=ALPHA(.05)
  /WSDESIGN=A.
```

Tests of Within-Subjects Effects

Measure:MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
A	Sphericity Assumed	49.000	3	16.333	11.627	.000	.624	34.881	.998
	Greenhouse-Geisser	49.000	1.859	26.365	11.627	.001	.624	21.609	.970
	Huynh-Feldt	49.000	2.503	19.578	11.627	.000	.624	29.100	.993
	Lower-bound	49.000	1.000	49.000	11.627	.011	.624	11.627	.831
Error(A)	Sphericity Assumed	29.500	21	1.405					
	Greenhouse-Geisser	29.500	13.010	2.268					
	Huynh-Feldt	29.500	17.520	1.684					
	Lower-bound	29.500	7.000	4.214					

a. Computed using alpha = .05

Kirk recommends using the Greenhouse-Geisser Option which multiplies the degrees of freedom by the Greenhouse-Geisser Epsilon value. 18

Tests of Within-Subjects Contrasts

Measure:MEASURE_1

Source	A	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
A	Linear	44.100	1	44.100	19.294	.003	.734	19.294	.962
	Quadratic	4.500	1	4.500	3.150	.119	.310	3.150	.336
	Cubic	.400	1	.400	.800	.401	.103	.800	.122
Error(A)	Linear	16.000	7	2.286					
	Quadratic	10.000	7	1.429					
	Cubic	3.500	7	.500					

a. Computed using alpha = .05

Tests of Between-Subjects Effects

Measure:MEASURE_1
Transformed Variable:Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Intercept	578.000	1	578.000	128.444	.000	.948	128.444	1.000
Error	31.500	7	4.500					

a. Computed using alpha = .05

Mauchly's Test of Sphericity[b]

Measure:MEASURE_1

Within Subject...	Mauchly's W	Approx. Chi-Square	df	Sig.	Epsilon[a] Greenhouse-Geisser	Huynh-Feldt	Lower-bound
A	.339	6.187	5	.295	.620	.834	.333

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.

a. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are displayed in the Tests of Within-Subjects Effects table.

b. Design: Intercept
Within Subjects Design: A

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Multivariate Tests[c]

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[b]
A	Pillai's Trace	.754	5.114[a]	3.000	5.000	.055	.754	15.341	.595
	Wilks' Lambda	.246	5.114[a]	3.000	5.000	.055	.754	15.341	.595
	Hotelling's Trace	3.068	5.114[a]	3.000	5.000	.055	.754	15.341	.595
	Roy's Largest Root	3.068	5.114[a]	3.000	5.000	.055	.754	15.341	.595

a. Exact statistic

b. Computed using alpha = .05

c. Design: Intercept
Within Subjects Design: A

Note. The above table tests the multivariate hypothesis that the means of the four variables are equal in the population.

The following output of the Tests of Means was obtained using the LSD test under Options.

Estimates

Measure:MEASURE_1

A	Mean	Std. Error	95% Confidence Interval Lower Bound	Upper Bound
1	3.000	.535	1.736	4.264
2	3.500	.327	2.726	4.274
3	4.250	.366	3.385	5.115
4	6.250	.750	4.477	8.023

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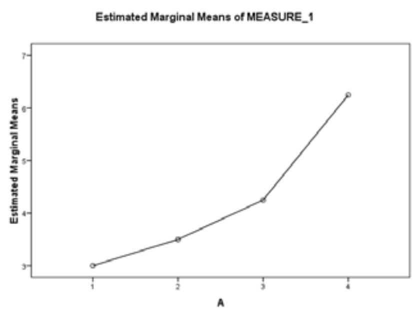
Plot of Means



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Pairwise Comparisons

Measure:MEASURE_1

(I) A	(J) A	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a] Lower Bound	Upper Bound
1	2	-.500	.327	.170	-1.274	.274
	3	-1.250*	.491	.038	-2.411	-.089
	4	-3.250*	.726	.003	-4.966	-1.534
2	1	.500	.327	.170	-.274	1.274
	3	-.750	.412	.111	-1.724	.224
	4	-2.750*	.773	.009	-4.579	-.921
3	1	1.250*	.491	.038	.089	2.411
	2	.750	.412	.111	-.224	1.724
	4	-2.000*	.681	.022	-3.611	-.389
4	1	3.250*	.726	.003	1.534	4.966
	2	2.750*	.773	.009	.921	4.579
	3	2.000*	.681	.022	.389	3.611

Based on estimated marginal means

a. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

*. The mean difference is significant at the .05 level.

These contrasts show the following pattern of results (means with a common line do not differ significantly from each other).

A1	A2	A3	A4
3.0	3.5	4.25	6.25

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A Note on Tests of Means

- Contrasts of means can be performed using the t, q, or F tests discussed previously. In this case, the Mean Square AS (i.e., Mean Square Residual) is used as the error term. That is, the t-test is:

$$t = \frac{\overline{X}_1 - \overline{X}_4}{\sqrt{\frac{2MS_{AS}}{n}}} = \frac{3.00 - 6.25}{\sqrt{\frac{2(1.405)}{8}}} = -5.48 \qquad df = 21$$

- The tests in SPSS GLM Repeated Measures use a different logic. Thus, the t-test uses the data only for the pair of cells concerned. That is:

$$t = \frac{\overline{d}}{S_{\overline{d}}} = \frac{-3.25}{.726} = -4.477 \qquad df = 7$$

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G*Power for single factor repeated measures

Select F-ratios and ANOVA: Repeated measures, within factors

Options for estimating sample size
(estimates based on example in slide 18 given mean r = .42)

Effect size	.99
α error prob	.05
Power	.80
Number of groups	1
Repetitions	4
Corr among rep measures	.42
Nonsphericity correction	1

Answer: Total sample size 4 (i.e., 16 observations)
power estimate = .95

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References

O'Brien, R.G. & Kaiser, M.K. (1985). MANOVA method for analyzing repeated measures designs: An extensive primer. *Psychological Bulletin, 97,* 316-333.

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