Research Design - - Topic 8

Hierarchical Designs in Analysis of Variance (Kirk, Chapter 11)

Experimental Design Approach
- General Rationale and Applications
- Rules for Determining Sources of Variance

Examples of Two factor Designs with B nested in A, and how to run in SPSS GLM
- Subjects nested in A and B
- Subjects nested in A
- Subjects crossed with A and B

Tests of Means

Variance Accounted for

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General Rationale and Applications

Hierarchical Design. At least one treatment factor is nested in at least one other factor. That is, a factor (B) is nested in another (A) if certain levels of B appear in only one level of A. This means that the factors are not crossed and that consequently there is no interaction involving those two factors.

Such designs are typically used when factor B is a nuisance variable. If it is a random factor it permits generalization to all possible levels of that factor and if this is the case the effects of other factors (e.g., A) apply to all possible levels of B.

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Example from Kirk, (1995, p. 480) to be used in each of the three designs

	B1	B2	B3	B4	B5	B6	B7	B8	
A1	3	1	5	2					
A1	6	2	6	3					$\bar{X}_{a1} = 3.50$
A1	3	2	5	4					
A1	3	2	6	3					
Means	3.75	1.75	5.5	3.0					
A2					7	4	7	10	
A2					8	5	8	10	$\bar{X}_{a2} = 7.25$
A2					7	4	9	9	
A2					6	3	8	11	
Means					7.0	4.0	8.0	10.0	

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Rules for Determining Sources of Variance

1. List all the factors, and their possible interactions indicating the nesting. Nesting is indicated by using a "/" between the factor and the one in which it is nested. For example, B nested in A is written B/A, Subjects nested in B which is nested in A is written S/B/A.
2. Eliminate all interactions containing elements where a factor is identified as interacting with another that is nested in it. This is indicated when a factor is shown both preceding (or not being associated with a /) and following a /.

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Example 1 has B nested in A with subjects nested in A and B. The sources of variance would be written as follows.

A
B/A
S/B/A
A*B/A
B/A*S/B/A
A*S/B/A
A*B/A*S/B/A

The second rule results in the following being eliminated:

A*B/A - - because - - A appears both before and after the /
B/A*S/B/A - - - - B appears both before and after a /
A*S/B/A - - - - A appears both before and after a /
A*B/A*S/B/A - - - - both A and B appear before and after a /

Thus, only the three main effect factors remain.

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Example 1. B nested in A, and Subjects nested in A and B. (A, B/A, S/B/A)

Definitional Formulae

Source	SS	df
A	$nb\sum^{a}\left(\bar{X}_{a}-\bar{G}\right)^{2}$	$(a-1)$
B/A	$n\sum^{a}\sum^{b}\left(\bar{X}_{b/a}-\bar{X}_{a}\right)^{2}$	$a(b-1)$
S/B/A	$\sum^{a}\sum^{b}\sum^{n}\left(X_{abi}-\bar{X}_{b/a}\right)^{2}$	$ab(n-1)$

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In this design, there are three Sums of Squares, but their relation to the sums of squares in the factorial design are as follows:

Hierarchical Design	Completely Randomized Factorial
SS_A	SS_A
$SS_{B/A}$	$SS_B + SS_{AB}$
$SS_{S/B/A}$	$SS_{S/AB}$

Source	SS	df	MS	F	E(MS)
A	112.50	1	112.50	6.46	$nb\theta_A^2 + n\left(1-b/B\right)\theta_{B/A}^2 + \sigma_\varepsilon^2$
B/A	104.50	6	17.42	22.62	$n\theta_{B/A}^2 + \sigma_\varepsilon^2$
S/B/A	18.50	24	0.77		σ_ε^2

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To analyze these data in SPSS GLM Univariate, perform the analysis as if it were a two factor completely randomized design. This would produce the following results. Use these values to do the hand calculations shown on the next slide.

Tests of Between-Subjects Effects

Dependent Variable: X

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	217.000[a]	7	31.000	40.216	.000
Intercept	924.500	1	924.500	1199.351	.000
A	112.500	1	112.500	145.946	.000
B	75.250	3	25.083	32.541	.000
A * B	29.250	3	9.750	12.649	.000
Error	18.500	24	.771		
Total	1160.000	32			
Corrected Total	235.500	31			

a. R Squared = .921 (Adjusted R Squared = .899)

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In order to calculate the SS for the hierarchical design, it would be necessary to do the following hand calculations.

Thus: $SS_{B/A} = 75.25 + 29.25 = 104.5 \quad MS_{B/A} = \frac{104.5}{6} = 17.42$

Where $df_{B/A} = 3 + 3 = 6$

And $F_A = \frac{MS_A}{MS_{B/A}} = \frac{112.50}{17.42} = 6.46, df = 1, 6, p < .05$

Assuming B is a random factor

Conclusion: A1 results in less activity (mean = 3.50) than A2 (mean = 7.25), and we can generalize this finding to all possible cages.

$$F_{B/A} = \frac{MS_{B/A}}{MS_{S/B/A}} = \frac{17.42}{.77} = 22.62 \quad df = 6, 24$$

Conclusion: There is significant variation among cages nested in A. Because B is a random factor, there would be no interest in comparing means for B nested in A.

Example 2. Subjects nested in A. For example A is two types of words, Concrete and Abstract, presented tachistoscopically to measure recognition time. B is Words (i.e., 4 exemplars drawn at random nested in types of words), and Subjects receive either the Concrete or Abstract lists.

A
B/A
S/A
A*B/A
B/A*S/A = BS/A
A*S/A
A*B/A*S/A

Following the second rule, the following would be eliminated:

A*B/A - - because - - A appears both before and after a /
A*S/A - - - - A appears both before and after a /
A*B/A*S/A - - - - A appears both before and after a /

Thus, four sources remain.

The following slide shows the defining formulae for this analysis, and the subsequent one presents the results obtained by applying these formulae to the data in Slide 3.

Example 2. B nested in A and Subjects nested in A (A,B/A, S/A)

Definitional formulae

Source	SS	df
A	$nb\sum^{a}(\bar{X}_a - \bar{G})^2$	$(a-1)$
B/A	$n\sum^{a}\sum^{b}(\bar{X}_{b/a} - \bar{X}_a)^2$	$a(b-1)$
S/A	$b\sum^{a}\sum^{n}(\bar{P}_{i/a} - \bar{X}_a)^2$	$a(n-1)$
BS/A	$\sum^{a}\sum^{b}\sum^{n}(X_{abi} - \bar{P}_{i/a} - \bar{X}_{b/a} + \bar{X}_a)^2$	$a(b-1)(n-1)$

Example 2
Numerical Example and Expected Mean Squares (Cornfield Tukey) (A,B/A, S/A)

Source	SS	df	MS	F	E(MS)
A	112.50	1	112.50	6.46	$nb\theta_A^2 + n(1 - b/B)\theta_{B/A}^2 + (1 - b/B)\sigma_{B\pi/A}^2 + \sigma_\varepsilon^2$
B/A	104.50	6	17.42	25.25	$n\theta_{B/A}^2 + \sigma_{B\pi/A}^2 + \sigma_\varepsilon^2$
S/A	6.00	6	1.00		$b\sigma_{\pi/A}^2 + \sigma_\varepsilon^2$
BS/A	12.50	18	0.694	0.69	$\sigma_{B\pi/A}^2 + \sigma_\varepsilon^2$

For this design, analyze the data with SPSS GLM Repeated as if you were running a split plot design. This will produce the following results for the Within Subjects Effect. **(Only the values from Sphericity Assumed are relevant.)**

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
B	Sphericity Assumed	75.250	3	25.083	36.120	.000
	Greenhouse-Geisser	75.250	2.256	33.351	36.120	.000
	Huynh-Feldt	75.250	3.000	25.083	36.120	.000
	Lower-bound	75.250	1.000	75.250	36.120	.001
B * A	Sphericity Assumed	29.250	3	9.750	14.040	.000
	Greenhouse-Geisser	29.250	2.256	12.964	14.040	.000
	Huynh-Feldt	29.250	3.000	9.750	14.040	.000
	Lower-bound	29.250	1.000	29.250	14.040	.010
Error(B)	Sphericity Assumed	12.500	18	.694		
	Greenhouse-Geisser	12.500	13.538	.923		
	Huynh-Feldt	12.500	18.000	.694		
	Lower-bound	12.500	6.000	2.083		

Compute values for B/A (i.e., SS and df) by summing the values for B and B*A, and computing F as shown in Slide 15.

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And the following table for the Between Subjects Effects.

Tests of Between-Subjects Effects

Measure: MEASURE_1
Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	924.500	1	924.500	924.500	.000
A	112.500	1	112.500	112.500	.000
Error	6.000	6	1.000		

In this case:
Error corresponds to S/A from Slide 12
Error (B) from Slide 13 corresponds to BS/A from Slide 12
And from Slide 12 $SS_{B/A}$ = 104.5 = 75.25 + 29.25 from Slide 13.

Thus, the three possible F-ratios that can be computed given the Expected Mean Squares on Slide 13 are as shown on the next slide.

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Examination of the E(MS) with B as a random factor yields the following :

$$F_A = \frac{MS_A}{MS_{B/A}} = \frac{112.50}{17.42} = \ 6.46 @ 1, 6\, df\, , p < .05$$

Conclusion: A1 (concrete words) recognized more quickly (mean = 3.50) than A2 (abstract words) (mean = 7.25), and we can generalize this finding to all possible concrete and abstract words.

$$F_{B/A} = \frac{MS_{B/A}}{MS_{BS/A}} = \frac{17.42}{.69} = 25.25 @ 6, 18\, df\, , p < .0001$$

Conclusion: There is significant variation in recognition speed of words within concrete and abstract lists. Because B is a random factor, there would be no interest in comparing means within either of the lists.

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Example 3. Subjects crossed with A and B. For example words from both lists (Concrete and Abstract) are administered tachistoscopically in random order to all subjects to measure recognition time.

A
B/A
S
A*B/A
B/A*S = BS/A
A*S
A*B/A*S

Following the second rule, the following would be eliminated:

A*B/A - - because - - A appears both before and after a /
A*B/A*S - - - - A appears both before and after a /

Thus, five sources remain.

The following slide shows the defining formulae for this analysis, and the subsequent one shows the results obtained by applying these formulae to the data presented in Slide 3.

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Example 3. B Subjects crossed with A and B (A,B/A, S) Definitional formulae

Source	SS	df
A	$nb\sum^{a}\left(\overline{X}_a - \overline{G}\right)^2$	$(a-1)$
B/A	$n\sum^{a}\sum^{b}\left(\overline{X}_{b/a} - \overline{X}_a\right)^2$	$a(b-1)$
S	$ab\sum^{n}\left(\overline{P}_i - \overline{G}\right)^2$	$(n-1)$
AS	$b\sum^{a}\sum^{n}\left(\overline{P}_{i/a} - \overline{P}_i - \overline{X}_a + \overline{G}\right)^2$	$(a-1)(n-1)$
BS/A	$\sum^{a}\sum^{b}\sum^{n}\left(X_{abi} - \overline{P}_{i/a} - \overline{X}_{b/a} + \overline{X}_a\right)^2$	$a(b-1)(n-1)$

Example 3 Numerical Example and Expected Mean Squares (Cornfield Tukey) (A,B/A, S)

Source	SS	df	MS	F	E(MS)
A	112.50	1	112.50	6.63	$nb\theta_A^2 + n\left(1-\frac{b}{B}\right)\theta_{B/A}^2 + \left(1-\frac{b}{B}\right)\sigma_{B\pi}^2 + b\sigma_{A\pi}^2 + \sigma_\varepsilon^2$
B/A	104.50	6	17.42	25.25	$n\theta_{B/A}^2 + \sigma_{B\pi}^2 + \sigma_\varepsilon^2$
S	5.25	3	1.75		$ab\sigma_\pi^2 + \left(1-\frac{b}{B}\right)\sigma_{B\pi}^2 + b\left(1-\frac{a}{A}\right)\sigma_{A\pi}^2 + \sigma_\varepsilon^2$
AS	0.75	3	0.25	.36	$b\sigma_{A\pi}^2 + \left(1-\frac{b}{B}\right)\sigma_{B\pi}^2 + \sigma_\varepsilon^2$
BS/A	12.50	18	0.69		$\sigma_{B\pi}^2 + \sigma_\varepsilon^2$

$$F_A = \frac{MS_A}{MS_{B/A} + MS_{AS} - MS_{BS/A}} \qquad df_1 = df_A \qquad df_2 = \frac{\left(MS_{B/A} + MS_{AS} - MS_{BS/A}\right)^2}{\frac{MS_{B/A}^2}{df_{B/A}} + \frac{MS_{AS}^2}{df_{AS}} + \frac{MS_{BS/A}^2}{df_{BS/A}}}$$

For example 3, subjects crossed with A and B, analyze the data with SPSS GLM Repeated as if you were running a randomized blocks factorial (repeated measures on both factors). This will yield the following Between Subjects table and the Within Subjects table on the next slide.

Tests of Between-Subjects Effects

Measure: MEASURE_1
Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	924.500	1	924.500	528.286	.000
Error	5.250	3	1.750		

The values for Error correspond to those for S in Example 3 (see Slide 18).

Output to be used for Example 3

(Only the values for Sphericity Assumed are relevant here)

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
A	Sphericity Assumed	112.500	1	112.500	450.000	.000
	Greenhouse-Geisser	112.500	1.000	112.500	450.000	.000
	Huynh-Feldt	112.500	1.000	112.500	450.000	.000
	Lower-bound	112.500	1.000	112.500	450.000	.000
Error(A)	Sphericity Assumed	.750	3	.250		
	Greenhouse-Geisser	.750	3.000	.250		
	Huynh-Feldt	.750	3.000	.250		
	Lower-bound	.750	3.000	.250		
B	Sphericity Assumed	75.250	3	25.083	37.625	.000
	Greenhouse-Geisser	75.250	1.138	66.105	37.625	.006
	Huynh-Feldt	75.250	1.372	54.865	37.625	.003
	Lower-bound	75.250	1.000	75.250	37.625	.009
Error(B)	Sphericity Assumed	6.000	9	.667		
	Greenhouse-Geisser	6.000	3.415	1.757		
	Huynh-Feldt	6.000	4.115	1.458		
	Lower-bound	6.000	3.000	2.000		
A * B	Sphericity Assumed	29.250	3	9.750	13.500	.001
	Greenhouse-Geisser	29.250	1.788	16.356	13.500	.009
	Huynh-Feldt	29.250	3.000	9.750	13.500	.001
	Lower-bound	29.250	1.000	29.250	13.500	.035
Error(A*B)	Sphericity Assumed	6.500	9	.722		
	Greenhouse-Geisser	6.500	5.365	1.212		
	Huynh-Feldt	6.500	9.000	.722		
	Lower-bound	6.500	3.000	2.167		

Examination of the E(MS) with B as a random factor yields the following :

There is no clear F-ratio for A, thus a quasi F-ratio must be computed using the formulae presented in Slide 19.

$$F_A = \frac{MS_A}{MS_{pooled}} = \frac{112.50}{17.42 + .25 - .69} = \quad 6.63 \,@\, 1, 5.7\, df,\, p < .05$$

Conclusion: A1 (concrete words) recognized more quickly (mean = 3.50) than A2 (abstract words) (mean = 7.25), and we can generalize this finding to all possible concrete and abstract words.

$$F_{B/A} = \frac{MS_{B/A}}{MS_{BS/A}} = \frac{17.42}{.69} = 25.25 @ 6, 18\, df,\, p < .0001$$

Conclusion: There is significant variation in recognition speed of words within concrete and abstract lists Because B is a random factor, there would be no interest in comparing means within either of the lists.

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We could compute two other F-ratios, though they may not be of much interest.

$$F_{AS} = \frac{MS_{AS}}{MS_{BS/A}} = \frac{.25}{.69} = .36\, ns$$

If it were significant this would indicate that there is an interaction between individual subjects and the type of word. Because Subject is a random factor, there would be no interest in testing differences between means.

And if A is fixed: $F_S = \frac{MS_S}{MS_{BS/A}} = \frac{1.75}{.69} = 2.54 \,@\, 3, 18\, df,\, ns$

Or if A is random $F_S = \frac{MS_S}{MS_{AS}} = \frac{1.75}{.25} = 7.00 \,@\, 3, 3\, df,\, ns$

If either were significant it would indicate that there are significant individual differences in recognition speed of words.

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Tests of Means

Because A is the only fixed factor in this example, only tests of the A means can be computed. They would not be necessary for this example because there are only two levels of A, but if there were more than two, the tests could be computed as follows (demonstrated only by the t-test, but the approach generalizes to all the tests of means).

$$t = \frac{\overline{X}_{a1} - \overline{X}_{a2}}{\sqrt{\frac{2MS_{error}}{bn}}}$$

Where: MS_{error} = $MS_{B/A}$ for examples 1 and 2
MS_{error} = $MS_{B/A}$ + MS_{AS} – $MS_{BS/A}$ for example 3 with the Satterthwaite estimate of degrees of freedom.

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Variance Accounted For

In order to compute estimates for ω² and ρ for hierarchical models, it is necessary to evaluate the Expected Mean Square Table where appropriate. For Examples 1 and 2, there are F-based formulae, but they are not directly comparable to previous examples.

Thus, the estimate for ω² for A in example 1 is:

$$\omega^2 = \frac{v_1(F_A - 1)}{v_1(F_A - 1) + \frac{N}{F_{B/A}}} = \frac{1(5.46)}{1(5.46) + \frac{32}{22.59}} = .79$$

The estimate for ω² for A in example 2 is:

$$\omega^2 = \frac{v_1(F_A - 1)}{v_1(F_A - 1) + \frac{N(MS_{S/A})}{MS_{B/A}}} = \frac{1(5.46)}{1(5.46) + \frac{32(1.00)}{17.42}} = .75$$

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