Quantum Incompatibility and Noncommutativity

Here is a way of looking at quantum incompatibility of properties, suggested by Basil Hiley, which brings noncommutativity into full focus.

We are given a bunch of objects each of which is either a sphere or a cube, and each of which is either red or blue. Thus the objects are classified under two attributes SHAPE and COLOUR. SHAPE further breaks down into the two properties **spherical** and **cubical,** and COLOUR into the properties **red** and **blue.**

We suppose that each of the four properties **spherical, cubical, red, blue** comes with a test for determining whether an object has the property in question. Thus, for example, if an object passes the test for **red,** it is deemed to have the property **red**; if not, it is deemed to have the property **blue;** in other words, it would have passed the **blue** test. We assume that, if an object passes a test for a given property, it will also pass a test for that property applied immediately afterwards (i.e. , without any intervening application of a test for a different property).

Now suppose that we test the objects for the property **spherical**, and assemble the objects passing the test into an ensemble **S.** Next, we subject the objects in **S** to the test for **red**, and assemble the objects passing this subsequent test into an ensemble which we shall write as **S ★ R.**

Now perform this procedure in the opposite order, first testing for **red** and then for **spherical.** The resulting ensemble will then, naturally, be **R ★ S.**

If the attributes SHAPE and COLOUR behave *classically*, we would expect the ensembles **S ★ R** and **R ★ S** to be identical, since then the operation ★ would simply correspond to the logical conjunction (∧) of properties (and of course ∧ is commutative). In other words, from a classical standpoint, ★ would be a commutative operation.

But in the quantum world ★ will not necessarily be a commutative operation, *and so will not necessarily correspond to the logical operation of conjunction*. For in the quantum world attributes such as SHAPE and COLOUR may be *incompatible*, that is, like position and momentum of electrons, not simultaneously ``testable`` . In that case **S ★ R** and **R ★ S** may *differ*, so that the ★ operation is *noncommutative.*

What this means is that, if we take an arbitrary object from **S ★ R** and subject it to the **red** test, it will always pass the test. But if we subject an arbitrary object from **R ★ S** to the same, **red**, test, sometimes it will not pass the test and *will then have to be deemed blue*, that is, it would have passed the **blue** test. In other words, **S ★ R** consists entirely of red objects, but **R ★ S** *will contain some blue objects.*

The startling nature of this conclusion can be made further evident by tracking the history of an object, *a,* in **R ★ S** which fails to pass the test for **red** and so is deemed **blue**. Now *a* initially passed the **red** test and so was, correctly, included in **R.** At this point *a* was definitely **red.** But then, for *a* to be included in **R ★ S,** it must subsequently pass the **spherical** test. Once *a* passes this test, it is then tested for **blue** and passes that test as well. In other words, applying the **spherical** test to the **red** object *a* has caused it to change its colour to **blue**!