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Modal Interpretations and Relativity

Wayne C. Myrvold[1]

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A proof is given, at a greater level of generality than previous "no-go" theorems, of the impossibility of formulating a modal interpretation that exhibits "serious" Lorentz invariance at the fundamental level. Particular attention is given to modal interpretations of the type proposed by Bub.

KEY WORDS: modal interpretations; relativity; Lorentz invariance.

1. INTRODUCTION

Modal interpretations of quantum mechanics posit that the state vector obeys linear, unitary evolution at all times, and supplement the state vector with a set of possessed properties sufficiently rich to account for the occurrence of definite events at the macroscopic level, including definite outcomes of experiments, but sufficiently restricted so as to avoid a Kochen–Specker contradiction. The question arises whether this can be done within the restrictions imposed by special relativity. In a relativistic context, the notion of an instantaneous state of a spatially extended system must be replaced by the notion of a state on a spacelike hyperplane, or, more generally, a spacelike hypersurface. Since hyperplanes belonging to distinct foliations will intersect, we must ask whether the definite properties assigned to systems on these intersecting hyperplanes can be made to mesh in a coherent way.

In connection with this question, two important "no-go" theorems must be mentioned. Dickson and Clifton[(1)] proved that the answer is negative for a broad class of modal interpretations. Berndl *et al.*[(2)] adapted an argument of Hardy[(9)] to show that no theory that shares with the Bohm

[1] Department of Philosophy, University of Western Ontario, London, ON Canada N6A 3K7; e-mail: wmyrvold@uwo.ca

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theory the attribution of definite positions at all times to particles can have the probability distributions for these positions match the quantum-mechanical probability distributions along every foliation. The Dickson–Clifton proof relies on an assumption concerning the transition probabilities for possessed values, the assumption they call "stability," but, as Arntzenius[3] has pointed out, the stability requirement is dispensable and the core of the proof concerns the nonexistence of certain joint distributions yielding the appropriate Born probabilities as marginals. The proof in the present paper is, in a sense, a generalization both of the proof of Berndl *et al.* and of Arntzenius' version of the Dickson–Clifton proof.

Bub[4] introduced a class of modal interpretations that single out some observable R as having a definite value at all times; this class includes the theories discussed by Berndl *et al.*, for which the preferred observable is position. As Dickson and Clifton (Ref. 1, p. 36) point out, it is possible for such an interpretation to evade their argument via a suitable choice of preferred observable. The existing "no-go" theorems, therefore, leave it open whether a Bub-type modal interpretation can be relativistically invariant. The question we want to ask is: for a suitable choice of preferred observable R, can the attribution of definite values to R be made in such a way that the probabilities concerning these definite values are given by the Born-rule probabilities yield by the quantum-mechanical state *along every foliation*? As will be shown below, the answer is negative, provided that the preferred properties are *local* properties and provided that certain transformations of the quantum state are possible. No assumptions about transition probabilities for possessed values will be made.

2. THE PROOF

Consider two systems, S_i, $i = 1, 2$, which, during the times that we are considering them, are localized (at least within the approximations permitted by relativistic quantum field theory) within regions that are large compared to their Compton wavelengths but small compared to the distance between them. We do not assume that they are at rest with respect to each other. Let α and β be two hyperplanes of simultaneity for some reference frame Σ. Let p_i be a small region on α in which the system S_i is located, and let q_i be a region on β in which S_i located (see Fig. 1). We assume that the two systems are sufficiently far apart that p_1 is spacelike separated from q_2, and p_2 is spacelike separated from q_1. Let γ be a spacelike hypersurface containing q_1 and p_2, and let δ be a spacelike hypersurface containing p_1 and q_2.

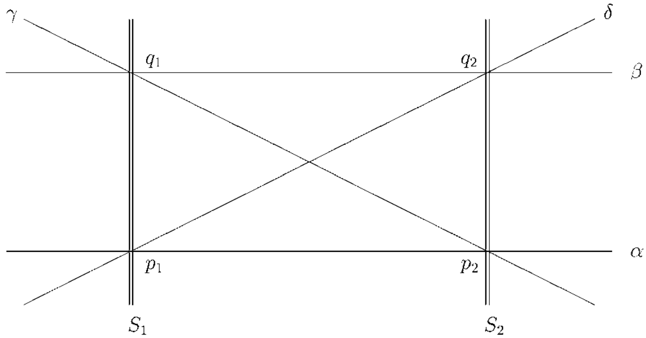

Fig. 1. The hypersurfaces used in the proof.

If S_1 and S_2 are isolated during the portion of their evolution between α and β, or if the parts of their environment with which they interact can be treated as effectively classical and these interactions are local, there will be unitary operators U_i such that the state of the combined system $S_1 \oplus S_2$ on β will be related to its state on α by,

$$\rho(\beta) = U_1 \otimes U_2\, \rho(\alpha)\, U_1^\dagger \otimes U_2^\dagger. \tag{1}$$

If the regions p_1, p_2, q_1, q_2 are sufficiently small, they may be treated as points, and we may regard γ and δ as hyperplanes of simultaneity for reference frames Σ', Σ'', respectively. Let $\rho'(\gamma)$ be the state according to Σ' of the system $S_1 \oplus S_2$ at $t' = t'_\gamma$, and let $\rho''(\delta)$ be the state according to Σ'' at time $t'' = t''_\delta$. We want to know how these states are related to the Σ-states.

Someone using Σ as a reference frame will judge that, if a measurement of an observable B_2 is performed on S_2 at time $t = t_\alpha$, and a measurement of an observable A_1 is performed on S_1 at time $t = t_\beta$, the expectation value of the product of the results of the measurements is

$$\mathrm{Tr}[\rho(\alpha)(U_1^\dagger A_1 U_1 \otimes B_2)]. \tag{2}$$

With respect to Σ', two such measurements occur simultaneously, at $t' = t'_\gamma$. The two reference frames must agree on the probabilities of the outcomes of the measurements. The expectation value of the product of the two measurements is, according to Σ',

$$\mathrm{Tr}[\rho'(\gamma)\, A'_1 \otimes B'_2], \tag{3}$$

where the operators A_1', B_2', are related to A_1, B_2 via the Lorentz transformation from Σ to Σ',

$$A_1' = \Lambda_1\, A_1\, \Lambda_1^\dagger, \qquad B_2' = \Lambda_2\, B_2\, \Lambda_2^\dagger,$$
$$A_1' \otimes B_2' = (\Lambda_1 \otimes \Lambda_2)(A_1 \otimes B_2)(\Lambda_1^\dagger \otimes \Lambda_2^\dagger) = \Lambda(A_1 \otimes B_2)\, \Lambda^\dagger. \tag{4}$$

(Although the argument here does not depend on the Lorentz transformation Λ being a factorizable operator, it can be proven[(1)] that it must, in fact, be factorizable.)

Since the two reference frames must agree on expectation values, we must have

$$\mathrm{Tr}[\rho(\alpha)(U_1^\dagger A_1 U_1 \otimes B_2)] = \mathrm{Tr}[\rho'(\gamma)\, A_1' \otimes B_2']. \tag{5}$$

A bit of algebraic manipulation yields

$$\mathrm{Tr}[(U_1 \otimes I_2)\, \rho(\alpha)(U_1^\dagger \otimes I_2)(A_1 \otimes B_2)] = \mathrm{Tr}[\Lambda^\dagger \rho'(\gamma)\, \Lambda(A_1 \otimes B_2)]. \tag{6}$$

Since this must hold for arbitrary A_1, B_2, we must have

$$(U_1 \otimes I_2)\, \rho(\alpha)(U_1^\dagger \otimes I_2) = \Lambda^\dagger \rho'(\gamma)\, \Lambda \tag{7}$$

or

$$\rho'(\gamma) = \Lambda(U_1 \otimes I_2)\, \rho(\alpha)(U_1^\dagger \otimes I_2)\, \Lambda^\dagger. \tag{8}$$

Similarly,

$$\rho''(\delta) = \Lambda'(I_1 \otimes U_2)\, \rho(\alpha)(I_1 \otimes U_2^\dagger)\, \Lambda'^\dagger, \tag{9}$$

where $\Lambda' = \Lambda_1' \otimes \Lambda_2'$ is the transformation from Σ to Σ''.

Now, the Lorentz boost operators Λ, Λ' merely effect a transformation from a state given with respect to one reference frame's coordinates to one given with respect to another reference frame's coordinates. In what follows, it will be more convenient to utilize the coordinate basis of one reference frame, Σ, for all states, even those on hypersurfaces that are not equal-time hyperplanes for Σ. We will therefore transform the states $\rho'(\gamma)$ and $\rho''(\delta)$ back into Σ's coordinate basis,

$$\rho(\gamma) = \Lambda^\dagger \rho'(\gamma)\, \Lambda = U_1 \otimes I_2 \rho(\alpha)\, U_1^\dagger \otimes I_2, \tag{10}$$
$$\rho(\delta) = \Lambda'^\dagger \rho''(\delta)\, \Lambda' = I_1 \otimes U_2 \rho(\alpha)\, I_1 \otimes U_2^\dagger. \tag{11}$$

For more general interactions of the system $S_1 \oplus S_2$ with its environment, the evolution of reduced state of the system will, provided that these interactions are local interactions, have a Kraus representation[5] consisting of factorizable operators (see Ref. 6 for a discussion):

$$\rho(\beta)=\sum_{m,n} K_{1m}\otimes K_{2n}\,\rho(\alpha)\,K_{1m}^{\dagger}\otimes K_{2n}^{\dagger}, \tag{12}$$

where

$$\sum_{k} K_{ik}^{\dagger}K_{ik}=I_i. \tag{13}$$

The corresponding states on γ and δ are given by

$$\rho(\gamma)=\sum_{m} K_{1m}\otimes I_2\,\rho(\alpha)\,K_{1m}^{\dagger}\otimes I_2, \tag{14}$$

$$\rho(\delta)=\sum_{n} I_1\otimes K_{2n}\,\rho(\alpha)\,I_1\otimes K_{2n}^{\dagger}. \tag{15}$$

Suppose that A_1 and A_2 are definite properties of S_1 and S_2, respectively, on α, and B_1 and B_2 are definite properties on β. If these are *local* properties—that is, properties possessed by the system irrespective of considerations of the rest of the universe—then the value of A_1 possessed by S_1 at p_1 is possessed by it without reference to the hypersurface containing p_1 being considered, and similarly for the other points of intersection p_2, q_1, q_2. (Indicating a particular outcome is, presumably, a local property of apparatus pointers. Being 100 km from New York City is not.)

We will require that the probability distributions for possessed values of local properties satisfy:

Relativistic Born Rule. For any spacelike hypersurface σ, if the quantum state of the combined system $S_1\oplus S_2$ on σ is $\rho(\sigma)$, and if X_1 and Y_2 are local definite properties of S_1 and S_2 on σ, then the probability that $X_1=x$ and $Y_2=y$ on σ is equal to $\mathrm{Tr}[P_{X_1}(x)\,P_{Y_2}(y)\,\rho(\sigma)]$, where $P_{X_1}(x)$ and $P_{Y_2}(y)$ are the projections onto the eigenspaces $X_1=x$ and $Y_2=y$, respectively.

Even if our modal interpretation is agnostic about transition probabilities, if the probabilities regarding the possessed values of the definite observables are to satisfy the Born rule on all four hypersurfaces, it must be possible for there to be a joint probability distribution over all four of our observables, that yields as marginals the Born probabilities on all four hyperplanes. Suppose, then, that there is such a distribution, $\Pr(a_{1i}, a_{2j}, b_{1k}, b_{2l})$, this being the probability that S_1 has $A_1=a_{1i}$ at p_1, S_2 has $A_2=a_{2j}$ at p_2, S_1 has $B_1=b_{1k}$ at q_1, and S_2 has $B_2=b_{2l}$ at q_2. We will make no assumption about this joint distribution other than that it yield the Born rule probabilities as marginals on all four hypersurfaces, α, β, γ, δ,

$$
\begin{aligned}
\sum_{k,l} \Pr(a_{1i}, a_{2j}, b_{1k}, b_{2l}) &= \mathrm{Tr}[P_{A_1}(a_{1i})\, P_{A_2}(a_{2j})\, \rho(\alpha)], \\
\sum_{i,j} \Pr(a_{1i}, a_{2j}, b_{1k}, b_{2l}) &= \mathrm{Tr}[P_{B_1}(b_{1k})\, P_{B_2}(b_{2l})\, \rho(\beta)], \\
\sum_{i,l} \Pr(a_{1i}, a_{2j}, b_{1k}, b_{2l}) &= \mathrm{Tr}[P_{B_1}(b_{1k})\, P_{A_2}(a_{2j})\, \rho(\gamma)], \\
\sum_{j,k} \Pr(a_{1i}, a_{2j}, b_{1k}, b_{2l}) &= \mathrm{Tr}[P_{A_1}(a_{1i})\, P_{B_2}(b_{2l})\, \rho(\delta)].
\end{aligned} \tag{16}
$$

Because of the relations between the states on the hyperplanes considered, the existence of such a joint distribution is equivalent to the existence of a joint distribution yielding, as marginals, the statistics in state $\rho(\alpha)$ for the observables $A_1 \otimes A_2$, $A_1 \otimes C_2$, $C_1 \otimes A_2$, $C_1 \otimes C_2$, where

$$C_i = U_i^\dagger B_i U_i \tag{17}$$

in the case of unitary evolution (1); in the case of non-unitary evolution (12), C_i is the "mixed observable,"

$$C_i = \sum_k K_{ik}^\dagger\, B_i\, K_{ik}. \tag{18}$$

It has long been recognized[7] that violation of a Bell inequality entails the nonexistence of such a joint distribution. If $\rho(\alpha)$ is a state such that a Bell inequality can be derived for the observables A_1, C_1, A_2, C_2, then, assuming the relativistic Born rule, it cannot be the case that A_1 is definite at p_1, A_2 is definite at p_2, B_1 is definite at q_1, and B_2 is definite at q_2.

Let us now apply these considerations to Bub's modal interpretation, which selects some observable R as always-definite. Let R_1, R_2 be always-definite observables of S_1 and S_2, respectively, such that the possession of any definite value of these observables is a local property of the system possessing it. We will assume that each R_i has at least two distinct eigenvalues, $\{r_i^+, r_i^-\}$. Let $\{|r_i^+\rangle, |r_i^-\rangle\}$ be corresponding eigenstates.

Suppose, now, that the system is prepared so as to be, on α, in the Hardy–Jordan state,[8]

$$|\psi(\alpha)\rangle = \frac{1}{2\sqrt{3}}\,(|r_1^+\rangle\,|r_2^+\rangle - |r_1^+\rangle\,|r_2^-\rangle - |r_1^-\rangle\,|r_2^+\rangle - 3\,|r_1^-\rangle\,|r_2^-\rangle). \tag{19}$$

Let us also assume that it is possible to effect a Hadamard transformation of the R-eigenstates,

$$
\begin{aligned}
U_i\,|r_i^+\rangle &= \frac{1}{\sqrt{2}}(|r_i^+\rangle + |r_i^-\rangle) \\
U_i\,|r_i^-\rangle &= \frac{1}{\sqrt{2}}(|r_i^+\rangle - |r_i^-\rangle).
\end{aligned}
\tag{20}
$$

Between α and β, we apply a Hadamard transformation to each system separately. The state on β of the combined system will then be given by

$$
|\psi(\beta)\rangle = \frac{1}{\sqrt{3}}(|r_1^+\rangle\,|r_2^-\rangle + |r_1^-\rangle\,|r_2^+\rangle - |r_1^+\rangle\,|r_2^+\rangle). \tag{21}
$$

The state on γ is

$$
\begin{aligned}
|\psi(\gamma)\rangle &= U_1 \otimes I_2\,|\psi(\alpha)\rangle \\
&= \frac{1}{\sqrt{6}}(|r_1^-\rangle\,|r_2^+\rangle + |r_1^-\rangle\,|r_2^-\rangle - 2\,|r_1^+\rangle\,|r_2^-\rangle).
\end{aligned}
\tag{22}
$$

The state on δ is

$$
\begin{aligned}
|\psi(\delta)\rangle &= I_1 \otimes U_2\,|\psi(\alpha)\rangle \\
&= \frac{1}{\sqrt{6}}(|r_1^+\rangle\,|r_2^-\rangle + |r_1^-\rangle\,|r_2^-\rangle - 2\,|r_1^-\rangle\,|r_2^+\rangle).
\end{aligned}
\tag{23}
$$

Suppose that, on α, R_1 and R_2 have the values (r_1^+, r_2^+). Since R_1 is, by assumption, a local property of S_1, S_1 must have the same value $R_1 = r_1^+$ on the hypersurface δ. The state (23) assigns probability zero to the pair of values (r_1^+, r_2^+), and so, on δ, R_2 must, with probability one, have the value r_2^-. Since R_2 is a local property of S_2, R_2 has the value r_2^- on β as well. A parallel argument leads to the conclusion that, if R_2 has the value r_2^+ on α, R_1 has the value r_1^- on β.

We therefore conclude that, if R_1 and R_2 have the values (r_1^+, r_2^+) on α, they have the values (r_1^-, r_2^-) on β. But, whereas (r_1^+, r_2^+) has probability $1/12$ on α, inspection of (21) shows that (r_1^-, r_2^-) has probability zero on β. Therefore, it is impossible to satisfy the Born-rule probabilities for possessed values of R_1 and R_2 on all four of the hypersurfaces α, β, γ, δ.

The above argument, as it stands, does not apply to those modal interpretations that use the Schmidt biorthogonal decomposition of the state to pick out the preferred observables. The argument can be made to apply with a simple modification. Associate with each of the systems S_i a second system A_i, among whose observables is a "pointer" observable with eigenstates $|p_i^{\pm}\rangle_{A_i}$ that can be made to interact with S_i in such a way that the values of the pointer observables become correlated with the values of R_i. Take the state of the system on α to be the state obtained from (19) by replacing $|r_i^{\pm}\rangle$ by $|r_i^{\pm}\rangle_{S_i}\,|p_i^{\pm}\rangle_{A_i}$. It is easy to check that the orthogonal decomposition of the reduced density operator for S_i is nondegenerate on all four hypersurfaces and yields R_i as definite properties on these hypersurfaces. The argument requires that we apply a Hadamard transformation to the combined system apparatus state,

$$\begin{aligned} U_i\,|r_i^+\rangle_{S_i}\,|p_i^+\rangle_{A_i} &= \frac{1}{\sqrt{2}}\left(|r_i^+\rangle_{S_i}\,|p_i^+\rangle_{A_i} + |r_i^-\rangle_{S_i}\,|p_i^-\rangle_{A_i}\right) \\ U_i\,|r_i^-\rangle_{S_i}\,|p_i^-\rangle_{A_i} &= \frac{1}{\sqrt{2}}\left(|r_i^+\rangle_{S_i}\,|p_i^+\rangle_{A_i} - |r_i^-\rangle_{S_i}\,|p_i^-\rangle_{A_i}\right). \end{aligned} \tag{24}$$

3. IDEALIZATIONS RELAXED

The above argument presumes that it is possible to keep the system isolated while performing a Hadamard transformation; this must be regarded as somewhat of an idealization, as no system is ever completely isolated from its environment. Bub[9], Sec. 5.2, has argued that the preferred observable should be stable with respect to environmentally induced decoherence. If this is the case, such decoherence will tend to turn coherent superpositions of distinct R-values into improper mixtures. Because of this the transformation invoked in the preceding section, which mixes distinct R_i-eigenspaces, may in practice be tremendously difficult. The issues with which we are concerned are, however, matters of principle; a theory that permits violations of the relativistic Born rule is not a relativistic theory even if situations that mandate such a violation are difficult to achieve in practice and the natural occurrence of such situations is extremely improbable. One might contemplate the possibility, however, of there being a limit in principle to the extent to which the system can be isolated from its environment; the always-definite observable might, for example, interact with the vacuum fields. We should, therefore, ask whether a version of the argument can survive such an ineliminable environmental interaction. We will still require that the relativistic Born rule be satisfied for arbitrary initial states, but will no longer assume that the system can be regarded as isolated while a Hadamard transformation is performed.

Suppose that we apply to S_i an external potential H_i. If H_i is much larger than the interaction of the system with its environment, then the evolution of the system will, for sufficiently short periods of time, be dominated by this term and will approximate the evolution that would obtain if there were no environmentally induced decoherence. It is therefore worth pointing out that a full Hadamard transformation is not necessary for a violation of the relativistic Born rule, and that this can be achieved, for a suitable initial state, by an arbitrarily small rotation of the state. To show this, we consider, not the Hardy–Jordan state, but the singlet state,

$$|\psi(\alpha)\rangle = \frac{1}{\sqrt{2}}(|r_1^+\rangle\,|r_2^-\rangle - |r_1^-\rangle\,|r_2^+\rangle). \tag{25}$$

Apply to the systems S_1 and S_2 potentials whose effect is to rotate the states in opposite directions:

$$H_1 = i\hbar\omega(|r_1^-\rangle\langle r_1^+| - |r_1^+\rangle\langle r_1^-|), \tag{26}$$

$$H_2 = -i\hbar\omega(|r_2^-\rangle\langle r_2^+| - |r_2^+\rangle\langle r_2^-|). \tag{27}$$

Take the time interval Δt between α and β to be sufficiently small that the effects of environmentally induced decoherence are negligible. We will then have the states on our other hypersurfaces given approximately by

$$|\psi(\gamma)\rangle = \frac{1}{\sqrt{2}}(\sin\phi\,|r_1^+\rangle\,|r_2^+\rangle + \cos\phi\,|r_1^+\rangle\,|r_2^-\rangle$$
$$-\cos\phi\,|r_1^-\rangle\,|r_2^+\rangle + \sin\phi\,|r_1^-\rangle\,|r_2^-\rangle), \tag{28}$$

$$|\psi(\delta)\rangle = \frac{1}{\sqrt{2}}(\sin\phi\,|r_1^+\rangle\,|r_2^+\rangle + \cos\phi\,|r_1^+\rangle\,|r_2^-\rangle$$
$$-\cos\phi\,|r_1^-\rangle\,|r_2^+\rangle + \sin\phi\,|r_1^-\rangle\,|r_2^-\rangle), \tag{29}$$

$$|\psi(\beta)\rangle = \frac{1}{\sqrt{2}}(\sin 2\phi\,|r_1^+\rangle\,|r_2^+\rangle + \cos 2\phi\,|r_1^+\rangle\,|r_2^-\rangle$$
$$-\cos 2\phi\,|r_1^-\rangle\,|r_2^+\rangle + \sin 2\phi\,|r_1^-\rangle\,|r_2^-\rangle), \tag{30}$$

where $\phi = \omega\Delta t$.

Let $R_i^+(x)$ be the proposition that R_i has value r_i^+ at spacetime point x, and similarly for $R_i^-(x)$. If there is a joint distribution over the possessed values of R_1 and R_2 on α and β, then we should have

$$0 \leqslant \Pr[R_1^+(p_1)\ \&\ R_2^-(q_2)] + \Pr[R_1^-(q_1)\ \&\ R_2^+(p_2)]$$
$$+\Pr[R_1^+(q_1)\ \&\ R_2^+(q_2)] - \Pr[R_1^+(p_1)\ \&\ R_2^+(p_2)] \leqslant 1. \tag{31}$$

Assuming that these probabilities are given by the Born rule, in our example this amounts to

$$0 \leqslant \cos^2 \phi + \tfrac{1}{2} \sin^2 2\phi \leqslant 1. \tag{32}$$

This is violated for $0 < |\phi| < \pi/4$, and hence for arbitrarily small ϕ.

4. LORENTZ INVARIANCE, SERIOUS AND OTHERWISE

"*Zur Elektrodynamik bewegter Körper*"[10] opens with the observation that electrodynamics, as it was understood at the time, leads to asymmetries in the theoretical description that are not present in the phenomena, in that the theoretical description distinguishes between bodies in motion and those at rest, in spite of the fact that the observable phenomena depend only on the *relative* motion of bodies. Such considerations, says Einstein, suggest that there is in fact nothing corresponding to absolute rest. He goes on in the paper to show how to reconcile electrodynamics with this suggestion; to do so involves rejecting the notion also that there is anything corresponding to absolute simultaneity of spatially separated events. The transformation between inertial coordinates, as measured by physical rods and clocks, must be given by the Lorentz transformation.

Now, it is certainly possible to suppose that there is a distinguished state of absolute rest; provided that this state is defined with respect to the matter in the Universe or some other physical structure, it is even possible for a theory that posits such a state to do this while preserving Lorentz invariance of the formulas of the theory. Similarly, a theory may introduce a preferred foliation in a Lorentz invariant manner. To do so, however, is to ignore the reasons why we should be interested in Lorentz invariance in the first place. The observable phenomena pick out neither a preferred rest frame nor a preferred relation of distant simultaneity. This is precisely what is to be expected if there is in reality no preferred state of rest and no distinguished relation of distant simultaneity, and so we hypothesize that this is, in fact, the case, and impose Lorentz invariance to ensure that an assumption of a preferred Lorentz frame is not concealed in our choice of coordinates. To introduce a preferred foliation in a Lorentz invariant manner is to abandon what Bell,[11] p. 180, calls "serious Lorentz invariance."

The "Lorentz-Covariant modal scheme" outlined by Dieks[12] evades the Dickson–Clifton proof by rejecting the Dickson–Clifton stability condition; it also evades the Arntzenius version of that proof, and the proof of the present paper, by rejecting the relativistic Born rule; on this scheme, the Born-rule probabilities do not give the probabilities for possessed values at all times for all foliations. Similarly, Dürr *et al.*[13] produce a covariant trajectory model by introducing, as part of the dynamical structure of the theory, a foliation with respect to which the "quantum equilibrium" condition $P = |\psi|^2$ is satisfied. As nothing in the observable phenomena depend on the particular choice of such a foliation, the distinguished foliation introduces into the theoretical description an asymmetry not present in the phenomena. The reasons for rejecting such a move, therefore, are precisely the same as the reasons for Einstein's dissatisfaction with a formulation of electrodynamics that invokes a preferred rest frame.

As Bell points out, we do not have a precise criterion for seriousness of Lorentz Invariance. It seems clear, however, that the relativistic Born rule should be satisfied by any interpretation of quantum mechanics with a claim to serious Lorentz invariance.

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