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Nonlinear adaptive control for multivariable chemical processes

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Abstract

Multivariable nonlinear models are common in chemical processes. Often many of the parameters change with time and some variables are not measured. To effectively control these processes, an adaptive scheme is developed here. This scheme combines the generic model control algorithm (GMC), with a nonlinear observer, which is able to track changes in the process model. This control scheme is shown to be quite robust due to the rapid convergence of the nonlinear observer coupled with integral action introduced by the GMC controller. Disturbance rejection is achieved with feedforward action and the combined observer/controller is easy to tune. Two examples demonstrate the effectiveness of this scheme. The first example is a simulation of an exothermic batch reactor where the algorithm is used for set point trajectory tracking and calorimetric estimation. The second example is a real-time application to a laboratory pressure tank, which is effectively controlled over a wide range. Both examples illustrate the ability of this nonlinear adaptive control strategy to provide good estimation and control of these nonlinear processes.

Keywords: Process model; Nonlinear control; Adaptive control; Parameter estimation; Observer design; Chemical processes

1. Introduction

The severity of the nonlinearities in chemical processes influences the selection of control algorithms for successful control of a process. Control strategies based on a linearized model may not yield satisfactory performance, if the process is subject to large disturbances or significant set point changes from, say, an on-line optimizer. In addition, the wide range of operating conditions encountered in start-up or shut-down of continuous processes and trajectory tracking of batch processes, in which there is the absence of a steady state, also pose an important challenge for the application of nonlinear control technologies. In recent years, a number of nonlinear control technologies have been developed, such as nonlinear control based on the differential geometric approach (Kravaris & Kantor, 1990), nonlinear model predictive control (Patwardhan, Rawlings, & Edgar, 1990), and generic model control (Lee & Sullivan, 1988). However, these approaches rely on the availability of a good process model, which is not always easy to obtain. In many reactors, for example, even the reaction kinetics are poorly understood, and lead to models with uncertain or time varying parameters. These cases are best handled with a nonlinear adaptive control strategy. Only limited studies have been reported concerning the development of nonlinear adaptive control strategies for chemical processes based on physical models. Henson and Seborg (1994) presented an adaptive nonlinear control strategy for a bench-scale pH neutralization system, which is obtained by augmenting a nonlinear controller (based on differential geometry approach) with a special indirect parameter estimation scheme (based on a recursive least-squares algorithm). Clarke-Pringle and MacGregor (1997) studied nonlinear adaptive temperature control of multi-product, semi-batch polymerization reactors. Their nonlinear adaptive controller consisted of a nonlinear controller (also based on the differential geometry approach) with an extended Kalman filter.

The purpose of this work is to develop an efficient and readily implementable nonlinear adaptive control strategy for a variety of chemical processes. It is recognized (Hua & Jutan, 2000) that among the reported nonlinear control technologies the generic model control (GMC,

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Lee & Sullivan, 1988) is relatively easy to apply, and has some important advantages which allow it to be used as a desirable framework for adaptive control. Generally, the on-line estimation of unknown parameters is critical to the success of the nonlinear controller over a broad range of conditions (Clarke-Pringle & MacGregor, 1997). Recently, Farza, Busawon, and Hammour, (1998) have successfully developed a simple nonlinear observer for the on-line estimation of the reaction rates in chemical or biochemical reactors. A main characteristic of this observer lies in the simplicity of its implementation and also in their calibration method. This nonlinear observer is extended here and is used as an on-line model parameter estimator for our case. A nonlinear adaptive control strategy is then obtained by combining a nonlinear GMC controller with a nonlinear parameter estimator.

Other difficulties in the control of batch reactors include the lack of direct measurements of the quality variables needed to be controlled, and unmeasured disturbances. Jutan and Uppal (1984) presented an interesting control method for solving these problems, which is called the "calorimetry method" (Schuler & Schmidt, 1992). In this method, only the energy balance is used in derivation of the control algorithm, in which the heat release term, coupled with the mass balance, is estimated on-line by easily available temperature measurements and the derivative of the temperature. They used simple numerical differentiation to get the derivative information, and designed a linear feedback controller with a feedforward controller to compensate for modeling errors. However, it may be difficult to apply this method to practical batch reactors because numerical differentiation is very sensitive to measurement noises, and linear control theory is not sufficient in many cases. There are many studies based on the calorimetry method (Schuler & Schmidt, 1992). Cott and Macchietto (1989) improved the performance of this control system by using a high-order difference equation for calculating the derivative and using GMC to design a nonlinear controller. In this work, we propose a nonlinear adaptive control strategy to improve control performance in batch reactors. Here, not only the heat released but also the heat-transfer coefficient, which is also a time varying parameter in many cases, are estimated on-line using a nonlinear observer.

Our nonlinear adaptive control scheme is applicable to both single and multi-input/output control problems. To illustrate its multi-input/output capabilities, the scheme is applied to a multivariable laboratory pressure tank.

This paper is arranged as follows. Firstly, a general form of nonlinear model, with a GMC control algorithm and nonlinear observer is outlined. The nonlinear adaptive control strategy is then developed in Section 2. The application of this strategy to a simulated batch reactor is shown in Section 3. Section 4 discusses an experimental application of this strategy to a laboratory pressure tank. Finally, some conclusions are given in Section 5.

2. Nonlinear adaptive control strategy

2.1. Control algorithm

Although there are several existing nonlinear control techniques, here we propose using the GMC controller. The formulation of the GMC controller is relatively straightforward, and its tuning is simple. Importantly, this approach has several advantages that are desirable for designing nonlinear adaptive control systems:

(1) The nonlinear (reduced) process model is directly incorporated in the control algorithm, allowing for the inherent nonlinearity of processes such as a batch reactor to be taken into account:
(2) The relationship between feedforward and feedback control is explicitly accounted for in the GMC algorithm:
(3) The GMC framework permits us to develop an adaptive control algorithm by updating parameters in the process model.

We consider a nonlinear system described by the following differential equation of the type:

$$\dot{x} = f(x,d)\theta + g(u,x,d),$$
$$y = cx, \tag{1}$$

where x is state vector of dimension n, u is input vector of dimension m, d the measurable disturbance vector of suitable dimension, y is the output vector of suitable dimension, and θ is model parameter vector of dimension n. In general, f and g are matrices of nonlinear functions. c is coefficient matrix. Here, we assume all states are measurable, and c is an unity matrix. According to the GMC basic principle (Lee & Sullivan, 1988), we derive the following control algorithm, which consists of three terms (dynamic process model, proportional action term and integral action term, respectively):

$$f(x,d)\theta + g(u,x,d) - K_1(y_{sp} - y)$$
$$-K_2 \int_0^t (y_{sp} - y)\,\mathrm{d}t = 0, \tag{2}$$

where y_{sp} is the set point of the output, K_1 and K_2 are diagonal $n \times n$ tuning parameter matrices. The control algorithm (2) is generally implicit. Hence, it is solved on-line by some iterative numerical method. If $g(u,x,d)$ is linear with respect to u, e.g. $g(u,x,d) = h(x,d)u$, then

Eq. (2) becomes an explicit control algorithm

$$u = (h(x,d))^{-1}(K_1(y_{sp} - y) + K_2 \int_0^t (y_{sp} - y)\,dt - f(x,d)\theta). \quad (3)$$

The values of the elements of K_1 and K_2 can be determined by the relationships (Signal & Lee, 1992)

$$K_{1(i,i)} = \frac{2\xi_i}{\tau_i}, \quad K_{2(i,i)} = \frac{1}{\tau_i^2}, \quad (4)$$

where ξ_i and τ_i determine the shape and speed of the desired closed-loop trajectory (the reference trajectory), respectively. The reference trajectory for a step change in the set point has a pseudo-second-order response. Yamuna and Gangiah (1991) showed that the formulae can be used to accurately calculate the specified response for any values of ξ_i and τ_i. When the values of ξ_i and τ_i, which correspond to the desired specified response, are selected, and K_1 and K_2 can be computed from Eq. (4).

2.2. Parameter estimation

In practice, there may be some mismatch between the model and the true process to be controlled. This process/model mismatch leads to deterioration in control performance. There are two general types of model mismatch: structural mismatch occurs when the process and the model are of different mathematical form; parameter mismatch occurs when numerical values of parameters in the model do not correspond with true values. Both structural and parameter mismatch affect controller performance. However, the effect of structural mismatch can be minimized if model parameters are regularly updated.

Here, model parameters in Eq. (1) are assumed to be time-varying. So, it is important to estimate the parameters on-line. There are some parameter estimation techniques, such as the least-squares method, and the Kalman filter. Here, however, the nonlinear observer (Farza et al., 1998) is used for parameter estimation. Also, we assume that the parameter dynamics in the nonlinear system Eq. (1) obey the following general first-order equation:

$$\dot{\theta} = \psi(u,x,d) + \varepsilon, \quad (5)$$

where ψ is a nonlinear function and ε is a function which may depend on x, θ, u, d, noise, etc. We assume that ε is an unknown but bounded function, and the disturbance d and its time derivative are bounded.

Rearrange the nonlinear system equations (1) and (5) in the following condensed form:

$$\dot{z} = \varphi(x,d)F(x,d)z + B(u,x,d) + \bar{\varepsilon},$$
$$y = Cz, \quad (6)$$

where

$$z = \begin{bmatrix} x \\ \theta \end{bmatrix}, \quad F(x,d) = \begin{bmatrix} 0 & f(x,d) \\ 0 & 0 \end{bmatrix},$$

$$B(u,x,d) = \begin{bmatrix} g(u,x,d) \\ \psi(u,x,d) \end{bmatrix}, \quad \bar{\varepsilon} = \begin{bmatrix} 0 \\ \varepsilon \end{bmatrix}$$

and $C = [I_n, \ 0]$, with I_n an $n \times n$ identity matrix. f and φ are, respectively, an $n \times n$ matrix and a scalar real-valued function which are differentiable and the corresponding partial derivatives are continuous.

According to Farza et al.'s (1998) method, the following nonlinear observer can be used to track the vector z:

$$\dot{\hat{z}} = \varphi(y,d)F(y,d)\hat{z} + B(u,y,d) - \varphi(y,d)\Lambda^{-1}(y,d)S^{-1}C^{\mathrm{T}}(C\hat{z} - y), \quad (7)$$

where

(1) $\hat{z} = \begin{bmatrix} y \\ \hat{\theta} \end{bmatrix} \in \Re^{2n}$, $\hat{\theta} \in \Re^n$ are the estimated vectors of state and parameter.

(2) $\Lambda(y,s) = \begin{bmatrix} I_n & 0 \\ 0 & f(y,d) \end{bmatrix}$.

(3) S is the unique symmetric positive-definite matrix which satisfies the algebraic Lyapunov equation

$$\lambda S + E^{\mathrm{T}}S + SE - C^{\mathrm{T}}C = 0, \quad (8)$$

where

$$E = \begin{bmatrix} 0 & I_n \\ 0 & 0 \end{bmatrix}, \quad S = \begin{bmatrix} \frac{1}{\lambda}I_n & -\frac{1}{\lambda^2}I_n \\ -\frac{1}{\lambda^2}I_n & \frac{2}{\lambda^3}I_n \end{bmatrix},$$

and $\lambda > 0$, λ is a design parameter.

It has been shown (Farza et al., 1998) that under the above conditions and when there exist finite real numbers α, β, α_h, β_h with $0 < \alpha \leqslant \beta$, $0 < \alpha_h \leqslant \beta$ such that for $t \geqslant 0$:

$$\alpha^2 I_n \leqslant f^{\mathrm{T}}(y,d)f(y,d) \leqslant \beta^2 I_n, \quad (9)$$

$$\alpha_h \leqslant |\varphi(x,d)| \leqslant \beta_h, \quad (10)$$

the convergence of the nonlinear observer can be guaranteed.

It can be seen from Eq. (7) that the tuning of the observer is reduced to the calibration of a single parameter λ. When $\varepsilon = 0$, the convergence of the observer error is exponential. In the case where $\varepsilon \neq 0$, the asymptotic error can be made arbitrarily small by choosing a sufficiently large value of λ. However, a very large value of λ may make the observer sensitive to noise. Thus, the choice of

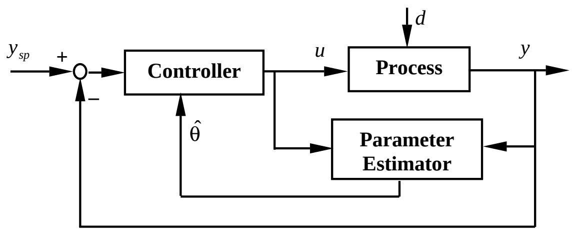

Fig. 1. Adaptive control system.

λ is a compromise between fast convergence and performance.

2.3. Adaptive control system design

The nonlinear adaptive control strategy is obtained by combining the nonlinear controller Eq. (2) with the nonlinear parameter estimator Eq. (7), as shown in Fig. 1. The adaptive control system design is straightforward, and tuning is simple. The two diagonal $n \times n$ matrices of K_1 and K_2 in the GMC controller Eq. (2) can be determined using Eq. (4), and need some tuning due to the fact there is an input saturation. The design parameter λ in the nonlinear estimator Eq. (6) can be tuned by trial and error. The single parameter tuning allows easy implementation of this observer.

It should be noted that this nonlinear adaptive control algorithm is applicable to cases where the number of parameters to be adapted is less than or equal to the number of controlled variables. This is not a serious restriction as the ability to adjust one parameter for each controlled variable to improve the controlled response is often sufficient.

In order to test out this nonlinear adaptive control algorithm, we apply it to two common nonlinear chemical processes.

3. Temperature control of a batch reactor

3.1. Batch reactor

The batch reactor in Cott and Macchietto (1989) is taken as an example. Their system has two parallel reactions, which occur in a jacketed batch reactor:

$$A + B \rightarrow C,$$

$$A + C \rightarrow D.$$

The control objective is to heat the reactor rapidly from a temperature of 20°C to its set point and to maintain the reactor at this set point. The result of this temperature profile is to maximize the desired product "C" and to minimize the production of the unwanted by-product "D".

The heat- and mass-transfer rates in the reactor are assumed to be high enough so that the system is essentially reaction rate limited. Therefore, the rate of production of C and D is only dependent on the reactant concentrations

$$R_1 = k_1 M_A M_B, \tag{11}$$

$$R_2 = k_2 M_A M_C, \tag{12}$$

where R_1 and R_2 are the production rates of C and D, respectively, and M_A, M_B, and M_C are the mole number of components A, B, and C present in the reaction at any given time. The rate constants, k_1 and k_2, are dependent on the reaction temperature through Arrhenius relation. Both reactions have a large heat of reaction ($\Delta H_1 = -41\,840$ kJ/kmol, $\Delta H_2 = -25\,105$ kJ/kmol), which makes the overall reaction system strongly exothermic.

Heating and cooling of the reactor contents is performed through the use of a single-pass jacket system. A temperature controller on the jacket inlet stream provides control of the jacket temperature.

They assume that the density of the reaction mixture is that of water. The nominal charge to the reactor is given as 360 kg of A and 1200 kg of B. The final reaction temperature is set to 95°C. The jacket temperature is limited to the range 20.0–120.0°C due to the heat-exchanger capacities, and the reaction mixture is constant at 20.0°C at time 0. Finally, because measurement errors are always present when working with real equipment, these were included in the simulation by adding noise to all temperature measurements. Here, a first-order moving average noise model was used to simulate the noise with a variance of 0.01°C.

Using a mass and energy balance, a detailed process model was developed. A full description of the reactor model and the values of the parameters used are given in Appendices A and B.

3.2. Temperature control based on calorimetry method

In general, we should design a nonlinear controller based directly on the process model given in Eqs. (A.1)–(A.6). However, due to lack of direct measurement of all the process variables and possible disturbances, control design for batch reactors requires additional consideration. Jutan and Uppal (1984) presented a calorimetry method in which the heat released in the reactor was estimated on-line by easily available temperature measurement and derivative information of the temperature. This concept allows just the energy balances around the jacket and reactor to be used for controller design.

It is commonly assumed that the amount of the heat retained in the walls of the reactor is negligible compared with the heat transferred in the reactor, and thus energy balances around the jacket and reactor are as follows:

$$\frac{\mathrm{d}T_r}{\mathrm{d}t} = \frac{Q + UA(T_j - T_r)}{WC_p}, \tag{13}$$

$$\frac{\mathrm{d}T_j}{\mathrm{d}t} = \frac{F_j\rho_j C_{Pj}(T_{\mathrm{jsp}} - T_j) - UA(T_j - T_r)}{V_j\rho_j C_{Pj}}, \tag{14}$$

where C_p and W are heat capacity and mass of the reactor contents (assumed constant), respectively, T_r is reactor temperature, T_j is jacket temperature, T_{jsp} is jacket temperature set point, A is the heat-transfer area, U is the heat-transfer coefficient, and Q is the heat released by the reaction, C_{pj} and V_j are heat capacity and volume of the jacket, respectively, F_j is flowrate, and ρ_j is density of the jacket contents. $Q = -\Delta H_1 R_1 - \Delta H_2 R_2$, which is a complex function of the reactor state variables such as M_A, M_B, M_C, and M_D, is estimated by the nonlinear observer due to the assumption of unknown reaction mechanisms.

In general, a conventional PID controller is used for jacket temperature control in the batch reactor. If the jacket temperature is considered as controller input, the formulation of a GMC controller for reactor temperature control is quite straightforward, and is based on Eq. (2). The reactor temperature, T_r, is the controlled variable, so we obtain the GMC controller as follows:

$$T_j = T_r + \frac{WC_p}{UA}\left(K_1(T_{\mathrm{rsp}} - T_r) + K_2\int_0^t (T_{\mathrm{rsp}} - T_r)\,\mathrm{d}t\right) - \frac{Q}{UA}. \tag{15}$$

However, Eq. (15) does not contain an explicit function of the jacket temperature set point, T_{jsp}, but instead the actual jacket temperature, T_j appears. In order to obtain the jacket temperature trajectory, some form of dynamic model of T_j was proposed (Cott & Macchietto, 1989). It is reasonable to assume that the dynamics of T_j be estimated by a nonlinear observer. The jacket temperature is approximately first order (Liptak, 1988) with time constant τ_j and, hence, the T_{jsp} is determined as

$$T_{\mathrm{jsp}} = \tau_j \frac{\mathrm{d}T_j}{\mathrm{d}t} + T_j, \tag{16}$$

where τ_j is the estimated time constant of the jacket. Hence, the solution of Eqs. (15) and (16) can be used to calculate the set point value for the jacket temperature.

3.3. On-line estimation of heat released and heat-transfer coefficient

In Jutan and Uppal's (1984) calorimetry method in which the heat released in the reactor was estimated based on temperature and derivative information, they used a simple first-order numerical difference approximation. It may be unwise to apply this method to practical batch reactors because simple numerical differentiation is relatively sensitive to measurement noise. Cott and Macchietto (1989) used a higher-order difference approximation for the derivative with improved results. In this work, we use a nonlinear observer to estimate heat released. This observer will have the appropriate filtering and tuning parameter available for estimating the heat released in the presence of noisy measurements. Since there may also be some fouling in the wall of the reactor during a run, the heat-transfer coefficient U may be time varying as well. It is thus important to track both these variables.

Models (13) and (14) can be rewritten as follows:

$$\begin{bmatrix} \dot{T}_r \\ \dot{T}_j \end{bmatrix} = \begin{bmatrix} \dfrac{1}{WC_p} & \dfrac{A(T_j - T_r)}{WC_p} \\ 0 & -\dfrac{A(T_j - T_r)}{V_j\rho_j C_{pj}} \end{bmatrix} \begin{bmatrix} Q \\ U \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{F_j(T_{\mathrm{jsp}} - T_j)}{V_j} \end{bmatrix}. \tag{17}$$

Our objective here is to estimate both the specific heat released Q and the heat-transfer coefficient U. Here, the temperature T_r and T_j are measured. The nonlinear observer for this batch reactor can then be developed by combining Eq. (7) with Eq. (17)

$$\begin{bmatrix} \dot{\hat{T}}_r \\ \dot{\hat{T}}_j \\ \dot{\hat{Q}} \\ \dot{\hat{U}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \dfrac{1}{WC_p} & \dfrac{A(T_j - T_r)}{WC_p} \\ 0 & 0 & 0 & -\dfrac{A(T_j - T_r)}{V_j\rho_j C_{pj}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{T}_r \\ \hat{T}_j \\ \hat{Q} \\ \hat{U} \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{F_j(T_{\mathrm{jsp}} - T_j)}{V_j} \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 2\lambda & 0 \\ 0 & 2\lambda \\ WC_p\lambda^2 & V_j\rho_j C_{pj}\lambda^2 \\ 0 & -\dfrac{V_j\rho_j C_{pj}\lambda^2}{A(T_j - T_r)} \end{bmatrix} \begin{bmatrix} \hat{T}_r - T_r \\ \hat{T}_j - T_j \end{bmatrix}, \tag{18}$$

where $\hat{T}_r$, $\hat{T}_j$, $\hat{Q}$, and $\hat{U}$ are estimates of T_r, T_j, Q, and U, respectively.

We verified that the convergence conditions for the nonlinear observer of the batch reactor are satisfied.

3.4. Simulation results

According to the tuning based on Eq. (4), we can easily choose the two parameters, K_1 and K_2, in controller algorithm Eq. (15), to follow a target profile of the controlled variable T_r. Appropriate values of these two constants are $K_1 = 0.0417\ \mathrm{s}^{-1}$ and $K_2 = 4.34 \times 10^{-8}\ \mathrm{s}^{-2}$. Here, the weight W, the mass heat capacity C_p of the reactor contents and the heat-transfer area A are 1560 kg, 1.8667 kJ/(kg °C), and 6.24 m^2, respectively.

In this batch reactor, the rate of change of the heat released, Q, and the heat-transfer coefficient U, are very different. Q changes rapidly with time, while U may change very slowly with time. As a consequence, it is better to estimate Q and U, using different tuning parameters, respectively. The parameter λ associated with Q was chosen as 0.02, while the parameter λ associated with U was chosen as 0.0033 after several trial and error iterations.

Fig. 2 shows the simulation results in the nominal case in which all the observer model parameters match the plant parameters. It can be seen from Fig. 2(a) that the estimate of the heat released is very satisfactory, and significant noise levels do not adversely affect the performance of the observer. Fig. 2(b) shows that the nonlinear observer also gives an excellent estimate of the heat-transfer coefficient. With these good estimates of the heat released and the heat-transfer coefficient, the GMC controller provided good control response as shown in Fig. 2(c).

The control system is then tested for robustness with respect to changes in process parameters or plant/model mismatch with the same controller and estimator parameters.

Fig. 3 shows the effect of decreasing the heat-transfer coefficient U by 26.6% to 0.5%. The GMC controller is able to still provide good control performance and the nonlinear observer quickly finds the new value for U. We also tested the controlled system by introducing a step disturbance change in the charge from the nominal 1560 –1300 kg. The results showed that the controlled system was robust and easily handled this change. The results, however, will not be given here due to space limitations.

The nonlinear adaptive controller/observer thus performed very well in this batch reactor profile control simulation. The second example involves real-time control of an experimental pressure tank.

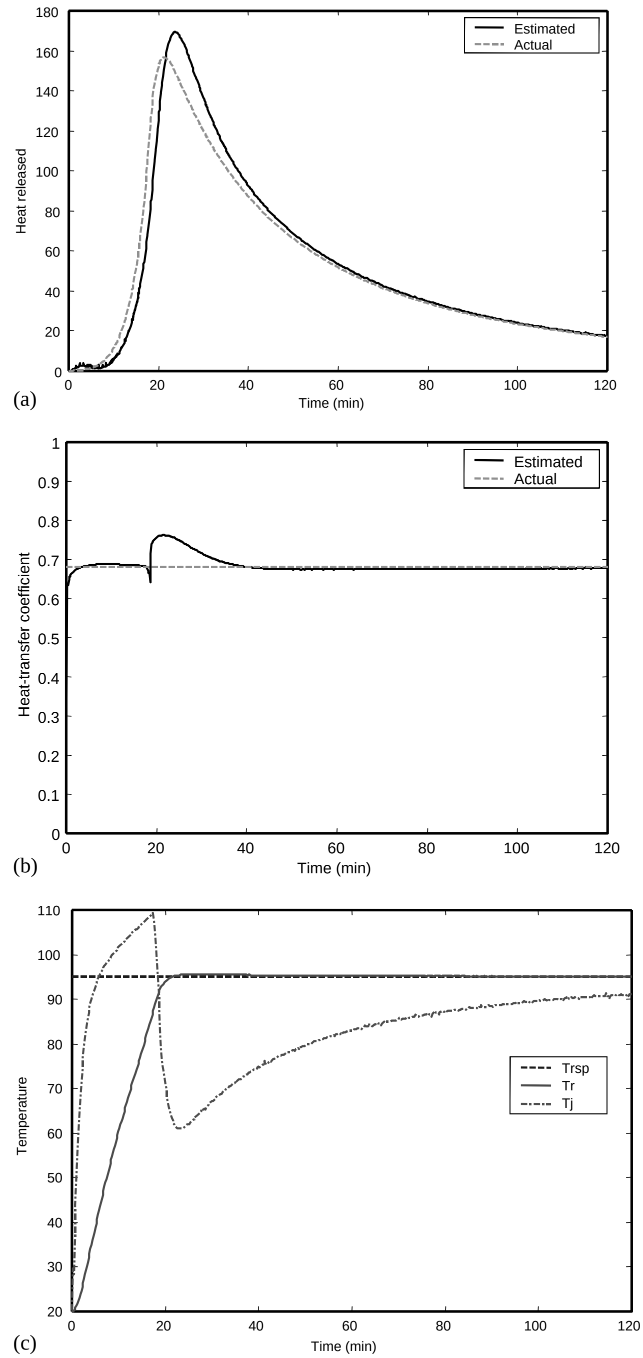

Fig. 2. (a) Estimates of heat released (nominal case). (b) Estimates of the heat-transfer coefficient (nominal case). (c) Response of the control system (nominal case).

4. Adaptive control of a pressure tank

4.1. Pressure tank

The laboratory system is a pressure tank (Fig. 4) through which the air flows from a regulated supply. Control valves are installed on both the inlet and the outlet of the tank. The pressure in the tank and the outlet flow rate are measured and transmitted to a computer. Data collection and system control are accompanied by use of a micro-computer with an input–output (I/O) interface board.

Here, the pressure in the tank and the outlet flow rate are controlled variables, and the control valve stem positions on both the inlet and the outlet of the tank are the manipulated variables. This is a two-input and two-output system. The variation in both inputs will influence both

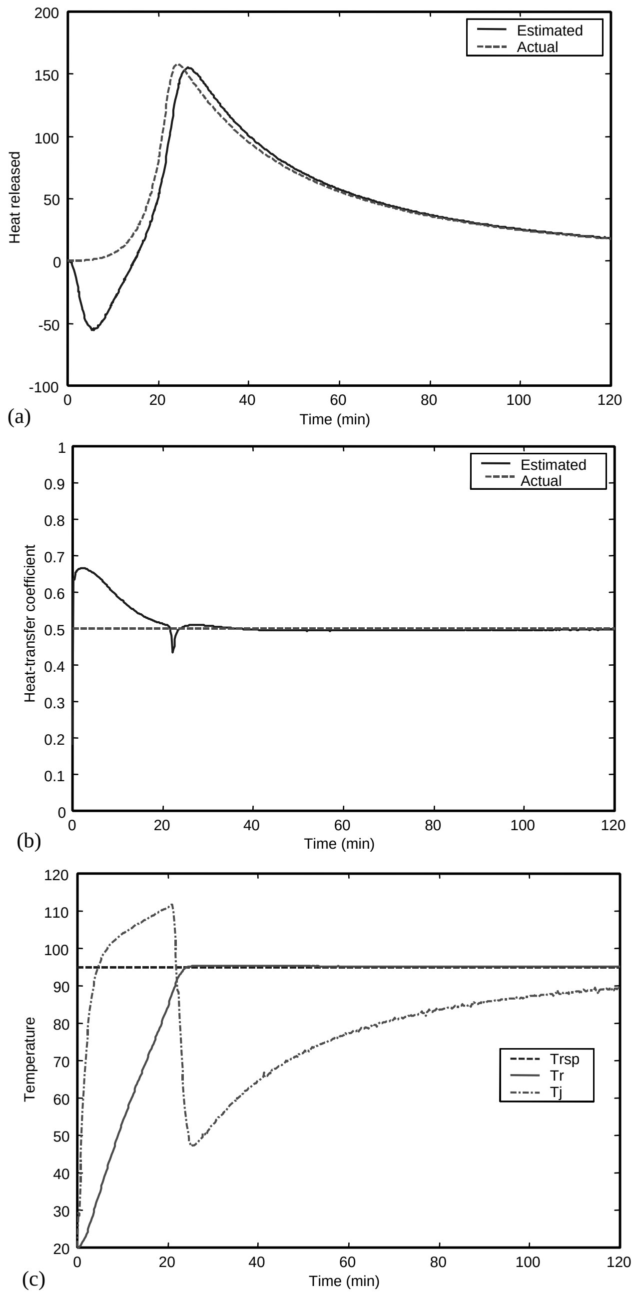

Fig. 3. (a) Estimates of heat released after a 26.6% step decrease in the heat-transfer coefficient. (b) Estimates of heat-transfer coefficient after a 26.6% step decrease in the heat-transfer coefficient. (c) Response of the control system after a 26.6% step decrease in the heat-transfer coefficient.

controlled variables. From the process design, it can be seen that this is a strongly interactive system. In fact, a relative gain analysis (RGA) of the process shows that the RGA elements are all close to 0.5. This implies that a control scheme based on multi-loop PID controllers would be very difficult to achieve (Zhu & Jutan, 1994).

Applying a mass balance to the pressure tank

$$\dot{m}_v = \dot{m}_i - \dot{m}_o, \tag{19}$$

where m_v, m_i, and m_o are the mass of air in the pressure tank, in the inlet and outlet, respectively. Assuming that air is an ideal gas, we can write

$$PV = \frac{m}{M} RT, \tag{20}$$

where P, V, m, M, R, and T are pressure, volume, mass, molecular weight, gas constant, and temperature of ideal gas, respectively. We assume that the control valves have a nonlinear characteristic governed by

$$\dot{m} = \frac{C_{V\,\max} l}{41616} \sqrt{\rho_a(-\Delta P)}, \tag{21}$$

where ρ_a and ΔP are density of air and differential pressure, respectively, and l is the valve opening. Substituting Eqs. (20) and (21) into Eq. (19) yields

$$\dot{P}_v = \frac{K_{vi} l_i}{V} \sqrt{\frac{RT}{M}} \sqrt{P_v(P_i - P_v)} - \frac{K_{vo} l_o}{V} \sqrt{\frac{RT}{M}} \sqrt{P_o(P_v - P_o)}, \tag{22}$$

where $K_{vi} = C_{vi\,\max}/41616$ is a flow parameter for the inlet valve, $K_{vo} = C_{vo\,\max}/41616$ is a flow parameter for the outlet valve, P_v, P_i, and P_o are the pressures in the pressure tank, inlet and outlet, respectively, and, l_i and l_o are the fractional openings of the inlet valve and the outlet valve, respectively. Applying Eqs. (20) and (21), we can obtain an expression for the volumetric flowrate at the outlet

$$\dot{V}_o = K_{vo} l_o \sqrt{\frac{RT}{M}} \sqrt{\frac{P_v - P_o}{P_o}}, \tag{23}$$

where $\dot{V}_o$ is the volumetric flowrate at the outlet.

4.2. Nonlinear adaptive controller for pressure tank

The controlled variables are pressure in the pressure tank and the volumetric flowrate at the outlet. The manipulated variables are the fractional openings of the inlet valve and the outlet valve. The relationship between the controlled variables and valve opening is nonlinear, moreover there is also severe hysteresis in the valves which is not directly modelled but would manifest itself as a change in K_{vi} or K_{vo}. This pressure tank represents a challenging multivariable real-time control problem for our algorithm.

The GMC controller for the pressure tank can be obtained by applying Eqs. (3), (22), and (23)

$$L = G^{-1}\left(K_1(y_{sp} - y_p) + K_2 \int_0^t (y_{sp} - y_p)\,\mathrm{d}t\right), \tag{24}$$

where L and y_p are the outputs of the controller and the pressure tank, respectively. K_1 and K_2 are tuning constant

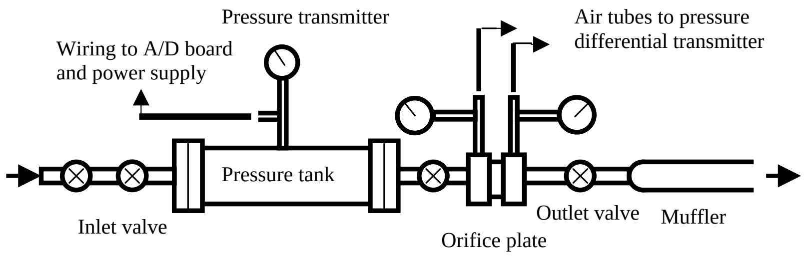

Fig. 4. Lab pressure tank.

matrices. So we have

$$L=\begin{bmatrix} l_i \\ l_o \end{bmatrix},\quad y_p=\begin{bmatrix} P_v \\ \dot{V}_o, \end{bmatrix},\quad K_1=\begin{bmatrix} k_{11} & 0 \\ 0 & k_{12} \end{bmatrix},$$

$$K_2=\begin{bmatrix} k_{21} & 0 \\ 0 & k_{22} \end{bmatrix},$$

$$G=\begin{bmatrix} \frac{K_{vi}}{V}\sqrt{\frac{RT}{M}}\sqrt{P_v(P_i-P_v)} & -\frac{K_{vo}}{V}\sqrt{\frac{RT}{M}}\sqrt{P_o(P_v-p_o)} \\ 0 & K_{vo}\sqrt{\frac{RT}{M}}\sqrt{\frac{P_v-P_o}{P_o}} \end{bmatrix}. \tag{25}$$

Due to valve hysteresis and sticking, the operating point is not easily reproducible, leading to poor repeatability in the dynamic data, and difficulties in model identification. This also causes the values of K_{vi} and K_{vo} to vary with the change of the inlet and outlet valve positions. Thus, for effective real-time control it is very important to estimate these two parameters. For the pressure tank, both the controlled variables of the system are measured. By applying Eq. (7) we can derive the nonlinear observer for estimating the K_{vi} and K_{vo} parameters

$$\begin{bmatrix} \dot{\hat{P}}_v \\ \dot{\hat{V}}_o \\ \dot{\hat{K}}_{vi} \\ \dot{\hat{K}}_{vo} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{l_i}{V}\sqrt{\frac{RT}{M}}\sqrt{P_v(P_i-P_o)} & -\frac{l_o}{V}\sqrt{\frac{RT}{M}}\sqrt{P_o(P_v-P_o)} \\ 0 & 0 & 0 & l_o\sqrt{\frac{RT}{M}}\sqrt{\frac{P_v-P_o}{P_o}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{P}_v \\ \hat{V}_o \\ \hat{K}_{vi} \\ \hat{K}_{vo} \end{bmatrix}$$
$$-\begin{bmatrix} 2\lambda & 0 \\ 0 & 2\lambda \\ \frac{V}{l_i}\sqrt{\frac{M}{RT}}\frac{\lambda^2}{\sqrt{P_v(P_i-P_o)}} & \sqrt{\frac{M}{RT}}\frac{P_o\lambda^2}{\sqrt{P_v(P_i-P_o)}} \\ 0 & \frac{\lambda^2}{l_o}\sqrt{\frac{M}{RT}}\sqrt{\frac{P_o}{P_v-P_o}} \end{bmatrix} \begin{bmatrix} \hat{P}_v-P_v \\ \hat{V}_o-V_o \end{bmatrix}, \tag{26}$$

where $\hat{P}_v, \hat{V}, \hat{K}_{vi}$, and $\hat{K}_{vo}$ are estimates of P_v, V, K_{vi}, and K_{vo}, respectively.

Again we verified that the convergence conditions for the nonlinear observer for the pressure tank are satisfied.

4.3. Experiment results

The real time control system is implemented in the MATLAB/SIMULINK environment. We tune the GMC controller based on choosing a target profile as shown in Eq. (4) for each of the controlled variables. We obtain the two tuning matrices for the GMC controller as well as a tuning parameter λ for the nonlinear observer, as shown in Table 1. Some fixed parameters for the pressure tank model are also given in Table 1. The sampling time Δt for the real time control system is 1 s.

We performed a real-time experiment in which the set point for pressure was changed from 11 to 20 psi at $t=250$ s and then back to 11 psi at $t=800$ s. The set point for flowrate is also changed from 30 to 50 ft^3/h at

Table 1
Process data and tuning parameters

$R = 8314.4$ J/kmol K	$P_o = 101$ kPa
$T = 25^\circ$C	$P_i = 377$ kPa
$M = 29$ kg/kmol	$V = 0.0215$ m^3
$K_{11} = 1.1111$ s^{-1}	$K_{21} = 0.1667$ s^{-1}
$K_{12} = 0.0086$ s^{-2}	$K_{22} = 1.929 \times 10^{-4}$ s^{-2}
$\lambda = 0.95$	$\Delta t = 1$ s

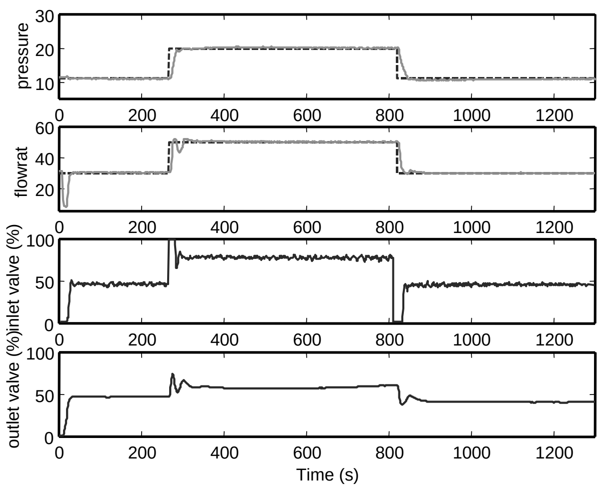

Fig. 5. Control results of pressure tank with set point change on both pressure and flowrate.

$t = 250$ s and then back to 30 ft^3/h at $t = 800$ s. This is intended to test the performance of the control system over different operating ranges.

Fig. 5 shows the control results for the pressure tank, in which step changes are applied to both controlled variables. Fig. 6 shows results of the related parameter estimation. Some control experiments are also performed, in which step changes are individually applied to the controlled variables of pressure and flowrate. The control results show the good decoupling performance of our controller. Figs. 7 and 8 give control results and parameter results when only the set point of pressure is changed, respectively. The control performance remains good, in spite of varying valve coefficients as the inlet and outlet valves change to different ranges. The control results demonstrate the ability of the adaptive controller based on an observer to control a nonlinear, interactive, multivariable process with time-varying process parameters.

5. Conclusions

In this paper, a nonlinear MIMO adaptive control strategy for chemical processes has been developed. The GMC is found to provide a framework for adaptive control of nonlinear processes. The nonlinear observer has been successfully applied to two nonlinear systems and is able to successfully track the time-varying parameters. The adaptive control strategy is easily implemented by combining the GMC controller and a nonlinear observer as an on-line parameter estimator. The control scheme has some important features, in particular, easy tuning of both the estimator and the controller. The adaptive control strategy was successfully applied to a simulated batch reactor and a laboratory pressure tank. Both examples demonstrate the effectiveness of this nonlinear adaptive control scheme in tracking time-varying parameters and providing good control.

Appendix A. Model equations for the batch reactor

$$dM_A/dt = -R_1 - R_2, \quad (A.1)$$

$$dM_B/dt = -R_1, \quad (A.2)$$

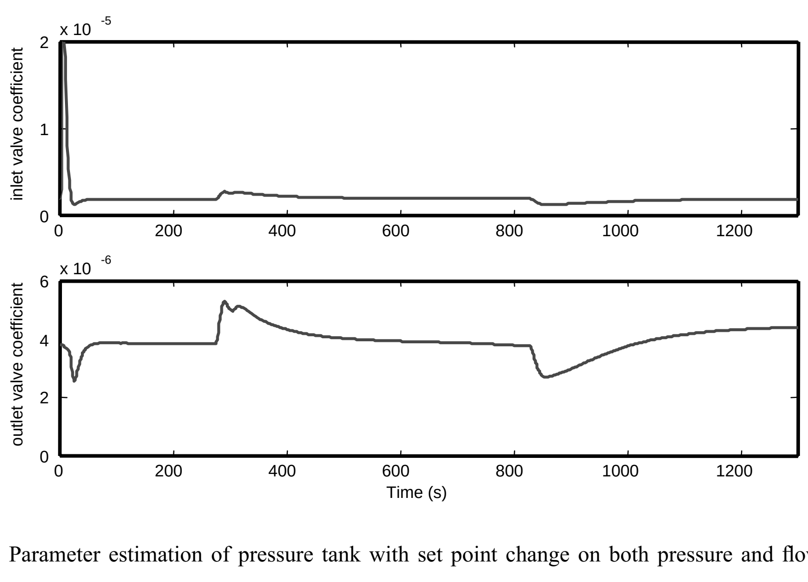

Fig. 6. Parameter estimation of pressure tank with set point change on both pressure and flowrate.

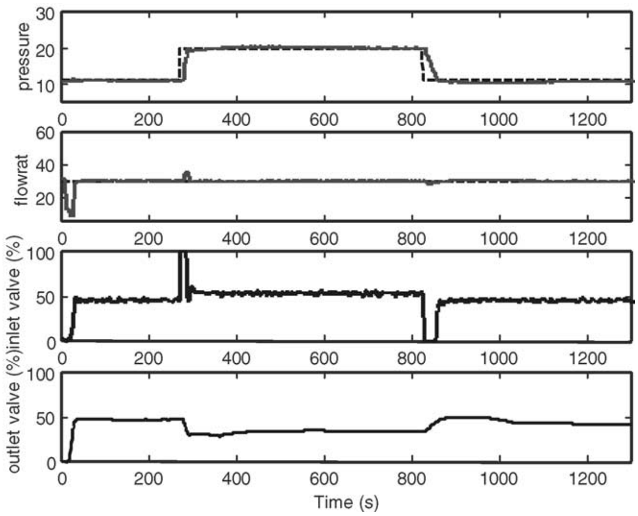

Fig. 7. Control results of pressure tank with set point change on pressure.

$$\mathrm{d}M_C/\mathrm{d}t = R_1 - R_2, \tag{A.3}$$

$$\mathrm{d}M_D/\mathrm{d}t = R_2, \tag{A.4}$$

$$\frac{\mathrm{d}T_r}{\mathrm{d}t} = \frac{UA(T_j - T_r) - \Delta H_1 R_1 - \Delta H_2 R_2}{M_r C_{pr}}, \tag{A.5}$$

$$\frac{\mathrm{d}T_j}{\mathrm{d}t} = \frac{F_j \rho_j C_{Pj}(T_{\mathrm{jsp}} - T_j) - UA(T_j - T_r)}{V_j \rho_j C_{Pj}}, \tag{A.6}$$

where

$$R_1 = k_1 M_A M_B, \tag{A.7}$$

$$R_2 = k_2 M_A M_C, \tag{A.8}$$

$$k_1 = \exp(k_1^1 - k_1^2/(T_r + 273.15), \tag{A.9}$$

$$k_2 = \exp(k_2^1 - k_2^2/(T_r + 273.15), \tag{A.10}$$

$$W = MW_A M_A + MW_B M_B + MW_C M_C + MW_D M_D, \tag{A.11}$$

$$M_r = M_A + M_B + M_C + M_D, \tag{A.12}$$

$$C_{Pr} = (C_{PA} M_A + C_{PB} M_B + C_{PC} M_C + C_{PD} M_D)/M_r, \tag{A.13}$$

$$V = W/\rho, \tag{A.14}$$

$$A = 2V/r. \tag{A.15}$$

Appendix B. Physical properties and process data for the batch reactor are given in Table 2

Table 2

$MW_A = 30$ kg/kmol	$k_1^1 = 20.9057$
$MW_B = 100$ kg/kmol	$k_1^2 = 10\,000$
$MW_C = 130$ g/kmol	$k_2^1 = 38.9057$
$MW_D = 160$ kg/kmol	$k_2^2 = 17\,000$
$C_{PA} = 75.31$ kJ/(kmol °C)	$\Delta H_1 = -41\,840$ kJ/kmol
$C_{PB} = 167.36$ kJ/(kmol °C)	$\Delta H_2 = -25\,105$ kJ/kmol
$C_{PC} = 217.57$ kJ/(kmol °C)	$\rho_j = 1000$ kg/m^3
$C_{PD} = 334.73$ kJ/(kmol °C)	$C_{pj} = 1.8828$ kJ/(kg °C)
$\rho = 1000$ kg/m^3	$F_j = 0.0058$ kg/s
$r = 0.5$ m	$V_j = 0.6912$ m^3
$U = 0.6807$ kW/(m^2 °C)	$A = 6.24$ m^2
Initial conditions	
$M_A^0 = 12$ kmol	$M_B^0 = 12$ kmol
$M_C^0 = 0$ kmol	$M_D^0 = 0$
$T_r^0 = 20$°C	$T_j^0 = 20$°C

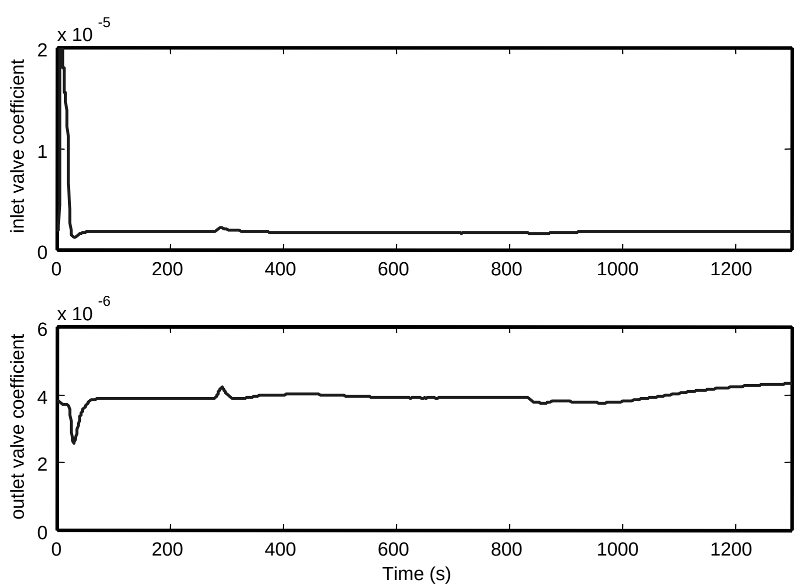

Fig. 8. Parameter estimation of pressure tank with set point change on pressure.

Notation

A	heat-transfer area, m^2
C_p	heat capacity of reactor contents, kJ/(kg °C)
C_{pi}	molar heat capacity of component i, kJ/(kmol °C)
C_v	control valve coefficient, $gpm/(psi)^{1/2}$
$C_{v\max}$	maximum control valve coefficient, $gpm/(psi)^{1/2}$
d	constant of nonlinear observer
k_i	rate constant for reaction i, 1/kmol/s
k_i^1	rate constant 1 for reaction i
k_i^2	rate constant 2 for reaction i
K_1	GMC controller constant or matrix 1
K_2	GMC controller constant or matrix 2
K_{vi}	flow coefficient for inlet valve
K_{vo}	flow coefficient for outlet valve
l_i	stem position for inlet valve
l_o	stem position for outlet valve
m	mass of air, kg
M	molecular weight of air
M_i	number of moles of component i, kmol
MW_i	molecular weight of component i, kg/kmol
P	absolute pressure, Pa
t	time, s
T	temperature, °C
Q	heat released in reactor, kw
r	radius of reactor, m
R	gas constant, J/kmol K
R_i	reaction rate of reaction i, kmol/s
u	manipulated variable
U	heat-transfer coefficient of reactor, $kw/(m^2\ °C)$
V	volume, m^3
$\dot{V}$	flowrate, ft^3/h
x	state
y	controlled variable
z	state

Greek letters

ΔH_i	heat of reaction for reaction i, kJ/kmol
ΔP	differential pressure, Pa
Δt	sampling period of GMC controller, s
θ	parameter
ξ	GMC tuning constant
ρ	density of reactor contents, kg/m^3
ρ_a	density of air, kg/m^3
τ	first-order time constant (s) or GMC tuning constant

Subscripts

1	reaction 1($A + B \rightarrow C$)
2	reaction 2($A + C \rightarrow D$)
A	component A
B	component B
C	component C
D	component D
i	order of matrix
j	jacket
r	reactor
sp	set point

Superscripts

ˆ	estimated
·	derivative
0	initial condition

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