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Analysis of Variance, Hierarchical Linear Modelling, and You

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Analysis of variance is a technique that permits you to determine whether the means for a series of conditions vary more than can be reasonably attributed to chance. This general statement applies to any analysis of variance procedure though, as you might realize, there are a few minor details involved. This discussion will focus on the single factor completely randomized design. In this form, the analysis of variance proceeds by calculating the relevant means, and using these means as the estimates of the means in the population. The variation due to error is the weighted mean of the cell variances, and the test of significance is the ratio of the mean square between groups to the mean square within groups. The underlying arithmetic model is referred to as ordinary least squares (OLS).

Hierarchical Linear Modelling is a technique that, at the simplest level, does exactly the same thing. That is, you could perform a Hierarchical linear modelling analysis where you had a number of groups, and you were interested in determining whether the means differed among the groups. This would be referred to as the "Persons within groups". Hierarchical Linear Modelling does not use the sample means directly, but instead estimates the population means and the variance of these means using maximum likelihood procedures. Thus, there are basically two differences between analysis of variance and hierarchical linear modelling. The major difference is that analysis of variance uses the sample means as estimates of the population means and uses these values to estimate the variance of the means, while hierarchical linear modelling estimates both the population means and the population variance of these means. The other is that analysis of variance uses an ordinary least squares procedure while hierarchical linear modelling uses maximum likelihood. Obviously, the tests of significance are different, even though in each case the question is whether the variation in the means in the population is something other than 0. This is demonstrated below using the data from Kirk (1995, p. 171). These data consist of four groups with a sample size of 8 observations per group.

Analyzing these data using SPSS Oneway

In analysis of variance, we can consider the Between groups factor fixed or random. In HLM it is treated as a random factor, so for comparability we will treat the between groups factor as random in our analysis using SPSS General Linear Model, Univariate Analysis. Following is the output for this analysis.

Tests of Between-Subjects Effects

Dependent Variable: x

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	Hypothesis	578.000	1	578.000	35.388	.009
	Error	49.000	3	16.333[a]		
b	Hypothesis	49.000	3	16.333	7.497	.001
	Error	61.000	28	2.179[b]		

a. MS(b)

b. MS(Error)

There are two tests of significance that might be of interest here. One is the test of the intercept. The F-ratio of 35.388 tests the hypothesis that the grand mean (4.25) differs significantly from 0. It has 1 degree of freedom in the numerator and 3 degrees of freedom in the denominator, and thus, the square root of this value would correspond to a t-test of 5.949. Often, this test is not of much interest. The other test for the between groups factor (b) is a test that the means vary more than can be reasonably attributed to chance. The F-ratio of 7.497 is significant indicating that it is reasonable to assume that the means in the population vary among themselves. That is, they are not all equal to the grand mean in the population. Because the factor is considered random, the test is technically a test of σ^2_b greater than 0.

Analyzing these data using HLM

HLM assumes that the groups factor is random, thus the test of variation among the groups is considered a random effect, while the test of the overall effect is considered fixed. The results for the Hierarchical Linear Modelling analysis performed using the HLM program (Raudenbush, Bryk, & Congdon, 2000) are as follows:

Final Estimation of fixed effects:

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx. df	p-value
For Intercept1, B0 Intercept2, G00	4.25	.714435	5.949	3	0.000

This part of the output tests the hypothesis that the intercept differs from 0. Note that in this case, the coefficient is in fact the estimate of the grand mean (4.25), and dividing this value by its standard error yields a single sample t-ratio of 5.949 with 3 degree of freedom (and note too its similarity to the results obtained in the analysis of variance).

Final estimation of variance components:

Random Effect	Standard Deviation	Variance Component	df	Chi-square	p-value
Intercept 1, U0 level-1, R	1.33017 1.47600	1.76935 2.17857	3	22.49180	0.000

This part of the output tests the hypothesis that the variance of the intercepts (i.e., means in this case) in the population is greater than 0. The estimate of the variance of the means is 1.76935, and this is tested with a Chi-square statistic with 3 degrees of freedom. The chi-square statistic is significant indicating that variance of the means in the population is greater than 0.

Comparing Analysis of Variance and Hierarchical Linear Modelling Analysis of these Data

These two approaches are concerned with testing the same hypothesis - - in this case, whether it is reasonable to conclude that the means differ in the population from which the samples were randomly drawn. They can yield very similar results, as in this example, or they can yield very different results. In the present example, the estimates of the population means are similar. The output for the two analyses yielded the following means:

Group	ANOVA Estimates (Sample Means)	HLM Estimates (Maximum Likelihood)
1	3.00	3.167
2	3.50	3.600
3	4.25	4.250
4	6.25	5.983
Grand Mean	4.25	4.25

The estimated means from the ANOVA are, of course, nothing more than the sample means, and are computed from the well known formula for the mean. The means for the HLM analysis are maximum likelihood estimates. They are not computed from a formula, but are instead estimated from the sample data along with the estimate of their variance in the population. It will be noted that the more extreme means in the ANOVA table are less extreme in the HLM table.

Obviously, these data analytic procedures are different and cannot be compared directly. For this data set, however, there are some interesting observations that can be made. Note, for example that we continually referred to variance of the means. If we were to compute the variance of the means estimated by the analysis of variance, we would obtain a value of 2.0415 for an unbiased estimate and 1.5312 for the biased estimate. The corresponding values for the maximum likelihood estimates are 1.5329 and 1.1496. It will be noticed that none of these values correspond to the variance of the means in the analyses - - nor should they.

Considering the analysis of variance as one involving a random factor, the expected mean square summary table is:

Source	Expected Mean Squares	Mean Square
Between (b)	$n\sigma_b^2 + \sigma_e^2$	16.333
Within (error)	σ_e^2	2.179

Thus, the estimate of the variance of the means from the analysis of variance is:

$$= \frac{n\sigma_b^2 + \sigma_e^2 - \sigma_e^2}{n} = \frac{16.333 - 2.179}{8} = 1.76925$$

It will be noted that this value is very similar to the value of the variance of the means estimated in the HLM analysis (1.76935). Of course, with other data, the values will not be this close, necessarily. The point is that the two procedures are testing comparable hypotheses. With analysis of variance, the tests make use of the sample values to make the test, while with HLM the tests of significance involve direct estimates of the population values based on maximum likelihood estimates.

Of course, hierarchical linear modelling is most frequently used in situations where one is concerned with the effects of some continuous variable(s) on an outcome variable while taking into account the fact that the subjects are members of groups. Extensions of this might also have values associated with the groups, and all of this analysis makes use of HLM. It can be shown that the points made here apply equally to any of these situations.

References

Kirk, R.E. (1995). *Experimental Design: Procedures for the Behavioral Sciences*. Pacific Grove, CA: Brooks/Cole.

Raudenbush, S., Bryk, T., & Congdon, R. (2000). *HLM 6 Hierarchical Linear and Nonlinear Modeling*. Scientific Softward International, Inc. www.ssicentral.com