The bibliography paradox revisited

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There is a well-known version of Russell's paradox concerning the bibliography of all bibliographies which fail to list themselves. The usual analysis of this paradox leads to the conclusion that such a bibliography is self-contradictory and so therefore cannot exist. However, as we show, a more searching analysis leads to a rather different conclusion.

Let us suppose that we are given two domains of things: titles and lists of titles, or bibliographies. We shall also suppose that each bibliography has been assigned a (unique) title. Now consider the bibliography $R$ which lists all and only titles of bibliographies which fail to list their own titles. Then $R$ lists its own title; for if not, then its title, call it $r$, has the property that there is a bibliography, namely $R$ itself, of which $r$ is the title, and which fails to list $r$. But, since $R$ lists all such titles, it must therefore list $r$. We conclude that $R$ lists its own title after all. Now since $R$ lists its own title $r$, it follows from the definition of $R$ that $r$ must also be the title of a bibliography $S$ which fails to list its own title. Then, $R$ and $S$ have the same title (namely, $r$) but they cannot be the same, since $R$ lists its own title but $S$ does not. We conclude therefore that there must exist two different bibliographies which are assigned the same title. And as we have seen, one of these bibliographies is $R$, which certainly exists.

Note the difference between the conclusion here and that of Russell's paradox. In the latter the conclusion is that there is a plurality which cannot be treated as a single thing. In the bibliographic case the conclusion is that, while each bibliography, considered as a plurality of titles, can be assigned a title - that is, in a certain sense treated as an individual - there will always exist two distinct pluralities which have to be treated as if they were the same thing. In both cases there are more pluralities than there are individuals.

This analysis of the bibliography paradox is (of course) a lightly disguised version of Cantor's theorem that, for any set $X$, there can be no one-to-one mapping of the power set of $X$ into $X$. 