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Completely Randomized Factorial Multivariate Analysis of Variance

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Purpose

To assess the effects of two or more factors in a completely randomized design analysis of variance with more than one dependent measure. It assesses the multivariate main and interaction effects, providing F-ratios in each case. SPSS presents four such tests, Pillais' Trace, Wilk's Lambda, Hotelling's Trace, and Roy's Largest Root. Pillais' Trace is generally preferred because it is less influenced by violations of assumptions. Often, multivariate analysis of variance is employed to control Type I error at nominal rates (i.e., .05) for each effect assessed, but this purpose has been shown to be incorrect (see Gardner & Tremblay, 2006 for a review of the issues). It also maximizes the possibility of identifying true multivariate effects by computing weighted aggregates of the dependent measures that maximally distinguish among levels of the treatment factor(s).

Applications

The archetypal design is where there exists two or more treatment factors where participants are randomly assigned to conditions and observations are taken on more than one dependent variable. It is also frequently used where at least one factor is based on a classification of respondents in term of some assumed prior individual difference variable (e.g., socio-economic status, gender, type of occupation, etc.).

General Rationale

Weighted aggregates of the dependent variables are obtained in such a way as to maximize the effects of interest. A weighted aggregate takes the form:

$$L_{abi} = C + w_1 X_{1abi} + w_2 X_{2abi} + \ldots + w_p X_{pabi}$$

where L_{abi} is the score for individual *i* in group <u>ab</u> on the weighted aggregate of the <u>p</u> dependent variables. The weights are determined based on the solutions of a series of determinantal equations for each effect. In each case, there are a number of aggregates that can be computed, depending on the univariate degrees of freedom for the effect and the number of measures.

Basic Mathematics

The Determinantal Equation for the main effects of one factor (i.e., A) is:

$$|SS_{within}^{-1}\ SS_A - \lambda I| = 0$$

This will produce a number of eigenvalues (λ), the number being the lesser of (<u>a</u>-1, <u>p</u>) where (<u>a</u>-1) equals the univariate degrees of freedom for the main effects of A, and p equals the number of dependent measures. SS^{-1}_{within} is the inverse of the SS_{within} covariance matrix and SS_A is the covariance matrix for the Between A effects. The value of each λ will equal the ratio of SS_A/SS_W for the aggregate score for the main effect of A.

To determine the weights, it is necessary to solve for the following equation:

$$[SS_{within}^{-1}\ SS_A\ -\ \lambda I][w]\ =\ 0$$

with the side condition that:

$$[w^T]\ MS_W\ [w]\ =\ 1$$

The corresponding equations for the main effects of B and the AB interaction take the above form using the appropriate covariance matrices and degrees of freedom estimates.

Assumptions

1. Independence of Observations

A basic assumption underlying analyses is that the observations are independent of one another within independent groups and between independent groups. Generally if subjects are randomly assigned to conditions and their scores on the dependent variables are not influenced by other individuals in the condition, this assumption will be satisfied. Violation of this assumption does, however, profoundly inflate Type I error rate. Procedures dealing with non-independence of observations are described for univariate tests by Myers and Well, 1991, pp. 327-329.

2. Multivariate Normality

A necessary, but not sufficient, condition is that each variable is normally distributed in each AB population. It is also necessary that any linear aggregate of the variables is normally distributed, and that all pairs of variables are bivariate normal (i.e., a scatter plot of any two variables will show the arrays to be normally distributed and the distribution to be normal around the regression line). Violation of this assumption has only a small effect on Type I error rates for moderate sample sizes (i.e., df error greater than 20-30) particularly if they are equal, although high kurtosis does have an equal effect on power.

3. Equivalence of Covariance Matrices

It is assumed that the within cell covariance matrices are equivalent in the population. Given p variables, this means that there will be a times b covariance matrices with p rows and columns with equal values in the corresponding cells for all pairs of groups. Of course, the sample values may not be identical, but it is assumed that they have equal values in the populations. Providing sample sizes are equal, the analysis is robust with respect to violations of this assumption except in very extreme cases. Unequal sample sizes and lack of equivalence of covariance matrices will affect the Type I error rate. If sample size is positively correlated with the generalized variance in the matrix (assessed in terms of the logarithm of the determinant for each group), Type I error rate will be underestimated (i.e., a conservative test) while the reverse is true for a negative correlation (i.e., a liberal test).

4. Null Hypotheses

There are three multivariate null hypotheses, each with a set of corresponding univariate hypotheses, one for each dependent variable.

Main Effects of A.

The univariate null hypothesis is that the means for any given dependent variable are identical in the population for all levels of A. This can be written as:

$$\mu_{a1} = \mu_{a2} = \mu_{a3} = \ldots = \mu$$

The corresponding multivariate null hypothesis is that the vector of means for each variable for each level of A is identical to that for every other level of A in the population.

Main Effects of B.

The null hypotheses for the main effects of B have the same structure as above.

Interaction of A and B.

The univariate null hypothesis for the AB interaction is:

$$\mu_{ab} - \mu_{a\cdot} - \mu_{\cdot b} + \mu = 0 \textit{ for all ab}$$

The corresponding multivariate null hypothesis is that the vector of such differences is the same for all AB combinations in the population.

Running SPSS GLM Multivariate

Consider the example by Tabachnick and Fidell (2001, p. 332). It is a 2 X 3 (AXB) completely randomized design with two independent variables, WRATR and WRATA. One factor (A) is a manipulation (Treatment vs. Control). The other (B) is a categorized individual difference variable, Degree of Disability (Mild, Moderate, Severe). To initiate the run, it is necessary to identify the two factors (A and B) and the dependent variables (WRATR and WRATA). Running the analysis results in both multivariate and univariate tests of significance.

Steps to Follow in Interpreting the Results

When conducting a multivariate analysis of variance, evaluate each multivariate hypothesis as follows:

1. If a multivariate main effect (A or B or both) is significant:

a. Evaluate the univariate effects using the univariate analysis of variance presented as part of the analysis.[1] For each significant univariate analysis of variance, do post-hoc contrasts of means. If the multivariate effect is not significant, do not consider the univariate tests.

2. If the multivariate interaction is significant:

a. Evaluate the univariate interactions for each variable and follow up the significant ones with post-hoc tests of simple main effects or treatment/contrast or contrast/contrast

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[1]It has been traditional to define the type I error rate for these tests as if only one dependent variable had been analyzed (e.g., .05), but the logic underlying this practice has been shown to be flawed and alternatives have been proposed (see Gardner & Tremblay, 2006 for a discussion).

interactions (but see footnote 1). If the multivariate effect is not significant, do not consider the univariate tests.

3. If it is a reasonable alternative, do step down analyses, ordering the variables in terms of their assumed contribution to the effects. Apply this only to effects that were significant at the multivariate level. Assuming there were three dependent variables, V1, V2, and V3, order them in terms of their presumed role. If it was expected that the major effect would be on variable V1, and that this might mediate the effects on V2 and perhaps V3, one would follow the univariate analysis of variance of variables V1 with analyses of covariance on variables V2 and perhaps V3. Main effect and interaction effects could be followed up with tests of the adjusted means.

Reference

Gardner, R. C. & Tremblay, P.F. (2006). *Essentials of Data Analysis: Statistics and Computer Applications.* London, ON: The University of Western Ontario