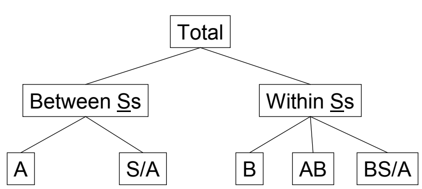
Research Design - - Topic 7
Split-Plot Factorial Designs

- General Description, Purpose, Example
- Univariate Approach
  - Experimental Design Model
- Multivariate Approach
- Running SPSS GLM REPEATED MEASURES
- Tests of Means
  - As presented in texts
  - As performed by SPSS Repeated

- The Split-plot Factorial Design consists of at least two factors, where one factor is based on independent observations and the other is based on correlated observations. It is sometimes referred to as a mixed design, or a mixed Between/Within design.
- There are two general sources of variation. One is the Between Subjects variation while the other is the Within Subjects (or Within Blocks) variation.
- The following diagram shows the breakdown of the Total sum of squares into the Between and Within Subject components

Schematic Breakdown of SS



- The following data set was adapted from Kirk (1995). It consists of one between Subjects factor (A) and one Within Subjects factor (B). The variable $\overline{P}_{i/A}$ is the mean for each Subject (or Block).

Split-Plot Example

A		B1	B2	B3	B4	$\overline{P}_{i/a}$
	1	3	4	7	7	5.25
	1	6	5	8	8	6.75
	1	3	4	7	9	5.75
	1	3	3	6	8	5.00
	2	1	2	5	10	4.50
	2	2	3	6	10	5.25
	2	2	4	5	9	5.00
	2	2	3	6	11	5.50
						$\overline{G} = 5.375$

Means

A	B1	B2	B3	B4	
1	3.75	4.00	7.00	8.00	5.6875
2	1.75	3.00	5.50	10.00	5.0625
	2.75	3.50	6.25	9.00	$\overline{G} = 5.375$

Questions to ask of the data.

- **Main Effect of A.** Do the means for the Between Subjects factor (A) vary more than can be reasonably attributed to chance?

- **Main Effect of B.** Do the means for the Within Subjects factor (B) vary more than can be reasonably attributed to chance?

- **Interaction Effects of A and B.** Do the AB-means vary from what you would expect given the values of the A-Means and the B-means?

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Experimental Design Model

The score for each individual is considered to be composed of parameters as follows:

$$X_{abi} = \mu + \alpha_a + \pi_{i/a} + \beta_b + \alpha\beta_{ab} + \beta\pi_{i/a} + \varepsilon_{ib/a}$$

Note , this is a non-additive model that assumes there is an interaction between B and Subjects nested in A (i.e., $\beta\pi_{i/a}$). (We could also write an additive model by eliminating $\beta\pi_{i/a}$.)

This model can be used to generate the Cornfield Tukey algorithm (see slide 10).

The following slide shows the definitional formulae for the Summary Table.

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Definitional Formulae

Source	Sum Of Squares	df
Between <u>S</u>s	$b\sum^{a}\sum^{n}\left(\overline{P}_{i/a} - \overline{G}\right)^2$	$an - 1$
A	$nb\sum^{a}\left(\overline{X}_a - \overline{G}\right)^2$	$a - 1$
S/A	$b\sum^{a}\sum^{n}\left(\overline{P}_{i/a} - \overline{X}_a\right)^2$	$a(n-1)$
Within <u>S</u>s	$\sum^{a}\sum^{b}\sum^{n}\left(X_{abi} - \overline{P}_{i/a}\right)^2$	$an(b-1)$
B	$an\sum^{b}\left(\overline{X}_b - \overline{G}\right)^2$	$b - 1$
AB	$n\sum^{a}\sum^{b}\left(\overline{X}_{ab} - \overline{X}_a - \overline{X}_b + \overline{G}\right)^2$	$(a-1)(b-1)$
BS/A	$\sum^{a}\sum^{b}\sum^{n}\left(X_{abi} - \overline{P}_{i/a} - \overline{X}_{ab} + \overline{X}_a\right)^2$	$a(b-1)(n-1)$
Total	$\sum^{a}\sum^{b}\sum^{n}\left(X_{abi} - \overline{G}\right)^2$	$abn - 1$

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Mean Squares

$MS_A \longrightarrow \dfrac{nb\sum^{a}\left(\overline{X}_a - \overline{G}\right)^2}{a-1}$

$MS_{S/A} \longrightarrow \dfrac{b\sum^{a}\sum^{n}\left(\overline{P}_{i/a} - \overline{X}_a\right)^2}{a(n-1)}$

$MS_B \longrightarrow \dfrac{an\sum^{b}\left(\overline{X}_b - \overline{G}\right)^2}{b-1}$

$MS_{AB} \longrightarrow \dfrac{n\sum^{a}\sum^{b}\left(\overline{X}_{ab} - \overline{X}_a - \overline{X}_b + \overline{G}\right)^2}{(a-1)(b-1)}$

$MS_{BS/A} \longrightarrow \dfrac{\sum^{a}\sum^{b}\sum^{n}\left(X_{abi} - \overline{P}_{i/a} - \overline{X}_{ab} + \overline{X}_a\right)^2}{a(b-1)(n-1)}$

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Applying the definitional formulae to the sample data produces the following Analysis of Variance Summary Table.

Source	SS	df	MS	F
Between Ss	12.500	7		
A	3.125	1	3.125	2.000
S/A	9.375	6	1.562	
Within Ss	223.000	24	16.333	
B	194.500	3	64.833	127.875
AB	19.375	3	6.458	12.738
BS/A	9.125	18	0.507	
Total	235.500	31		

Note. This analysis assumes a fixed effects model (i.e., A and B are fixed factors).

Cornfield Tukey Algorithm

Source	E (MS)
Between Ss	
A	$nb\theta_A^2 + b\sigma_{\pi/A}^2 + n(1-b/B)\theta_{AB}^2 + (1-b/B)\sigma_{B\pi/A}^2 + \sigma_\varepsilon^2$
S/A	$b\sigma_{\pi/A}^2 + (1-b/B)\sigma_{B\pi/A}^2 + \sigma_\varepsilon^2$
Within Ss	
B	$na\theta_B^2 + n(1-a/A)\,\theta_{AB}^2 + \sigma_{B\pi/A}^2 + \sigma_\varepsilon^2$
AB	$n\theta_{AB}^2 + \sigma_{B\pi/A}^2 + \sigma_\varepsilon^2$
BS/A	$\sigma_{B\pi/A}^2 + \sigma_\varepsilon^2$
Total	

Note. "a" refers to the number of levels of A, "b" to the number of levels of B, and "n" to the number of individuals in each level of A. θ, σ, and the sampling fractions are defined as before.

Formal Expected Mean Squares and F-ratios for the fixed effects model

Source	E(MS)	
Between Ss		
A	$\frac{nb\sum\alpha_a^2}{(a-1)} + b\sigma_{\pi/A}^2 + \sigma_\varepsilon^2$	$F_A = \frac{MS_A}{MS_{S/A}}$
S/A	$b\sigma_{\pi/A}^2 + \sigma_\varepsilon^2$	
Within Ss		
B	$\frac{na\sum\beta_b^2}{(b-1)} + \sigma_{B\pi/A}^2 + \sigma_\varepsilon^2$	$F_B = \frac{MS_B}{MS_{BS/A}}$
AB	$\frac{n\sum\sum\alpha\beta_{ab}^2}{(a-1)(b-1)} + \sigma_{B\pi/A}^2 + \sigma_\varepsilon^2$	$F_{AB} = \frac{MS_{AB}}{MS_{BS/A}}$
BS/A	$\sigma_{B\pi/A}^2 + \sigma_\varepsilon^2$	

Formal Expected Mean Squares and F-ratios when A is fixed and B is random

Source	E (MS)	
Between Ss		
A	$\frac{nb\sum\alpha_a^2}{(a-1)} + b\sigma_{\pi/A}^2 + n\sigma_{AB}^2 + \sigma_{B\pi/A}^2 + \sigma_\varepsilon^2$	No obvious F-ratio for A (but see next slide).
S/A	$b\sigma_{\pi/A}^2 + \sigma_{B\pi/A}^2 + \sigma_\varepsilon^2$	
Within Ss		
B	$na\sigma_B^2 + \sigma_{B\pi/A}^2 + \sigma_\varepsilon^2$	$F_B = \frac{MS_B}{MS_{BS/A}}$
AB	$n\sigma_{AB}^2 + \sigma_{B\pi/A}^2 + \sigma_\varepsilon^2$	$F_{AB} = \frac{MS_{AB}}{MS_{BS/A}}$
BS/A	$\sigma_{B\pi/A}^2 + \sigma_\varepsilon^2$	

Computing quasi F-ratios

Determine an appropriate error term for A by adding and subtracting various Mean Squares.

$MS_{S/A}$	$b\sigma^2_{\pi/A} + \sigma^2_{B\pi/A} + \sigma^2_\varepsilon$
$+\ MS_{AB}$	$+\quad n\sigma^2_{AB} + \sigma^2_{B\pi/A} + \sigma^2_\varepsilon$
$-\ MS_{BS/A}$	$-\quad \sigma^2_{B\pi/A} + \sigma^2_\varepsilon$
	$b\sigma^2_{\pi/A} + n\sigma^2_{AB} + \sigma^2_{B\pi/A} + \sigma^2_\varepsilon$

Therefore:

$$F_A' = \frac{MS_A}{MS_{S/A} + MS_{AB} - MS_{BS/A}} = \frac{\frac{nb\sum\alpha_a^2}{a-1} + b\sigma^2_{\pi/a} + n\sigma^2_{AB} + \sigma^2_{\beta\pi/A} + \sigma^2_\varepsilon}{b\sigma^2_{\pi/a} + n\sigma^2_{AB} + \sigma^2_{\beta\pi/A} + \sigma^2_\varepsilon}$$

$$= \frac{3.125}{1.562 + 6.458 - .507} = .416$$

Numerator df = (a-1) = 1

Denominator df = (Satterthwaite, 1946)

$$\frac{(MS_{S/A} + MS_{AB} - MS_{BS/A})^2}{\frac{MS^2_{S/A}}{df_{S/A}} + \frac{MS^2_{AB}}{df_{AB}} + \frac{MS^2_{BS/A}}{df_{BS/A}}} = \frac{7.513^2}{\frac{1.562^2}{6} + \frac{6.458^2}{3} + \frac{.507^2}{18}} = 3.94$$

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Assumptions

Independent Random Sampling. Ss are randomly and independently obtained from the Between Subjects factor.

Normality. The observations in the AB populations are normally distributed.

Homogeneity. There are 3 aspects:

1. Homogeneity of variance of means for Subjects or Blocks across A (for test of Main Effects for A).
2. Equivalence of covariance matrices for the A factor.
3. Circularity of the pooled covariance matrix.

Null Hypotheses:

A $\quad \mu_{a1} = \mu_{a2}$

B $\quad \mu_{b1} = \mu_{b2} = \mu_{b3} = \mu_{b4}$

AB $\quad \mu_{ab} - \mu_a - \mu_b + \mu = 0 \quad \}$ for all AB

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Tests of Homogeneity

Box's M test of equivalence of the covariance matrices. If this test is significant, it indicates that the covariance matrices are not equivalent.

Mauchly's test of Sphericity. If this test is significant, it indicates that the pooled covariance matrix does not satisfy the assumption of circularity.

Generally, these tests are not robust with respect to violations of normality and it is recommended that regardless of the results of these tests, the degrees of freedom for the within subjects effects be reduced using an epsilon multiplier. Kirk recommends using the Greenhouse-Geisser estimate. SPSS GLM presents the degrees of freedom for the case where the assumptions are satisfied as well as when the epsilon value is applied.

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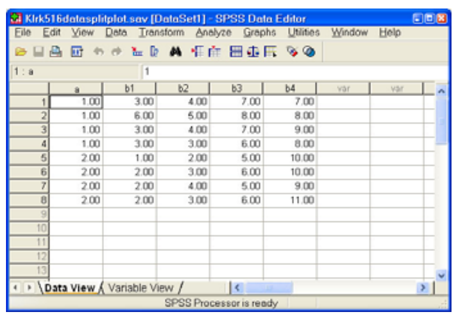
Running SPSS GLM Repeated Measures

Data Editor



	a	b1	b2	b3	b4	var	var
1	1.00	3.00	4.00	7.00	7.00		
2	1.00	6.00	5.00	8.00	8.00		
3	1.00	3.00	4.00	7.00	9.00		
4	1.00	3.00	3.00	6.00	8.00		
5	2.00	1.00	2.00	5.00	10.00		
6	2.00	2.00	3.00	6.00	10.00		
7	2.00	2.00	4.00	5.00	9.00		
8	2.00	2.00	3.00	6.00	11.00		
9							
10							
11							
12							
13							

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Cloping the appropriate choices produces the Syntax file.

```
GET
  FILE='C:\PSYCH540\kirk516data.sav'.
DATASET NAME DataSet2 WINDOW=FRONT.
GLM
  b1 b2 b3 b4 BY a
  /WSFACTOR = b 4 Polynomial
  /METHOD = SSTYPE(3)
  /PRINT = ETASQ OPOWER HOMOGENEITY
 /CRITERIA = ALPHA(.05)
  /WSDESIGN = b
  /DESIGN = a .
```

The following 4 slides present the major output.

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Homogeneity Tests

• 1. **Box's M test of Equivalence of Covariance matrices**. This test cannot be produced for this example because there are fewer than two non-singular covariance matrices.

• 2. **Levene's test.** This tests whether the variances for each level of B are heterogeneous over the A groups. In this example, only b1 is significant (p<.25).

Levene's Test of Equality of Error Variances[a]

	F	df1	df2	Sig.
b1	3.600	1	6	.107
b2	.000	1	6	1.000
b3	.000	1	6	1.000
b4	.000	1	6	1.000

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a.
Design: Intercept+a
Within Subjects Design: b

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• 3. **Mauchly's Test of Sphericity of the Pooled Covariance Matrix**

Mauchly's Test of Sphericity[b]

Measure: MEASURE_1

					Epsilon[a]		
Within Subjects Effect	Mauchly's W	Approx. Chi-Square	df	Sig.	Greenhouse-Geisser	Huynh-Feldt	Lower-bound
b	.315	5.449	5	.372	.584	.943	.333

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.

a. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are displayed in the Tests of Within-Subjects Effects table.

b.
Design: Intercept+a
Within Subjects Design: b

Mauchly's test is not significant, indicating that the assumption of circularity is satisfied. Nonetheless, it is customary to adjust the degrees of freedom for the repeated measures F-ratios by multiplying them by an epsilon multiplier.

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Univariate Tests of the Within Subjects Effects

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
b	Sphericity Assumed	194.500	3	64.833	127.890	.000	.955	383.671	1.000
	Greenhouse-Geisser	194.500	1.752	110.992	127.890	.000	.955	224.113	1.000
	Huynh-Feldt	194.500	2.830	68.738	127.890	.000	.955	361.879	1.000
	Lower-bound	194.500	1.000	194.500	127.890	.000	.955	127.890	1.000
b * a	Sphericity Assumed	19.375	3	6.458	12.740	.000	.680	38.219	.998
	Greenhouse-Geisser	19.375	1.752	11.056	12.740	.002	.680	22.325	.969
	Huynh-Feldt	19.375	2.830	6.847	12.740	.000	.680	36.048	.998
	Lower-bound	19.375	1.000	19.375	12.740	.012	.680	12.740	.843
Error(b)	Sphericity Assumed	9.125	18	.507					
	Greenhouse-Geisser	9.125	10.514	.868					
	Huynh-Feldt	9.125	16.978	.537					
	Lower-bound	9.125	6.000	1.521					

a. Computed using alpha = .05

As indicated earlier, it is typical to interpret the results using the Greenhouse-Geisser adjustment. Thus, the results for B would be written as $F(2,11)=127.89$, $p<.0004$), rounding the degrees of freedom to the next highest integer; those for AB would be written as $F(2,11)=12.74$, $p<.002$.

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Univariate Tests of the Between Subject Effects

Tests of Between-Subjects Effects

Measure: MEASURE_1
Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Intercept	924.500	1	924.500	591.680	.000	.990	591.680	1.000
a	3.125	1	3.125	2.000	.207	.250	2.000	.223
Error	9.375	6	1.563					

a. Computed using alpha = .05

The test of the Intercept is a test that the grand mean deviates significantly from 0, *F*(1,6) = 591.68, *p*<.0004. The test of A indicates that the effects due to A are not significant, *F*(1,6) = 2.00, ns.

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The Multivariate Approach

The Within Subjects components of the Split-plot Factorial design can also be investigated from a multivariate perspective, where the data are considered to be a set of **b** variables administered to **a** groups of subjects. There are consequently two classes of effects.

- **The main effects of B.** This test is comparable to the test of effects for the single factor repeated measures design but in this case the **a** groups are collapsed so that there is only one group with **an** observations in each mean.
- **The interaction of A and B.** This tests the equivalence of the contrasts between the **b** means at each level of the **a** factor.
- **The tests of significance.** In SPSS GLM Repeated, 4 statistics are given, Pillai's Trace, Wilks' Lambda, Hotelling's Trace, and Roy's Largest Root. When the number of levels of the Between Subjects factor is 2, the F-ratio corresponds to Hotelling's T^2 for both the main and interaction effects. For more than 2 levels, the statistics produce different F-ratios and degrees of freedom for the interaction.

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Assumptions

Assumptions for the repeated measures effects are:

- **Independent random sampling.** Ss are independently and randomly sampled from the Between Subjects factor
- **Multivariate Normality.** This assumption applies to each level of the Between Subjects factor.
- **Equivalence of the Covariance Matrices.** The covariance matrices for the Between Subjects factor are the same in the population.

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Multivariate Null Hypotheses for Within Subjects Effects

Main Effect for B

$$\begin{bmatrix} \mu_{b1} \\ \mu_{b2} \\ \mu_{b3} \end{bmatrix} = \begin{bmatrix} \mu_{b4} \\ \mu_{b4} \\ \mu_{b4} \end{bmatrix}$$

Degrees of freedom: v1 = b-1 v2 = N-(a-1) -(b-1)

Interaction Effect for AB

$$\begin{bmatrix} \mu_{a1b1} - \mu_{a1b4} \\ \mu_{a1b2} - \mu_{a1b4} \\ \mu_{a1b3} - \mu_{a1b4} \end{bmatrix} = \begin{bmatrix} \mu_{a2b1} - \mu_{a2b4} \\ \mu_{a2b2} - \mu_{a2b4} \\ \mu_{a2b3} - \mu_{a2b4} \end{bmatrix}$$

Degrees of freedom: v1 = (a-1)(b-1) v2 = N - a – b + 2

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The multivariate tests are appropriate for the Within Subjects Effects. The Between Subjects Effects are assessed using the univariate approach as presented in Slide 21.

Multivariate Tests[c]

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
b	Pillai's Trace	.973	47.192[b]	3.000	4.000	.001	.973	141.577	1.000
	Wilks' Lambda	.027	47.192[b]	3.000	4.000	.001	.973	141.577	1.000
	Hotelling's Trace	35.394	47.192[b]	3.000	4.000	.001	.973	141.577	1.000
	Roy's Largest Root	35.394	47.192[b]	3.000	4.000	.001	.973	141.577	1.000
b * a	Pillai's Trace	.856	7.906[b]	3.000	4.000	.037	.856	23.718	.700
	Wilks' Lambda	.144	7.906[b]	3.000	4.000	.037	.856	23.718	.700
	Hotelling's Trace	5.930	7.906[b]	3.000	4.000	.037	.856	23.718	.700
	Roy's Largest Root	5.930	7.906[b]	3.000	4.000	.037	.856	23.718	.700

a. Computed using alpha = .05

b. Exact statistic

c.
Design: Intercept+a
Within Subjects Design: b

The main effect for B is significant, $F(3,4) = 47.19$, $p<.001$ indicating that the B-means vary more than can be reasonably attributed to chance.

The AB interaction is significant, $F(3,4) = 7.91$, $p<.037$ indicating that some contrasts in A1 differ from the corresponding ones in A2. This is equivalent to a univariate interaction.

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Tests of Means
As presented in most textbooks (cf., Kirk, 1995)

The formulae are written for unequal n's for the general case. With equal n's (as is the case here) , the denominators can be written more simply as 2 times one of the elements.

Main Effects of A

$$t = \frac{\bar{X}_{a1} - \bar{X}_{a2}}{\sqrt{\frac{MS_{S/A}}{bn_1} + \frac{MS_{S/A}}{bn_2}}} = \frac{5.6875 - 5.0625}{\sqrt{\frac{1.562}{16} + \frac{1.562}{16}}} = \frac{.6250}{.442} = 1.414 \quad at\ df = 6$$

Main Effects of B

$$t = \frac{\bar{X}_{b1} - \bar{X}_{b2}}{\sqrt{\frac{MS_{BS/A}}{an_1} + \frac{MS_{BS/A}}{an_2}}} = \frac{2.75 - 3.50}{\sqrt{\frac{.507}{8} + \frac{.507}{8}}} = \frac{-.75}{.3560} = -2.106 \quad at\ df = 18$$

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Tests of Cell Means

1. Tests of Simple main Effects
2. Tests of Interaction Effects

1. Simple Main Effects

To determine the appropriate error term, ask 2 questions:

1. What is the error term for the interaction?
2. What is the error term for the factor being varied?

If the answer is the same, use that one Mean Square as the error term.
If the answer is not the same calculate a pooled error term by adding the sums of squares for the two error terms and dividing by the sum of their degrees of freedom.

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Simple Main Effects of B at each level of A. The answer to each question is $MS_{BS/A}$. Therefore:

$$t = \frac{\bar{X}_{a1b1} - \bar{X}_{a1b2}}{\sqrt{\frac{MS_{BS/A}}{n_1} + \frac{MS_{BS/A}}{n_2}}} = \frac{3.75 - 4.00}{\sqrt{\frac{.507}{4} + \frac{.507}{4}}} = \frac{-.25}{.5035} = -.497 \quad at\ df = 18$$

Simple Main Effects of A at each level of B. The answer to question 1 is $MS_{BS/A}$ while that for question 2 is $MS_{S/A}$. Therefore:

$$MS_{pooled\ error} = \frac{SS_{S/A} + SS_{BS/A}}{df_{S/A} + df_{BS/A}} = \frac{9.375 + 9.125}{6 + 18} = \frac{18.5}{24} = .771$$

$$df_{pooled\ error} = \frac{(SS_{S/A} + SS_{BS/A})^2}{\frac{SS^2_{S/A}}{df_{S/A}} + \frac{SS^2_{BS/A}}{df_{BS/A}}} = \frac{(9.375 + 9.125)^2}{\frac{9.375^2}{6} + \frac{9.125^2}{18}} = \frac{342.25}{14.648 + 4.626} = 17.759$$

$$t = \frac{\bar{X}_{a1b1} - \bar{X}_{a2b1}}{\sqrt{\frac{MS_{pooled\ error}}{n_1} + \frac{MS_{pooled\ error}}{n_2}}} = \frac{3.75 - 1.75}{\sqrt{\frac{.771}{4} + \frac{.771}{4}}} = \frac{2.00}{.621} = 3.221 \quad at \quad df = 17.759$$

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2. Interaction Effects

a. Treatment/Contrast Interactions

b. Contrast/Contrast Interactions

These are conducted as discussed in Topic 5. **Note** that the error term in each case would be the error term for the interaction because these are pure interaction effects and not confounds of main and interaction effects as is the case when performing tests of simple main effects.

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Tests of Means in SPSS GLM Repeated Measures

Main Effects of A. SPSS uses the formula described earlier (see slide 26) for this test.

Main Effects of B. Compute a pooled estimate of the variance of the difference for each level of B over all levels of A. This requires computing the variance of the difference (i.e., b1-b2) in each A group, and pooling them as follows:

$$S^2_{pooled(b1-b2)} = \frac{(n_{a1}-1)S^2_{a1(b1-b2)} + (n_{a2}-1)S^2_{a2(b1-b2)}}{n_{a1}+n_{a2}-2} = \frac{3(.9166)+3(.25)}{6} = .583$$

$$t = \frac{\overline{X}_{b1} - \overline{X}_{b2}}{\sqrt{\frac{S^2_{pooled(b1-b2)}}{an}}} = \frac{2.75-3.50}{\sqrt{\frac{.583}{8}}} = \frac{-.75}{.270} = -2.778 \quad at\ df = n_{a1}+n_{a2}-2 = 6$$

Note. This is different from that presented in slide 26

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Simple Main Effects of B at each level of A. Use the pooled error term described on slide 30.

$$t = \frac{\overline{X}_{a1b1} - \overline{X}_{a1b2}}{\sqrt{\frac{S^2_{pooled(b1-b2)}}{n}}} = \frac{3.75-4.00}{\sqrt{\frac{.583}{4}}} = \frac{-.25}{.382} = -.654 \quad at\ df = n_{a1}+n_{a2}-2 = 6$$

Simple Main Effects of A at each level of B. Compute a pooled error term for each level of B (over each level of A).

$$S^2_{pooled(b1)} = \frac{(n_{a1}-1)S^2_{a1b1} + (n_{a2}-1)S^2_{a2b1}}{n_{a1}+n_{a2}-2} = \frac{3(1.5^2)+3(.5^2)}{6} = 1.25$$

Compute t.

$$t = \frac{\overline{X}_{a1b1} - \overline{X}_{a2b1}}{\sqrt{\frac{S^2_{pooled(b1)}}{n_{a1}} + \frac{S^2_{pooled(b1)}}{n_{a2}}}} = \frac{3.75-1.75}{\sqrt{\frac{1.25}{4}+\frac{1.25}{4}}} = \frac{2.00}{.791} = 2.5284 \quad at\ df = n_{a1}+n_{a2}-2 = 6$$

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SPSS Output for Tests of Means

Main Effect A-Means

Estimates

Measure: MEASURE_1

a	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
1.00	5.688	.313	4.923	6.452
2.00	5.063	.313	4.298	5.827

Tests of Main Effects for A-Means

Pairwise Comparisons

Measure: MEASURE_1

(I) a	(J) a	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a]	
					Lower Bound	Upper Bound
1.00	2.00	.625	.442	.207	-.456	1.706
2.00	1.00	-.625	.442	.207	-1.706	.456

Based on estimated marginal means

a. Adjustment for multiple comparisons: Bonferroni.

Note. These tests are the same as described on slide 26.

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Main Effect B-Means

Estimates

Measure: MEASURE_1

b	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
1	2.750	.395	1.783	3.717
2	3.500	.289	2.794	4.206
3	6.250	.250	5.638	6.862
4	9.000	.289	8.294	9.706

Tests of Main Effects of B

Pairwise Comparisons

Measure: MEASURE_1

(I) b	(J) b	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a] Lower Bound	Upper Bound
1	2	-.750	.270	.193	-1.793	.293
	3	-3.500*	.270	.000	-4.543	-2.457
	4	-6.250*	.489	.000	-8.141	-4.359
2	1	.750	.270	.193	-.293	1.793
	3	-2.750*	.250	.000	-3.716	-1.784
	4	-5.500*	.456	.000	-7.263	-3.737
3	1	3.500*	.270	.000	2.457	4.543
	2	2.750*	.250	.000	1.784	3.716
	4	-2.750*	.323	.001	-3.997	-1.503
4	1	6.250*	.489	.000	4.359	8.141
	2	5.500*	.456	.000	3.737	7.263
	3	2.750*	.323	.001	1.503	3.997

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

a. Adjustment for multiple comparisons: Bonferroni.

Note. These tests are different from those described on slide 26, but are the same as those described on slide 30.

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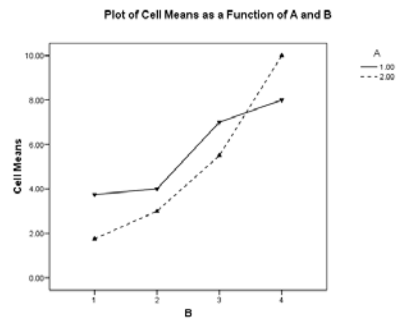
Examining the Interaction Effect

AB Means

Estimates

Measure: MEASURE_1

a	b	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
1.00	1	3.750	.559	2.382	5.118
	2	4.000	.408	3.001	4.999
	3	7.000	.354	6.135	7.865
	4	8.000	.408	7.001	8.999
2.00	1	1.750	.559	.382	3.118
	2	3.000	.408	2.001	3.999
	3	5.500	.354	4.635	6.365
	4	10.000	.408	9.001	10.999



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Tests of Simple Main Effects of B at each level of A.

Pairwise Comparisons

Measure: MEASURE_1

a	(I) b	(J) b	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a] Lower Bound	Upper Bound
1.00	1	2	-.250	.382	1.000	-1.725	1.225
		3	-3.250*	.382	.001	-4.725	-1.775
		4	-4.250*	.692	.005	-6.924	-1.576
	2	1	.250	.382	1.000	-1.225	1.725
		3	-3.000*	.354	.001	-4.366	-1.634
		4	-4.000*	.645	.005	-6.494	-1.506
	3	1	3.250*	.382	.001	1.775	4.725
		2	3.000*	.354	.001	1.634	4.366
		4	-1.000	.456	.426	-2.763	.763
	4	1	4.250*	.692	.005	1.576	6.924
		2	4.000*	.645	.005	1.506	6.494
		3	1.000	.456	.426	-.763	2.763
2.00	1	2	-1.250	.382	.102	-2.725	.225
		3	-3.750*	.382	.000	-5.225	-2.275
		4	-8.250*	.692	.000	-10.924	-5.576
	2	1	1.250	.382	.102	-.225	2.725
		3	-2.500*	.354	.002	-3.866	-1.134
		4	-7.000*	.645	.000	-9.494	-4.506
	3	1	3.750*	.382	.000	2.275	5.225
		2	2.500*	.354	.002	1.134	3.866
		4	-4.500*	.456	.000	-6.263	-2.737
	4	1	8.250*	.692	.000	5.576	10.924
		2	7.000*	.645	.000	4.506	9.494
		3	4.500*	.456	.000	2.737	6.263

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

a. Adjustment for multiple comparisons: Bonferroni.

Note. These tests differ from those on slide 28 but agree with slide 31.

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Tests of Simple Main Effects of A at B.

Pairwise Comparisons

Measure: MEASURE_1

b	(I) a	(J) a	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a] Lower Bound	Upper Bound
1	1.00	2.00	2.000*	.791	.045	.066	3.934
	2.00	1.00	-2.000*	.791	.045	-3.934	-.066
2	1.00	2.00	1.000	.577	.134	-.413	2.413
	2.00	1.00	-1.000	.577	.134	-2.413	.413
3	1.00	2.00	1.500*	.500	.024	.277	2.723
	2.00	1.00	-1.500*	.500	.024	-2.723	-.277
4	1.00	2.00	-2.000*	.577	.013	-3.413	-.587
	2.00	1.00	2.000*	.577	.013	.587	3.413

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

a. Adjustment for multiple comparisons: Bonferroni.

Note. These tests differ from those described on slide 28 but are the same as those on slide 31.

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References

Satterthwaite, F.E. (1946). An approximate distribution of estimates of variance components. *Biometrics Bulletin, 2,* 110-114.

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