Joel Bennhall: Scholar, Musician, Mathematician:

An Unfinished Portrait

Joel Bennhall was born in 1870 in Dublin. His father, Sean McDermot Bennhall, an engineer and amateur mathematician, had known Hamilton in his youth and had heard from Hamilton himself the story of his revelation in 1840 of the laws governing quaternions and his inscription of the same on Brougham Bridge. Bennhall's mother, Daniella Sophia Wilde Bennhall, a talented pianist and a passionate advocate of Irish independence, was a distant cousin of Oscar Wilde. Bennhall recalled having met Oscar in 1887 on the occasion of a rare visit by his mother’s famous cousin. While acknowledging Oscar’s brilliance, Sean had always regarded the man as fundamentally unsound, and discouraged his wife’s efforts to renew contact with him.

The name "Joel" was the result of an irony on his father's part. In his youth Sean had been a pessimist, and Sophocles’ dictum “Not to be born is, past all prizing, best” he took as axiomatic. But on meeting Daniella he was swept away, his pessimism dissolved by her zest, her beauty, her irreducibility, her otherness. But his pessimistic nature was momentarily reactivated when, a year or so after his marriage, Daniella suddenly announced her pregnancy. Sean's response was to quote the book of Joel, “The sun and the moon shall be darkened, and the stars shall withdraw their shining.” So it was that the child came to be christened Joel. But that was merely an irony, rather than a malediction, on Sean’s part, since he came to delight in his son. Nevertheless, in his later years he reverted to pessimism, committing suicide in 1892, leaving a considerable estate. He is known to have observed, long before Jean-Paul Sartre, that the greatest gift a father can give his son is his early death. And, in accordance with his principles, he followed through.

While JB cannot be said to have been a child prodigy in the received sense, he quickly showed himself to possess something more than the considerable gifts – linguistic,
musical, mathematical - his accomplished parents regarded as his birthright. On his 7th birthday he delighted his mother by sitting down at her Blüthner and launching into a passable rendition of Beethoven’s Für Elise. Around that time he startled his father by asking him, right out of the blue, “Daddy, why doesn’t dy/dx simplify to y/x by cancelling the d’s?” His father - unwilling, and, for all his authority as paterfamilias, probably not equipped to embark on a deconstruction of the intricacies of the differential calculus - was stymied. But later his mother rose to the occasion when young Joel stopped the conversation at dinner by suddenly piping up, as the blancmange was served, that the word “desserts” spelled in reverse was “stressed”. In his unpublished (and as he ruefully observed, unpublishable) memoir Total Oblivion Joel records, with a lingering pride in his mother’s perspicacity, that she, refusing to be upstaged by a mere child, retorted, “But, dear Joel, there’s no need to invoke the plural, since after all the reverse of ‘dessert’ is ‘tressed’, and am I not?” This incident, mercifully, was not to lead to Oedipal fixation on JB’s part, but to his enduring obsession with palindromes and the delightful tyranny of language. His later speculations on the nature of memory and his numerous unpublished works on the nature of reality can be traced to the same source.

Despite its Euclidean initial conditions of privilege, JB’s worldline was soon to display both curvature and torsion. The early deviations he experienced – the curvature- from the straightness of the trajectory prescribed for him by his parents blossomed in his mind into an enlarged conception of the world which would be suited to accommodate the twists and turns - the torsion - of his burgeoning imagination. As he later put it Total Oblivion “I had to find my own road to originality, overcoming the unspoken assumptions of my parents that I would extend their virtuous path into the future and at the same time rejecting the idea that my own trajectory would be orthogonal to theirs. I realized that the answer was to construct a new space in which my worldline would actually be skew to theirs.”
Chronology of JB

1870. Birth in Dublin. Privately educated by a series of tutors. In a letter to JB's father one of these tutors observes: "I enjoy teaching your son, whose intelligence and enthusiasm would delight any pedagogue. But it seems to me that - probably in an effort to impress you and your wife, as well as himself- he is actually working beyond his native intellectual ability." JB comes across this letter and is greatly disturbed by it, leading eventually to his consulting Freud in 1909.

1884. His parents send him to Copenhagen to study music with the aging Danish composer Dag Henrik Erum-Hellerup. Many years later JB reflects in Total Oblivion: "When I started my studies with Hellerup I must have been about 14, and he was well into his eighties. In those days children were brought up to revere their elders and I was no exception. I certainly respected Hellerup, and not merely for his advanced age. Nevertheless, I had begun, with adolescent pessimism, to wonder what was the point of aging? This may have been triggered by my learning of John Cay's epitaph: Life's a jest/And all things show it/I thought so once/And now I know it. Youth can grasp the pointlessness of aging, even if the concrete details are beyond it: no special capacity is required, just a certain inherent pessimism which, like one's eye colour, is a permanent feature of one's constitution. On the other hand, the fact that life furnishes no cosmic revelations, no gaudy epiphanies—the fact, in other words, that one's pessimism is finally confirmed—shows that this "pessimism" is absurd, just as a child's doubts that Santa Claus will turn up at Christmas are absurd (although these are touching). To expect nothing should not lead to pessimism! In any case one can have experiences resembling epiphanies—moments musicaux, for instance—which, while falling short of gaudiness, at least serve to moisten the eyes. But of course these have only local significance, since, in the final analysis, locality is all."
1889-92. Wins a scholarship to read mathematics at Trinity College, Cambridge. Bertrand Russell is one of his contemporaries, with whom he strikes up a friendship. Undergoes what he later termed "the customary undergraduate depression". As he reports in Total Oblivion: "Each of my days contained a surfeit of hours. Every morning the miracle of returning consciousness was quickly polluted by the certain knowledge that I would soon be staring into the void, unrelieved by a providential knock on the door by Bertie, who, with his talent to engage, would, I thought with a stab of envy, be engaging elsewhere. Out of sheer self-pity I would now and then succumb to this feeling of abandonment by lying face down on the floor of my rooms in Great Court. Normally I would quickly become bored with my self-posturing, get up, and reface the outer world. But on one occasion I was still lying prone on the floor when the gyp for my staircase, one Tubbs, came suddenly through the door and stumbled over me. 'Blimey, young sir,' he exclaimed. 'There you are, flat on the floor, and as far as I can tell, you ain't even drunk.' Which of course I wasn't - just trying to overcome ennui. Despite these self-indulgences JB manages to graduate as 11th Wrangler in the Tripos examinations. He is disappointed with this (aut Gauss, aut nihil!) and abandons the idea of an academic career. On receiving his inheritance from his father's estate, he becomes a wealthy idler, and moves to Paris.

1892-1903 His Paris years. He meets Huysmans, to whom he suggests the idea of writing a novel about Satanism, which leads to Huysmans writing Là Bas. He also befriends the youthful Paul Valéry, with whom he is delighted to find that he has much in common. As reported in Total Oblivion, in October 1892 he accompanies Valéry on a trip to Italy to visit relatives. On the night of 4 October 1892 Valéry wakes up from uneasy dreams, suffering from what he later describes as an existential crisis. He rouses JB, who proposes to him that he devote himself to la vie de l'esprit. JB suggests that a suitable way of doing this would be to get up around 5 each morning and jot down his waking thoughts in a journal. This would afford him the right to be stupid for the remainder of the day. Valéry adopts this precept and adheres to it for the rest of his life, eventually leading to the famous Cahiers. Later JB meets the great mathematician Henri Poincaré, and remarks to
him that if, as he says, mathematics is the art of calling different things by the same name, then, by the same token, poetry is the art of calling the same thing by different names. Poincaré replies that he's already heard that observation from Valéry. In October 1900 JB visits his distant cousin Oscar Wilde - not long before the latter's death at age 46 - who has been reduced to confinement in a shabby Paris hotel. In Total Oblivion JB reports that he succeeded in persuading Wilde to venture outside his prison to partake of an apéritif, over which Wilde declares, with (as JB describes it) "ironic despondency" My wallpaper and I are fighting a duel to the death. One of us has to go.

1904-1910. JB decides that his sojourn in Paris, while a delight to the senses, has been essentially frivolous. He resolves to press the musical talent of his youth into an effort to re-establish his nontriviality in his own eyes. Accordingly early in 1904 he travels to Vienna to study composition with Schoenberg, where he meets fellow-students Berg, Webern and Wellesz, Schoenberg's brother-in-law Alexander Zemlinsky, Gustav Mahler and his wife Alma, the child prodigy composer Erich Wolfgang Korngold, and the pianist Artur Schnabel. JB becomes intimate with Berg, whose urbanity, charm, self-irony and striking facial resemblance to JB's distant cousin Oscar Wilde, appeals greatly to him. He tells Berg about his final meeting with Wilde, and Wilde's wallpaper declaration. Berg's response: "How sad that such a genius be reduced to such degradation. But I hope that, should I eventually find myself in such dire circumstances, I would be able to make light of it so wittily" Many years later JB learns that his friend had acquitted himself admirably in that regard. For a few days before Berg's death of septicemia in December 1935 at the age of 50, he had received a blood transfusion, which resulted in a temporary improvement. He asked to meet the donor, an ordinary young Viennese, in order to thank him, remarking to a friend, who later became his biographer, "I hope I don't turn into a composer of operettas now!" But JB is also struck by a certain obsessiveness in Berg's character. As JB records in Total Oblivion, one evening, over a glass of the Niersteiner to which Berg was partial, he confides to JB that the he has developed what he describes as "an unhealthy fascination" with the number 23. JB, tongue in cheek, responds: "23 is
entirely harmless, an unremarkable number whose primality is its sole distinction. I would be seriously concerned, though, if you had told me that you had become partial to the number 37. Over and above its primality, this number has the diabolical property of being a factor of 666, the number of the Beast." JB reports his surprise at Berg's response: "Yes, it's true, I was initially attracted to 37, but I soon saw that it was a factor of 666, and so best avoided. I'm glad to hear that you regard my new numerical obsession with 23 as harmless." Berg later produces a number of compositions based on the number 23, including the Lyric Suite. JB also reports that Berg had been struggling throughout 1907 - 08 to finish his Piano Sonata (which was to become his opus 1). He had completed the first movement but, as he told JB, he was "bereft of inspiration" as to the composition of the two further movements he had originally envisaged. Struck by the beauty and originality of the one movement that Berg had completed, and at the same time envious of Berg's achievement (as he later admits in Total Oblivion) JB declares that it could stand alone, adding that, "the lack of appendages to Schubert's Quartettsatz and Unfinished Symphony has not impeded their progress in the musical world." Schoenberg, characteristically, puts the matter more bluntly, telling Berg that his inability to complete the work showed that he had said all he had to say. While JB is deeply impressed with Schoenberg's indomitable spirit and passionate conviction as a composer, he reports in a letter to one of his Irish cousins that he finds Schoenberg "something of a martinet, issuing orders to his students in the manner of a drill-sergeant. In AS's eyes, to disobey an order amounts to nothing less than a betrayal. He wants disciples, not students." Still, JB reports in Total Oblivion that Schoenberg's great expressionist works Three Piano Pieces, Op. 11 and Five Pieces for Orchestra, Op. 16, "came as a total revelation to me, as indeed they did to my fellow-students." The impact of these works leads JB to compose his Seven Episodes for Eleven Instruments and Chromatic Miniatures for Solo Harmonica.

In Total Oblivion JB reports that, during a tutorial held sometime in 1906, Schoenberg suddenly exclaimed: "To make progress in music you must feel as if you have suddenly been plunged into a bathtub of boiling water. If you do not feel this, you will never
produce anything but kitsch!" JB recalls this remark when, many years later, he learns of Schoenberg's account, in a 1947 speech in Los Angeles, of his life as a composer: "Personally I had the feeling as if I had fallen into an ocean of boiling water, and not knowing how to swim or to get out in another manner, I tried with my legs and arms as best as I could. I do not know what saved me; why I was not drowned or cooked alive. I have perhaps only one merit: I never gave up! But how could I give up in the middle of an ocean...." JB observes: "Poor Schoenberg: out of the bathtub and into the ocean! But I admire his courage since I myself have done little more than dangle a toe in that vast cauldron. Nevertheless even my hesitant probings have served to confirm to me the existence of Schoenberg’s boiling ocean, along with its temperature and turbulence."

In 1909 JB consults Freud about his feeling, placed in his youthful mind through his reading of his tutor's letter, and which continues to haunt him, that he is "working beyond his ability”. As JB records in Total Oblivion, "Upon being ushered into Freud's consulting-room, complete with the couch already well-known to the Viennese intelligentsia, he immediately asked me, with much the same directness I had come to expect from Schoenberg, 'So what can I do for you?' I told him, with what I felt was equal directness, that I had never been able to overcome the fear that I was "working beyond my ability." I was startled, and then delighted at his response. With a wave of his cigar, he said, "You should regard yourself as lucky. Those who are not truly working beyond what others characterize as their native ability will produce nothing but platitudes."

Many years later, Bennhall penned the following essay, which was eventually published in the Festschrift of John Lane Bell, a minor mathematician and philosopher, whom he had met in London not long before his death, and on whom he had made an indelible impression.
Chapter 26
Inscrutable Harmonies: The Continuous and the Discrete as Reflected in the Playing of Jascha Heifetz and Glenn Gould

Joel Bennhall

We owe the Pythagoreans the revelation that the harmonies of music derive from number, that is, from the discrete. This must be seen as a triumph of inscrutability. Inscrutable indeed is the resulting subsumption of music within mathematics, a colourless, forbidding subject to most, indeed the polar opposite of music, whose gaudy, yet profound, epiphanies offer a striking contrast. The Pythagoreans are to be blamed for the fact that music came to find itself in the embrace of such unlikely bedfellows as arithmetic, geometry and astronomy, the other members of the mathematical quadrivium.

Still, this evidence might cause a Marxist to respond that, while the basis of musical organization is to be located in the discrete, its means of production originate in the realm of the continuous. For is not sound itself nothing more or less than a continuous vibratory excitation of the atmospheric envelope, whether induced by blowing, plucking, or striking a tensed string, beating a drum, blowing down a tube, or straining one's vocal cords? The Pythagorean discovery, at bottom, is an instance of ex continuo discretem.

Musical instruments may be classed as continuous or discrete according to the manner in which individual notes are sounded. Thus the voice, bowed string instruments, and slide trombones are naturally identified as continuous, while valved wind instruments such as the clarinet or oboe, plucked or struck string instruments such as the lute or dulcimer, and keyboard instruments such as the harpsichord or piano may be classified as discrete.

J. Bennhall (ed.)
Musicologist and Composer

Editors' note: The musicologist and composer Joel Bennhall was briefly, when very young, a pupil of the Danish composer Dag Henrik Eerum-Hellerup. Later he studied with Schoenberg. His fascination with the relationship between music and mathematics led him to create topomusicological analysis, at one time a rival in musicological circles to Hans Keller’s better known functional analysis. In his old age Bennhall met the man celebrated in this volume at a meeting of the London Melomanicaal Society, at which he read the present paper; sadly he was prevented from publishing it owing to his sudden demise (at the age of 98). The Editors are pleased to provide it with a suitable resting place.

Accomplished vocalists and players of continuous instruments have considerable freedom both in determining the quality of individual notes and in the shaping of the “line” engendered by the succession of notes. This freedom is manifested above all in the case of the violin. In the violin the continuous and the discrete are truly united. For while the violinist’s bow is the source par excellence of continuous sound, of a variable intensity controlled by subtle alterations in pressure of the fingers on the strings, in the hands of a virtuoso that very bow is also employed to spectacular effect in engendering discreteness: witness, for example, spiccato, staccato and col legno bowing.

With the violinist’s left hand the order of continuity and discreteness is reversed, since the violinist’s digits are employed in the first instance to produce separate discrete notes through “stopping” the strings. But just as the bow can generate discreteness, so can the left hand generate continuity, e.g. through vibrato, the continuous minute oscillation of pitch of a single note\(^1\); the portamento, the subtle continuous movement from one note to another by gently gliding the finger along the string; and the shift, the violinist’s equivalent of the mathematician’s continuous change of coordinate system.

While the discrete instruments lack these refinements, they have one great advantage over their continuous counterparts, namely, their capacity to support simultaneity. A mere tyro on the guitar plays multiply voiced chords as a matter of course, while even a competent violinist may have difficulty in playing double-stops in tune. With the keyboard instruments this natural capacity to engender simultaneity has achieved its highest development in the polyphonic structures created by the composers of the Baroque period, and above all by J.S. Bach. Bach raises keyboard polyphony to undreamt-of heights, and certainly to a level far surpassing that achievable on any single stringed instrument. Even that most elaborate four-part fugue in the C major solo sonata cannot compare in complexity with the cyclopean edifice of the Art of Fugue!

Now let us turn to consider two modern masters of their respective instruments—Jascha Heifetz, the violinist whose technical command of the instrument is widely regarded as supreme—and Glenn Gould, the wizard of piano polyphony. The one, the master of the continuous, the other, the master of the discrete.

Jascha Heifetz—a name with which to conjure. For the present writer the name evokes a cluster of associations, each of which involves the continuous in one way or another. One recalls for example the famous exchange at the young Heifetz’s New York debut between two members of an audience packed with musicians eager to hear the new Wunderkind—Mischa Elman, great violinist, and Leopold Godowsky, equally great pianist. To Elman’s ingenuous observation, “It’s hot in here, isn’t it?”, no wittier response is conceivable than Godowsky’s “Yes, but not for pianists!” In this instance the discrete could afford to smile at the embarrassment of

\(1\) The trill—the rapid alternation of the main note with that a tone or semitone above it—is of course a discrete effect.
the continuous. (And yet it must be recalled that pianist-composers such as Liszt and Chopin were strongly influenced by the virtuosity of Paganini.)

Consider also Heifetz’s celebrated definition of a Russian: one Russian—an anarchist; two Russians—a game of chess; three Russians—a revolution; four Russians—the Budapest String Quartet. Here one sees a striking progression from the pure discreteness of the unit to the continuity of stringed instruments.

Even Virgil Thomson’s nastily dismissive description of Heifetz’s repertoire as “silk underwear music” brings the continuous to mind.

But in truth Heifetz was the supreme master of silkiness, indeed of that ultimate form of continuity that mathematicians call smoothness. This quality is best heard in his recordings of the 1930s and 1940s, most arresting in encore pieces and lightweight concertos such as those of Korngold and Gruenberg. In these latter works the silky smoothness of Heifetz’s tone—endlessly imitated but never duplicated by generations of violinists under his spell—almost surpasses belief.

Mathematicians have introduced the concept of a smooth topos, a mathematical “world” in which all correlations are arbitrarily many times differentiable, there are no jagged edges and in which, in Leibniz’s words, natura non facit saltus. There is no question that Heifetz would be the canonical fiddler in such a world.

But Heifetz was also a master of the discrete effects achievable on the violin; above all he could play detached notes on his preferred Guarnerius with blinding celerity. A remarkable example of his facility in this respect is provided by his recording of the Sinding Suite, in which the first, presto movement is despatched with truly hair-raising speed and accuracy. There could not be a more striking contrast between this glittering flurry of notes and the smooth, yet sweetly earnest and heartfelt manner in which Heifetz delivers the second, adagio movement.

The present writer did not have many opportunities to see Heifetz on the concert platform, but they remain treasured experiences. Especially memorable was the recital at the Royal Festival Hall in the 1950s at which Heifetz was due to begin with the Vitali Chaconne. This piece, familiar to all violinists, begins with a G minor chord, so when Heifetz picked up his violin and struck a G major chord the ranks of violinists occupying the first few rows of seats fell back in shock. Heifetz nonchalantly went on to play the English national anthem in G major.

Fortunately there are in existence a handful of filmed performances of Heifetz. One of the most remarkable of these is the rendition of that famed encore piece the Hora Staccato, transcribed by Heifetz from a Rumanian original. The writer once had the experience of hearing this exacting morsel played by a gypsy fiddler in an Amsterdam restaurant. While adequate, the performance could not compare with that of Heifetz, who contrives to make the staccato effect sound discrete but appear to the eye as continuous.

Now let us turn to Glenn Gould. While he was of course a master of the keyboard, with an unexampled command of polyphonic technique, one suspects that he may have envied the string player and the singer their immediate contact with the continuous. Grounds for this surmise are provided by his admitted inability to suppress the vocalise which invariably accompanied his piano playing, and which was such a source of vexation to critics and listeners alike.
On the other hand, for Gould, in the final analysis, polyphony was all, and polyphony, on the piano at least, is achieved by the systematic exploitation of discreteness. So it is reasonable to suppose that at some point Gould made the conscious decision to celebrate the discreteness of the piano, to avoid the mimicking of continuity by what he saw as contrived and hackneyed effects such as overpedalling and the gratuitous use of legato. Thus he strove for a secco, détaché sound, with each individual note rejoicing in its separateness. This approach is extraordinarily effective in Baroque works; and also with the twentieth century composers Gould most admired: Schoenberg, Webern, Krenek, Hindemith. In the present writer's opinion, the approach is also effective in Beethoven, especially in the early works of that master. With Mozart, however, the result is, to this writer's ears at least, nothing short of disastrous—but this was, of course, exactly what Gould, who disliked Mozart's music (with the conspicuous exception of the early sonatas K. 279–284 and the Fantasy and Fugue K. 394) was trying to achieve. Here Gould carried discrete deconstruction to the point of destruction.

It is to Gould's transcendent performances of works by composers he esteemed that one turns again and again. And above all, of course, to the compositions of J.S. Bach. Although Gould was most famed for his recordings of the Goldberg Variations—a fame that led this work to be identified by his fans as the "Gouldberg" Variations—the composition of Bach's he revered above all others was the Art of Fugue. And yet Gould produced no complete recording of this supreme, but alas, unfinished, masterpiece of the polyphonist's art. For this writer the most exciting rendition of any part of this work is Gould's 1967 Canadian radio broadcast of Contrapuncti IX, XI and XIII. Here Gould's playing achieves what can only be described as an ecstatic seamlessness fusing discreteness and continuity in an almost Hegelian Aufhebung.

Finally, we must consider the question of how Heifetz and Gould would have sounded had they played together. Would these supreme exponents of continuity and discreteness have achieved a harmonious union?

The vast majority of Heifetz's duo recordings were made with contract pianists—able, but somewhat colourless. An exception is the magnificent recording of Brahms' op. 108 sonata Heifetz made in the 1950s with the brilliant American pianist William Kapell (who died tragically young). Here the power of the pianist's playing comes close to matching Heifetz's, driving the latter to peaks even he did not always attain with his usual accompanists.

As for Gould, he made only a handful of recordings with violinists. One recalls the curious rencontre with Yehudi Menuhin during which Gould persuaded the violinist to play the Schoenberg Fantasy op. 47, a work to which, like all of Schoenberg's output, Gould was partial, but which Menuhin later said he found totally incomprehensible. In this connection it is pertinent to recall Heifetz's similar antipathy to Schoenberg's oeuvre. Heifetz actually commissioned Schoenberg's Violin Concerto op. 36 but on seeing the score instantly rejected it, giving the scarcely credible excuse that to play it would require him to grow a sixth finger. "I can wait," Schoenberg is reputed to have replied.
Gould did record the Bach violin and keyboard sonatas with Jaime Laredo, a good violinist, but who takes a back seat to Gould. The one string player who really stood up to Gould in duet performance was the cellist Leonard Rose, who, in their recording of Bach's sonatas for cello and keyboard gives a robust performance fully matching Gould's powerful rendition.

The upshot is that we can only imagine the sound of a Heifetz/Gould recital. One's musical tongue waters at the idea of recordings by these two masters of the Bach, Beethoven, or Brahms sonatas. The nearest approach we can make to this ideal is to listen to the pair of Bach violin concertos (in E major and A minor) and their keyboard transcriptions (in D major and G minor) as recorded, respectively, by Heifetz and Gould. It is a rare treat to hear Bach's sublime lines played first continuously, and then with discrete elaboration.

If only Heifetz and Gould had collaborated! That would have been the ultimate synthesis of the continuous and the discrete.