Analysis of Variance with Categorical and Continuous Factors: Beware the Landmines

R. C. Gardner
Department of Psychology

Sometimes researchers want to perform an analysis of variance where one or more of the factors is a continuous variable and the others are categorical, and they are advised to use multiple regression to perform the task. The intent of this article is to outline the various ways in which this is normally done, to highlight the decisions with which the researcher is faced, and to warn that the various decisions have distinct implications when it comes to interpretation.

This first point to emphasize is that when performing this type of analysis, there are no means to be discussed. Instead, the statistics of interest are intercepts and slopes. More on this later.

Models.

To begin, there are a number of approaches that one can follow, and each of them refers to a different model. Of primary importance, each model tests a somewhat different hypothesis, and the researcher should be aware of precisely which hypothesis is being tested. The three most common models are:

Model I. This is the unique sums of squares approach where the effects of each predictor is assessed in terms of what it adds to the other predictors. It is sometimes referred to as the regression approach, and is identified as SSTYPE3 in GLM. Where a categorical factor consists of more than two levels (i.e., more than one coded vector to define the factor), it would be assessed in terms of the *F*-ratio for change when those levels are added to all other effects in the model. If there is more than one degree of freedom, it would be necessary to test the R-squared change resulting from adding the relevant collection of vectors to all other vectors. Where the categorical factor is a dichotomy and the associated effect involves only one degree of freedom, this *F*-ratio would be equal to the square of the *t*-ratio for the significance of the regression coefficient.

Model II. In SPSS GLM, this is referred to as SSTYPE2, and involves tests of models in a series of hierarchical steps. In step 1, all of the vectors defining the contributions of the individual factors are entered in a block. This is often referred to as entering the main effects first. To determine whether any one factor is significant, it would be necessary to determine whether the block of vectors defining that factor results in a significant R-squared change when compared to the other vectors in that block (i.e., test to see if each main effect adds significantly to the other main effects). If all factors have one degree of freedom (i.e., the categorical factors are all dichotomous), the *t*-test for the associated regression coefficient is a test of this R-squared change. In step 2, the two-way interaction vectors are then entered in a block, and R-squared change for each interaction is tested for its contribution to the R-squared for this block. Again, if each interaction is based on one degree of freedom, the *t*-tests for the regression coefficients for these vectors are tests of this effect. In step 3, the three-way interaction vectors are added, and so on.

Model III. This is often referred to as the hierarchical approach, where effects are entered one at a time. Thus, if there were 3 factors, the primary 'main effect' one would be entered first, then the next, then the next, etc., until all 'main effect' vectors are added. Tests of significance would be the R-squared change after each effect is added. After all of the 'main effects' vectors are entered, the two way interaction terms would be added one at a time based on some prior defined order, and the R-squared change effects tested for significance at each step, and then the three way-interaction vectors, etc...

Types of Coding

To make use of categorical factors in multiple regression, it is necessary to code the factors in such a way that each level of a factor is uniquely defined. There are a number of ways in which factors can be coded. The two most common are Dummy coding and Effect coding.

1. Dummy Coding. Dummy coding involves the use of 1 and 0 values to identify the levels of each factor. If there are only two levels of the factor, there would be one vector with those in one level of the factor coded 1, and those in the other level, coded 0. If there are more than two levels for a factor, there would be two or more vectors, each one comprised of 0's and 1's with no levels of that factor having the value of 1 more than once. One level of the factor will have all 0's for the vectors identifying that factor.

2. Effect coding. Effect codes consist of a series of 1's and -1's, and 0's if there are more than two levels for the factor and thus more than one vector. Thus, one level of the factor would be defined as -1 across all vectors for that factor.

For the continuous variable, one can use it as it is (i.e., uncentred), or it can be centred (i.e., the grand mean of the variable is subtracted from each value of the variable, so that the mean of the centred variable is 0. This has no effect on the slopes that are computed but it does influence the value of the intercepts.

Proceeding with the Analysis.

Regardless of the model followed, the values computed when all predictors are included in the model are the intercepts and slopes of the various groups identified by the factors. That is, if the design consisted of two dichotomous factors, there would be four groups, and the intercepts and slopes would be the values obtained if one were to compute bivariate regressions for each of these groups. If, it there were three crossed dichotomous factors, there would be 8 groups, and so on. (As an aside, it should be noted that if there were two continuous variables, the regression coefficients for each group would correspond to the regression coefficients for each continuous variable in each group if a multiple regression were computed for that group and the two continuous variables were treated as predictors.) Thus, from this, one might conclude correctly that the type of coding or the type of model is inconsequential. *This does not mean, however, that this is true for various steps along the way*. The purpose of the rest of this article is to demonstrate precisely what hypotheses are tested by the various models and types of coding. We will limit discussion to Dummy and Effect coding for Models I and II because these are the most frequently used models. The generalization to Model III is straight-forward. Furthermore, we will focus on dichotomous categorical factors (specifically a 2X2) and one continuous centred variable; it is a straight-forward generalization to factors with more than two levels.

Sample data are presented in Appendix A. The two categorical factors are identified as *A* and *B*, and the centered continuous variable is *C*. The intercepts and slopes for the data in each of the four groups are as follows (you can prove this to yourself by taking the data for each of the four groups and running them through the bivariate regression program).

Cell Intercepts

	B1	B2
A1	13.438	21.330
A2	17.313	15.113

Cell Slopes

	B1	B2
A1	1.00	-1.316
A2	-1.00	.600

For what its worth, the univariate analyses indicate that each of the intercepts is significantly different from 0, but only the slope for A1B2 is significantly different from 0.

Model I - - Effect Coding.

The following table presents the effect coding for a 2X2 design:

Factor Levels		Effect Codes	
A	B	*ea*	*eb*
1	1	1	1
1	2	1	-1
2	1	-1	1
2	2	-1	-1

The multiple correlation for the 7 predictors defining the factors and their interaction is .931 The regression coefficients are given in Table 1. The factors are:

c - - the continuous variable

ea - - the effect coded vector for the A factor

eb - - the effect coded vector for the B factor

eaeb - - the product term for the effect codes for A and B

eac - - the product term for the effect code for A and the C factor

ebc - - the product term for the effect code for B and the C factor

eaebc - - the product term for the effect codes for A and B and the C factor

Inspection of the table will reveal that the regression coefficients for the B factor and the interaction of AXB are significant ($p < .05$) and that the three way interaction of AXBXC is marginally significant, $p < .057$. For the purpose of this example, we will assume that this is significant.

Table 1

Regression Coefficients for Model I using Effect coding

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	16.798	.448		37.493	.000
	c	-.179	.441	-.062	-.406	.696
	ea	.586	.448	.172	1.307	.228
	eb	-1.423	.448	-.418	-3.176	.013
	eaeb	-2.523	.448	-.741	-5.631	.000
	eac	.021	.441	.007	.047	.963
	ebc	.179	.441	.062	.406	.696
	eaebc	.979	.441	.336	2.218	.057

a. Dependent Variable: x

The unstandardized coefficients can be used to construct the regression equation:

Y = 16.798 + .586*ea - 1.423*eb - 2.523*ea*eb -.179*c + .021*ea*c + .179*eb*c + .979*ea*eb*c

These terms can be rearranged to distinguish between the intercept and slope as follows:

Y = 16.798 + .586*ea - 1.423*eb - 2.523*ea*eb + (-.179 + .021*ea + .179*eb + .979*ea*eb)c

Thus, the intercepts and slopes for each group can be calculated using the following formulae and substituting the appropriate values for *ea* and *eb*.

Intercept = 16.798 + .586 * ea - 1.423 * eb - 2.523 * ea * eb

Slope = -.179 + .021 * ea + .179 * eb + .979 * ea * eb.

To compute the corresponding marginal values, it is necessary to set the *ea* or *eb* values to 0 for the vector to be eliminated. All of these values are shown in the following two tables. As can be seen, the cell values correspond to the values presented previously, and the marginal values are simply the unweighted means of the corresponding cell values.

Cell Intercepts

	B1	B2	Marginal A Intercepts
A1	13.438	21.330	17.384
A2	17.313	15.113	16.213
Marginal B Intercepts	15.376	18.222	16.798

Cell Slopes

	B1	B2	Marginal A Slopes
A1	1.00	-1.316	-.158
A2	-1.00	.600	-.200
Marginal B Slopes	0	-.358	-.179

These values are instructive because they show the precise definitions of the regression coefficients in Table 1. Thus:

1. Constant = 16.798 (the grand mean of the intercepts)
2. *c* = -.179 (the grand mean of the slopes)
3. *ea* = 17.384 - 16.798 = .586 (i.e., main effect of A)
4. *eb* = 15.376 - 16.798 = -1.423 (i.e., main effect of B)
5. *eaeb* = 13.438 - 17.384 - 15.375 + 16.798 = -2.523 (i.e., interaction of A and B)
6. *eac* = -.158 - (-.179) = .021 (i.e., interaction of A and C)
7. *ebc* = 0 - (-.179) = .179 (i.e., interaction of B and C)
8. *eaebc* = 1.0 - (-.158) - 0 + -.179 = .979 (i.e., three way interaction of A, B, and C expressed as a two-way interaction of the slopes)

As can be seen, therefore, the tests of significance of the regression coefficients shown in Table 1 are tests of significance of the mean intercept and the mean slope as well as the effects (i.e., the contrasts indicated above). Because each categorical factor has only two levels (1 degree of freedom), the effects correspond to the relevant main effects and interactions.

Model I - - Dummy Coding.

Model I should not be used with Dummy coding because many of the tests of significance are not as they seem. Nonetheless, the analysis was done here to show precisely what null

hypotheses are tested. The tests of significance of the regression coefficients for the so-called main effects are not tests of these effects, but instead are tests of contrasts involving the cell values. This can have serious implications for the interpretation of tests of significance, especially if any of the categorical factors has more than two levels. The following table presents the dummy coding:

Factor Levels		Effect Codes	
A	B	*da*	*db*
1	1	1	1
1	2	1	0
2	1	0	1
2	2	0	0

The multiple correlation for the 7 predictors defining the factors and their interaction is .931 The regression coefficients are given in Table 2. The factors are:

c - - the continuous variable

da - - the dummy coded vector for the A factor

db - - the dummy coded vector for the B factor

dadb - - the product term for the dummy codes for A and B

dac - - the product term for the dummy code for A and the C factor

dbc - - the product term for the dummy code for B and the C factor

dadbc - - the product term for the dummy codes for A and B and the C factor

Table 2

Regression Coefficients for Model I Using Dummy Coding

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	15.113	.884		17.091	.000
	c	.600	.555	.209	1.080	.311
	da	6.217	1.247	.912	4.985	.001
	db	2.200	1.270	.323	1.732	.121
	dadb	-10.092	1.792	-1.283	-5.631	.000
	dac	-1.916	.979	-.369	-1.958	.086
	dbc	-1.600	.962	-.322	-1.663	.135
	dadbc	3.916	1.765	.417	2.218	.057

a. Dependent Variable: x

It will be noted that the values of the regression coefficients are different and would yield the following equations for the intercept and slope.

Intercept = 15.113 + 6.217 * da + 2.200 * db - 10.092 * da * db

Slope = .600 + (-1.916) * da + (-1.600) * db + 3.916 * da * db.

These equations will produce the same values for the intercepts and slopes for the cells as shown in the earlier tables but they will not yield the correct marginal values. If the same strategy as discussed above is used to calculate the marginal values, it will be found instead that the values obtained are identical to those given for the A2 row and B2 column. That is, what one might consider the marginal values in fact are not the marginal values, but rather the values obtained when the coded value is 0 (i.e., for A2 and B2).

Using this information, we can show that the regression coefficients shown in Table 2 are:

1. Constant = 15.113 (the value for A2B2 in the table of intercepts)

2. *c* = .600 (the value for A2B2 in the table of slopes)

3. *da* = 21.330 - 15.113 = 6.217 (A1B2 - A2B2 in the table of intercepts)

4. *db* = 17.313 - 15.113 = 2.200 (A2B1 - A2B2 in the table of intercepts)

5. *dadb* = 13.438 - 21.330 - 17.313 + 15.113 = -10.092 (A1B1-A1B2-A2B1+A2B2 for the intercepts)

6. *dac* = -1.316 - .600 = -1.916 (A1B2 - A2B2 in the table of slopes)

7. *dbc* = -1.00 - .600 = -1.600 (A2B1 - A2B2 in the table of slopes)

8. *dadbc* = 1.00 - (-1.316) - (-1.00) + .600 = 3.916 (A1B1-A1B2-A2B1+A2B2 for the slopes)

Thus, the tests of significance of the regression coefficients shown in Table 2 are tests of significance of both the intercept and slope for group A2B2 as well as contrasts based on the cell values. None of them reflect tests of main effects or interaction effects directly. Note, however, that two of the tests of significance (i.e., for *dadb* and *dadbc*) are identical to those obtained with Effect Coding (i.e., *eaeb* and *eaebc*). This is because these two interactions are the highest possible ones in this design. That is, *dadb* is the highest interaction involving only the two categorical factors and *dadbc* is the highest order interaction involving the three factors. Where the categorical factors are all dichotomous, these two classes of highest order interactions will always agree with those obtained with Effect coding. If any of the categorical factors have more than two levels, however, this will not be the case; in this type of situation none of the regression coefficients can be interpreted as describing interaction effects. Instead, they all describe specific contrast/contrast interactions involving the groups coded by all 0's.

Model II - - Effect Coding.

Model 2 is a hierarchical procedure in which vectors are entered in blocks beginning with the “main effect” vectors, then the two-way product vectors, then the three-way product terms, and so on. For the three-factor design discussed here, the regression coefficients for all terms in the final equation are identical to those already presented, thus discussion will be limited to the first two blocks. Table 3 presents the regression coefficients for step 1 (referred to as Model 1 in the table) and step 2 (Model 2 in the table).

Table 3

Regression Coefficients for Model II using Effect Coding

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	16.625	.862		19.284	.000
	c	.190	.730	.066	.260	.799
	ea	.637	.863	.187	.738	.475
	eb	-1.512	.863	-.444	-1.751	.105
2	(Constant)	16.616	.528		31.484	.000
	c	-.363	.519	-.126	-.698	.503
	ea	.561	.537	.165	1.046	.323
	eb	-1.544	.533	-.453	-2.897	.018
	eaeb	-2.559	.536	-.751	-4.770	.001
	eac	-.356	.488	-.124	-.730	.484
	ebc	-.219	.483	-.076	-.453	.662

a. Dependent Variable: x

The regression equation for Model 1 in this analysis would produce the following cell and marginal values for the intercepts but only the one value of the slope = .190.

Cell Intercepts

	B1	B2	Marginal A Intercepts
A1	15.750	18.774	17.262
A2	14.476	17.500	15.988
Marginal B Intercepts	15.113	18.137	16.625

These intercepts are the adjusted values of the intercepts for each cell once you have taken into account the other main effect variables, while the slope is the best estimate of the slope once you have taken into account the two categorical main effect factors. These estimates are made on the assumption that all two-way and the three way interactions are 0. They could be considered the results of mutual analyses of covariance. The marginal values are simply the unweighted means of the cell values. Thus, the regression coefficients tested for significance are:

1. Constant = 16.625 (the grand mean of the adjusted intercepts)
2. *c* = .190 (the adjusted slope)
3. *ea* = 17.262 - 16.625 = .637 (the main effect of the adjusted intercepts)
4. *eb* = 15.113 - 18.137 = -1.512 (the main effect of the adjusted slopes)

The regression equation for Model 2 yields the following values for the intercepts and slopes.

Cell Intercepts

	B1	B2	Marginal A Intercepts
A1	13.074	21.280	17.177
A2	17.070	15.040	16.055
Marginal B Intercepts	15.072	18.160	16.616

Cell Slopes

	B1	B2	Marginal A Slopes
A1	-.938	-.500	-.719
A2	-.226	.212	-.007
Marginal B Slopes	-.582	-.144	-.363

The regression coefficient for B is significant indicating that the adjusted intercept for B1 is significantly lower than that for B2. Furthermore, the interaction between the A and B is significant as indicated by the four cell intercepts. As can be seen, the intercepts for A1 increase from B1 to B2 while those for A2 decrease from B1 to B2. As before, it would be shown as a consequence that the regression coefficients are:

1. Constant = 16.616(the grand mean of the adjusted intercepts)
2. *c* = -.363 (the adjusted slope)
3. *ea* = 17.177 - 16.616 = .561 (the main effect of the adjusted intercepts)
4. *eb* = 15.072 - 16.616 = -1.544 (the main effect of the adjusted slopes)
5. *eaeb* = 13.074 - 17.177 - 15.072 + 16.616 = -2.559 (the interaction of adjusted intercepts)
6. *eac* = -.719 - (-.363) = -.356 (interaction of adjusted slopes with A)
7. *ebc* = -.582 - (-.363) = -.219 (interaction of adjusted slopes with B)

Model II - - Dummy Coding.

Table 4 presents the regression coefficients for step 1 and step 2 when Dummy coding is used.

Table 4

Regression Coefficients for Model 2 using Dummy Coding

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	17.500	1.493		11.720	.000
	c	.190	.730	.066	.260	.799
	da	1.274	1.727	.187	.738	.475
	db	-3.024	1.727	-.444	-1.751	.105
2	(Constant)	15.040	1.059		14.205	.000
	c	.212	.632	.074	.336	.744
	da	6.240	1.494	.916	4.176	.002
	db	2.030	1.519	.298	1.337	.214
	dadb	-10.237	2.146	-1.301	-4.770	.001
	dac	-.713	.976	-.137	-.730	.484
	dbc	-.437	.966	-.088	-.453	.662

a. Dependent Variable: x

It will be noted that none of the regression coefficients, with the exception of the constant, are significant at Step 1, while two plus the constant are significant at Step 2. It will be noted too that although the unstandardized regression coefficients at step 1 in Table 4 are different from those in Table 3, the t-values are the same, as are the standardized regression coefficients. In point of fact, except for the constant, the unstandardized regression coefficients in Table 4, step 1, are simply twice the value of those in Table 3. In short, in step 1, Model II, both Effect coding and Dummy coding lead to identical conclusions.

The regression coefficients for step 1 can be used to form the regression equation, and if it is solved it will be noted that the cell values for the intercept and the slopes are the same as those obtained when effect coding was used. If it were attempted to compute the marginal values using the logic described earlier, however, it would be discovered that in fact the marginal values are not obtained; instead this procedure would present the values for the A2 row and the B2 column. As with the previous example using Dummy coding, the regression coefficients can be shown to equal:

1. Constant = 17.500 (the intercept for A2B2)
2. *c* = .190 (the same value obtained for step 1 using Effect coding)
3. *da* = 18.774 - 17.500 = 1.274 (A1B2 - A2B2 adjusted intercepts)
4. *db* = 14.476 - 17.500 = -3.024 (A2B1 - A2B2 adjusted slopes)

Note that the constant is different from that obtained for step 1 using effect coding, but that the value for *c* is identical while those for da and db are twice those obtained earlier.

The regression coefficients for Model 2 are presented in Table 4, and as before they can be used to estimate the cell values, though the attempt to estimate the marginal values will not be successful, estimating the A2 row values and the B2 column values instead. Of course, the estimates will be identical with those obtained in step 2 for the effect coded vectors. Also, the regression coefficients can be shown to equal:

1. Constant = 15.040 (the grand mean of the adjusted intercepts)
2. *c* = .212 (the adjusted slope)
3. *da* = 21.280 - 15.040 = 6.240 (A1B2 - A2B2 adjusted intercepts)
4. *db* = 17.070 - 15.040 = 2.030 (A2B1 - A2B2 adjusted slopes)
5. *dadb* = 13.074 - 21.280 - 17.070 + 15.040 = -10.236 (adjusted intercept interaction)
6. *dac* = -.500 - .212 = -.712 (A1B2 - A2B2 slopes)

7. *dbc* = -.226 - .212 = -.438 (A2B1 - A2B2 slopes)

Note that both the unstandardized and standardized regression coefficients are different from those obtained in step 2 for Effect coding, but that the tests of significance for the three interaction terms (*dadb*, *dac*, and *dbc*) are identical to those obtained in step 2 for Effect coding. Note too, that except for rounding error, the unstandardized regression coefficient for *dadb* is 4 times that for *eaeb*, while those for *dac* and *dbc* are 2 times the value of their counterparts for Effect coding. This relationship does not carry over to the standardized regression coefficients, however.

These results would appear to suggest that as long as the researcher limited interpretation to the results obtained at the appropriate step, the results would be the same for Effect coding and Dummy coding using Model II. This is true for situations where all the categorical factors are dichotomous, but not so when at least one of the categorical factors has more than two levels. Under these conditions, the interpretation of the regression coefficients refers clearly to main and interaction effects of adjusted intercepts and slopes when Effect coding is used, but not when Dummy coding is used. In this case, the various effects describe cell contrasts or cell contrast/contrast interactions, etc., involving the category identified by all 0's. Attempting to interpret regression coefficients directly can be like walking in a field with landmines, if attention is not paid to the precise meaning of the coefficient.

Using SPSS GLM Univariate to Obtain the Model I Results.

It can be shown that one can perform an analysis of variance where one factor is continuous by using SPSS GLM Univariate. This is done by setting up the analysis as an analysis of covariance where the covariate is the centred continuous variable, and the factors are identified by their levels (i.e., not the dummy or effect codes, but the actual level numbers). For a 2X2 design, this would create a Syntax file for a 2X2 analysis of covariance. To change it to an analysis of variance with one continuous factor, paste the original run into the Syntax file, then open that file and add the appropriate interactions to the design statement, as indicated by the additions in capitals as shown in the design statement in the following Syntax file.

```
UNIANOVA
  x  BY a b  WITH c
  /METHOD = SSTYPE(3)
  /INTERCEPT = INCLUDE
```

```
/PRINT = PARAMETER
/CRITERIA = ALPHA(.05)
/DESIGN = c a b a*b A*C B*C A*B*C.
```

Running this file with these data presents the summary table for the analysis of variance shown in Table 5.

Table 5

Summary of Analysis of Variance with a Continuous Factor

Tests of Between-Subjects Effects

Dependent Variable: x

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	161.077[a]	7	23.011	7.461	.006
Intercept	4335.327	1	4335.327	1405.719	.000
c	.508	1	.508	.165	.696
a	5.268	1	5.268	1.708	.228
b	31.114	1	31.114	10.089	.013
a * b	97.805	1	97.805	31.713	.000
a * c	.007	1	.007	.002	.963
b * c	.508	1	.508	.165	.696
a * b * c	15.177	1	15.177	4.921	.057
Error	24.673	8	3.084		
Total	4608.000	16			
Corrected Total	185.750	15			

Examination of this table will show that the tests of significance are identical to those obtained with Effect coding for Model I, but are different from those obtained with Dummy Coding for Model I, emphasizing once again that if one is interested in the unique contributions of the various effects when using multiple regression, Effect coding is the one of choice.

It is interesting to note that when listing the parameters for this model, SPSS GLM Univariate presents the regression coefficients for Dummy Coding. These are presented in Table 6. It will be noted that these are identical to those presented in Table 2. That is, although SPSS GLM

Univariate uses Dummy Coding, it is able to obtain the correct *F*-ratios, though you can't if you try to do the same analysis using multiple regression.

Table 6

Regression Coefficients Obtained from SPSS GLM Univariate

Parameter Estimates

Dependent Variable: x

Parameter	B	Std. Error	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Intercept	15.113	.884	17.091	.000	13.073	17.152
c	.600	.555	1.080	.311	-.681	1.881
[a=1.00]	6.217	1.247	4.985	.001	3.341	9.093
[a=2.00]	0[a]	.	.	.	.	.
[b=1.00]	2.200	1.270	1.732	.121	-.729	5.129
[b=2.00]	0[a]	.	.	.	.	.
[a=1.00] * [b=1.00]	-10.092	1.792	-5.631	.000	-14.225	-5.960
[a=1.00] * [b=2.00]	0[a]	.	.	.	.	.
[a=2.00] * [b=1.00]	0[a]	.	.	.	.	.
[a=2.00] * [b=2.00]	0[a]	.	.	.	.	.
[a=1.00] * c	-1.916	.979	-1.958	.086	-4.173	.341
[a=2.00] * c	0[a]	.	.	.	.	.
[b=1.00] * c	-1.600	.962	-1.663	.135	-3.818	.618
[b=2.00] * c	0[a]	.	.	.	.	.
[a=1.00] * [b=1.00] * c	3.916	1.765	2.218	.057	-.155	7.987
[a=1.00] * [b=2.00] * c	0[a]	.	.	.	.	.
[a=2.00] * [b=1.00] * c	0[a]	.	.	.	.	.
[a=2.00] * [b=2.00] * c	0[a]	.	.	.	.	.

a. This parameter is set to zero because it is redundant.

On the Significance of the Regression Coefficients.

We will use the results from Model I, Effect coding to demonstrate how to present the results. In Table 1, it was shown that the following coefficients were significant:

1. Constant = 16.798. This is the unweighted mean of the intercepts. The *t*-test is significant

indicating that the mean intercept is significantly greater than 0. In most cases, there is not too much of interpretative value in this finding.

2. *eb* = -1.423. This is the value of the difference between the unweighted mean of the intercepts for B1 and the unweighted mean of the intercepts (15.376 - 16.798 = -1.422 (rounding)). This difference is significant, but since there are only two levels of B, it is equivalent to indicating that B1 is significantly less than B2.

3. *eaeb* = -2.523. This is the value of the interaction of the intercepts (13.438 - 17.384 - 15.375 + 17.798 = -2.524 (rounding)) because there is only 1 degree of freedom (i.e., it's a 2X2 table). Because the data were centred, these values are what would be predicted for the particular group for individuals scoring at the mean of the continuous variable. These values could be presented in graphical form just like means in traditional analysis of variance.

If they were it would be noted that for A1 the intercepts increase from B1 to B2, while for A2 they decrease from B1 to B2. If it were desired to perform tests of simple effects, one could do so much like performing tests of simple main effects in traditional analysis of variance. Alternatively, one could use the estimates of standard error from the four bivariate regressions and perform t-tests of the differences in the intercepts for the simple main effect or interest.

4. *eaebc* = .979. This is the three way interaction between A, B, and C, but since C is a continuous variable and there are only two levels of both categorical factors, the test of significance of this three-way interaction is a test of the two-way interaction of the slopes. That is, it is defined as 1.00 - (-.158) - 0 + (-.179) = .979. It must be remembered that each value is an index of the linear relationship between the dependent variable and the continuous variable for that particular AB condition. Because each categorical variable has only two levels, the test of significance of this coefficient is equivalent to a linear by linear by linear contrast in traditional analysis of variance. The values in the cells of the table (i.e., 1.0, -1.316, -1.00 and .600) are the four slopes that would be obtained if one were to perform bivariate regression analyses for each of the AB conditions. When presenting these values, it is typically the case that this is done in an AXB table, though they could be presented in a figure similar to that for intercepts. One reason for not doing this is that the values themselves refer to linear regressions (i.e., slopes) and it may be confusing for some readers.

Nonetheless, it could be instructive in this case because for A1, the slope of the dependent variable on the continuous variable is positive at B1 but definitely negative at B2 . For A2, on the other hand, the relationship between the dependent variable and C is clearly negative at B1 but slightly positive at B2. That is, the slopes do interact with the treatment conditions.

If it were desired, one could test the significance of each of the regression coefficients from 0 (note we already saw that this could be done by analysing the data for each group using bivariate regression and using the tests of significance from this analysis). Alternatively, one could conduct tests of simple main effects for pairs of slopes. One way to do this is to use the slopes and their standard errors from the bivariate linear regression (see, for example, McNemar, 1969, pp. 161). Another would be to follow the procedures recommended by Aiken and West (1991).

Conclusions.

The intent of this paper was to demonstrate the meaning of tests of significance for Models I and II when using Effect Coding and Dummy Coding. It was demonstrated that when the categorical factors all have two levels, the differences are very clear. Though not demonstrated here, the overall conclusions apply equally when any categorical factor has more than two levels. The following generalizations pertain:

1. One should never use Dummy Coding for Model I (the Unique Sums of Squares Approach) because it will provide incorrect tests of significance for all but the highest order effect. This is true regardless of the number of levels of the categorical factors, even though it was demonstrated here only by consideration of the tests of significance of the regression coefficients (which in this case are equivalent to the square roots of the *F*-ratios of the effects).

2. In Model II, both Effect coding and Dummy coding yield the same tests of significance at each step in the case where each categorical variable has only two levels. This is not the case when a categorical variable has more than two levels, even though the *F*-ratios for *R*-change within each step for each factor would be equivalent.

3, The estimates from Model II for both Effect coding and Dummy coding at each step are based on setting the effects at 0 for all higher order effects. That is, the values cannot be derived directly from the sample data and reflect the equivalent of predicted estimates from an analysis of covariance. In this data set, using Model II we would conclude that there is not a significant effect for the main effect of B, while for Model I and Effect coding we would conclude there is a significant effect of B. Both models would conclude that there is a significant AB interaction, though the estimates of the cell intercepts would differ slightly.

References

Aiken, L.S. & West, S. G. (1991). *Multiple regression: Testing and interpreting interactions.* Newbury Park, CA: Sage.

McNemar, Q. (1969). *Psychological statistics* (Fourth Edition). New York, NY: Wiley.

Appendix A

Sample Data Used for the Analyses

A	B	C	X
1	1	-1.19	12
1	1	.81	14
1	1	-.19	12
1	1	-.19	15
1	2	-19	21
1	2	-1.19	23
1	2	-.19	22
1	2	1.81	19
2	1	-.19	17
2	1	.81	16
2	1	-1.19	19
2	1	1.81	16
2	2	-2.19	14
2	2	.81	19
2	2	-1.19	13
2	2	1.81	14