What Broad Banks Do, and Markets Don’t: Cross-subsidization

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Abstract

We show that interbank markets are a poor substitute for “broad” banks that operate across regions or sectors. In the presence of regional or sectoral asset and liquidity shocks, interbank markets can distribute liquidity efficiently, but fail to respond efficiently to asset shocks. Broad banks can condition on the joint distribution of both shocks and, hence, achieve an efficient internal allocation of capital. This allocation involves the cross-subsidization of loans across regions or sectors. Compared to regional banks that are linked through well-functioning interbank markets, broad banks lead to higher levels of aggregate investment, higher output, and less fluctuations within regions. However, broad banks generate endogenously aggregate uncertainty.

Keywords: Banking Restrictions; Interbank Markets; Universal Banking; Endogenous Uncertainty

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1 Introduction

Restrictions on the geographical and sectoral scope of U.S. banks’ operations have been successfully relaxed over the last two decades. In the wake of these changes, a number of banks have extended their operations across states either via mergers or by opening branches. Similarly, a reduction in Euro-area regulatory barriers has led to increased cross-national banking. However, while interbank markets in Europe have largely become integrated, substantial barriers to a fully integrated European banking sector remain (Trichet (2004)). As a result, European regulators today face a similar question to what American regulators faced twenty years ago: If interbank markets can be freely accessed by banks, are there gains from removing restrictions on the scope of banks? The answer to this question depends upon whether interbank markets can replicate internal capital allocations within banks that operate broadly across regions or sectors. In other words, are anonymous interbank markets where banks can borrow funds from other banks when faced with unexpected liquidity or asset shocks a substitute for “broad” intermediaries which operate across regions or sectors?

We argue that interbank markets are a poor substitute for broad banks in the presence of regional or sectoral asset and liquidity shocks. We adopt the standard view of banks as intermediaries that provide liquidity to depositors and make risky loans to entrepreneurs. To secure investment funds, entrepreneurs must pledge collateral whose value is risky, since they cannot commit to repay loans. Whenever the return on entrepreneurs’ projects is not perfectly correlated with the value of collateral, the return on investment differs for depositors and entrepreneurs. In turn, regional shocks to collateral values and liquidity cause these differences in return (or wedges) to vary across regions. Regions with high liquidity needs and high collateral values have low wedges in return on investment, but require the withdrawal of funds to meet liquidity needs of lenders. Conversely, regions with low liquidity needs and low collateral values have large wedges in returns leading to the liquidation of investment projects and excess funds for investment in these regions. Interbank markets can distribute liquidity efficiently. This prevents the termination of projects due to liquidity needs in regions where the returns for depositors and entrepreneurs are similar. However, interbank markets fail to take into account the differential return on investment across regions. Broad banks that operate across regions can condition on the realized joint distribution of shocks to both
satisfy the liquidity needs of lenders and efficiently allocate investment across regions. This internal allocation of capital involves the cross-subsidization of some projects ex-post that have high returns to entrepreneurs, but relatively low returns to depositors. As a result, broad banks lead to higher levels of aggregate investment and higher output than regional or sectoral banks with well-functioning interbank markets.

To formalize this argument we analyze an environment with multiple “regions” and region-specific liquidity and asset shocks. The basic environment has five key features. First, lenders face stochastic liquidity needs (modeled as preference shocks) which generates a deposit feature. Second, entrepreneurs have projects with returns exceeding the return on a storage technology. Third, lending contracts are subject to enforcement frictions. We assume that the repayment of debt contracts is enforced solely by the threat of seizing a collateral good. This leads to the financing of entrepreneurial projects via collateralized debt. The fourth feature is that the value of collateral is stochastic. As a result, low realizations of the collateral value induce depositors to reduce or stop lending, since they will be unable to collect on their loans.¹ Finally, to capture the idea of “regions”, we replicate our basic environment \( N \geq 2 \) times. Motivated by the observation that bank lending and deposit taking tends to be regionally or functionally specialized, we assume that depositors and entrepreneurs in each region are spatially separated from their counterparts in other regions.

We analyze two banking structures. In an interbank market arrangement, banks in each region are restricted to taking deposits and making loans to entrepreneurs in their region. There is a competitive interbank market where banks from different regions can borrow and lend taking the interbank interest rate as given. Interbank borrowing is fully backed by collateral as the rights to seize collateral are transferred between banks. The second structure formalizes broad banks. Banks are able to operate branches in every region that take deposits in a region and make loans to entrepreneurs in the same region. Hence, banking activity is still local. However, banks are now able to directly transfer resources across branches or regions. When making transfers, banks are constrained by the threat of potential competition from region-specific banks.² We model this threat as an outside option for lenders and entrepreneurs

¹The assumption that collateral is only valued by the entrepreneur is made to simplify the analysis. What is essential is only that the entrepreneur value the collateral good more than anyone else.

²One interpretation is that banking is a “contestable” market in the sense that new regional banks can
to form a region-specific coalition to finance projects locally.

In the interbank market setup, banks in regions with sufficiently bad asset shocks terminate all projects, as the amount that can recovered from entrepreneurs is less than the return on the storage technology. These banks are lending on the interbank market. Banks in regions with a high realization of the asset shock wish to borrow on the interbank market so as to avoid the liquidation of projects. The allocation of funds within a broad bank is more complicated as it depends upon the joint realization of the asset and liquidity shocks. As in the interbank market, broad banks transfer funds from regions with bad asset shocks to regions with good asset shocks and high liquidity needs. However, the broad bank is able to extract some of the surplus from entrepreneurs on islands receiving liquidity loans and use it to cross-subsidize projects in regions with bad asset shocks that would have been terminated with liquidity provision through interbank markets. Here it is the threat of exclusion from liquidity motivated loans that induces depositors and entrepreneurs in a region to stay with the broad bank despite their outside option and to agree with the transfers to cross-subsidize projects in other regions. Since the cost of exclusion depends upon the joint realization of the shocks, a negative correlation between liquidity and asset shocks lowers the outside option and leads to higher transfers, more cross-subsidization and higher aggregate investment.\(^3\)

Our analysis generates a novel aspect for the relationship between aggregate uncertainty and banking restrictions. While broad banks reduce idiosyncratic regional risk, they also create endogenous aggregate uncertainty. The reason is that, with broad banks, the level of aggregate investment depends upon the extent of cross-subsidization which is a function of the realized joint distribution of asset and liquidity shocks. Purely idiosyncratic shocks across regions thus lead to endogenous aggregate fluctuations in the level of investment and output. With interbank markets and regional banks, investment depends solely upon regional asset shocks. Thus, idiosyncratic asset shocks lead to variations in the level of regional investment, but not aggregate investment.

The model has then two interesting testable implications based on the mechanism of freely enter the market place and offer contracts to local entrepreneurs and depositors.\(^3\)It is worth emphasizing that the relative inefficiency of interbank markets is not due to limited diversification, but limitations on cross-subsidizing regional investment.
allocating capital internally. First, broad bank lending within a region should respond less to regional asset shocks than that of regional banks. Second, the removal of barriers to broad banks should lead to a reduction in idiosyncratic output and investment fluctuations across regions. As we discuss in more detail at the end of the paper, these predictions are consistent with the existing empirical evidence on the presence and effect of internal capital markets within banks.

Closest in spirit to our environment is recent work by Diamond and Rajan (2001) and Kashap, Rajan and Stein (2002) that addresses the question of what makes banks “special” compared to other types of types of financial intermediaries and markets. Kashap, Rajan and Stein (2002) argue that banks exist to economize on liquidity. They show that when both borrowers and lenders are subject to imperfectly correlated liquidity needs that a bank is able to reduce the amount of reserves needed to provide for sufficient liquidity. Diamond and Rajan (2001) examine an environment with liquidity and asset shocks. They argue that the financial fragility of banks (whereby banks borrow short and lend long) helps banks to commit not to hold-up depositors. Their work is complementary to ours. We addresses the question of restricting the scope of bank operations whereas these other papers provide a rational for the existence of banks. We also differ from these papers in the way we model asset shocks. Building upon Kocherlakota (2001), we assume that limited enforcement requires lending to be secured with a risky collateral good.

Also related to our work are Battacharya and Gale (1987), Chari (1989) and Battacharya and Fulghieri (1994). These papers find that interbank markets cannot provide liquidity efficiently and achieve an optimal diversification of liquidity risk when a bank’s true liquidity needs or investment returns are private information. Apart from assuming full information, our work differs from these papers in two dimensions. First, we analyze a model where both stochastic liquidity needs and co-insurance against asset risk interact to shape interbank relationships. Second, we look at the question whether alternative arrangements such as broad banks could improve upon interbank markets.\(^4\)

\(^4\)Allen and Gale (1997, 2000) also argue that banks (or financial intermediaries) can improve upon market outcomes, but through a different channel. By accumulating reserves as a buffer against aggregate (generational) shocks, intermediaries are able to better smooth intertemporal consumption.
The remainder of the paper is organized as follows. The next section sets out the environment, concentrating on the contractual framework and the enforcement frictions. Section 3 analyzes the provision of liquidity via competitive markets, while Section 4 discusses the failure of the market solution in achieving efficiency and highlights the benefits of broad banks. Section 5 discusses the implications of our model and outlines empirical evidence which confirms our results. The last section concludes. All proofs are relegated to the appendix.

2 The Basic Model

The economy is composed of $N \geq 2$ locations or “islands” and lasts for three periods $t = 0, 1, 2$. Each location is populated by a measure one of both entrepreneurs and investors who face a basic financing problem with the possibility of default. Entrepreneurs have an investment opportunity with positive net present value, but have no funds. Investors can finance the project, but face random needs for liquidity. The key friction in the loan market is that entrepreneurs cannot directly commit to repay any borrowed funds. However, lenders can seize an entrepreneurs collateral good if they default on repayment of a loan. The value of this collateral is stochastic, which generates equilibrium default.

2.1 An Investment Problem with Default and Demand for Liquidity

Investors have a single unit of a consumption good and can access a storage technology with gross return of 1 at any time. With probability $\tilde{\theta} (1 - \tilde{\theta})$ lenders only value consumption at $t = 1$ ($t = 2$), where $\tilde{\theta}$ is a random variable with support $\Theta \subset (0, 1)$. Formally, preferences of lenders are defined by

$$u_L(c^1_{L,i}, c^2_{L,i}) = \begin{cases} c^1_{L,i} & \text{if type 1} \\ c^2_{L,i} & \text{if type 2} \end{cases}$$

where $c^t_{L,i}$ is the amount consumed by an investor at $t$ on location $i$. The random variable $\tilde{\theta}_i$ represents a location-specific liquidity shock a la Diamond and Dybvig (1983). A realization $\theta_i$ is interpreted as the fraction of investors on island $i$ that have a need for liquidity, since
they only value consumption at \( t = 1 \).

Entrepreneurs are endowed with an investment project and an indivisible special good, called collateral. Their project must be initialized at \( t = 0 \) and be financed continuously through \( t = 2 \) to yield any return. An amount \( x \in [0, 1] \) of the consumption good invested in the project at \( t = 0 \) yields a deterministic return of \( Rx \) units of the consumption good at \( t = 2 \).\(^5\) If funds are withdrawn from the project at \( t = 1 \), the project is terminated and yields no return at \( t = 2 \). New investments at \( t = 1 \) are not productive.

Collateral is special since only the individual entrepreneur derives utility from it.\(^6\) With probability \( \tilde{\pi} \) \((1 - \tilde{\pi})\) the collateral’s value is \( V(0) \), where \( \tilde{\pi} \) is a random variable with support \( \Pi \subset (0, 1) \). We assume that entrepreneurs consume only at \( t = 2 \), and preferences at \( t = 2 \) depend on the realized value of collateral. Preferences are given by

\[
    u_B(c_{B,i}^V, c_{B,i}^0) = \begin{cases} 
    c_{B,i}^V + V & \text{if collateral has value} \\
    c_{B,i}^0 & \text{if collateral has no value}
    \end{cases}
\]

where \( c_{B,i} \) denotes the entrepreneurs’ consumption of the good.

In our environment, collateral plays an essential role due to an enforcement problem. Entrepreneurs cannot commit to payments to lenders at \( t = 2 \). They can, however, pledge collateral which investors can seize in the event of non-repayment of loaned funds. Even though lenders do not value collateral, it allows entrepreneurs to credibly commit to repay loans, since losing valuable collateral is costly. Making the value of the collateral stochastic for entrepreneurs creates a default problem where the random variable \( \tilde{\pi} \) represents a location-specific collateral or asset shock. Assuming a law of large numbers, a realization \( \pi_i \) is the fraction of borrowers with valuable collateral on island \( i \). It is this default problem together with the random need for liquidity that is essential for our results, whereas the particular

\(^5\) Note that total investment per project is capped at 1. This is without loss of generality as all projects on all islands are indistinguishable as of period \( t = 0 \).

\(^6\) The assumption that collateral is only valued by the entrepreneur is made to simplify the analysis. What is essential (as in Lacker (2001) and Bester (1985)), is only that the entrepreneur value the collateral good more than anyone else. This is often the case in small and medium business lending where personal (or business related) assets are used to secure loans from banks. Such asset have a larger value for the entity posting the collateral than for a lender that cannot use the collateral directly.
assumptions concerning the valuation of the collateral are not.\footnote{For a detailed discussion, see Section 3.}

\section*{2.2 Timing and Uncertainty}

To simplify the analysis, we assume that the random variables $\tilde{\theta}$ and $\tilde{\pi}$ can each take on $N$ different values. Furthermore, we assume that each value occurs exactly once. A state $s$ of the economy is thus given by an assignment of the $N$ values for each random variable to the $N$ islands. In other words, a state is a joint distribution of the $N$ values for both random variables across the $N$ islands. One can assume without loss of generality that all states occur with equal probability. Hence, there is no aggregate uncertainty per se and individual islands face only idiosyncratic shocks. We relegate the formal construction of the appropriate probability space to the Appendix.

We can now summarize the timing of the model. At $t = 0$ investors on island $i$ invest $x^0_i$ on their island and store the remaining goods $(1 - x^0_i)$. This initial investment can be seen as a loan commitment to entrepreneurs, made before information about the liquidity and asset shock is known.

At $t = 1$, both shocks, $\tilde{\pi}_i$ and $\tilde{\theta}_i$, are realized for each island. This implies that at this stage the investors learn their types and the fraction of entrepreneurs with valued collateral on each island are publicly known. However, at this point investors and entrepreneurs do not know which entrepreneurs will value collateral at $t = 2$. Hence, $\tilde{\pi}_i$ is a perfect signal about the overall default problem on island $i$. Based on state $s$ and their type, lenders decide on the new investment $x^1_i(s)$ and ask for withdrawals $x^0_i - x^1_i(s) \geq 0$. This captures the reversibility of loan commitments once more information has become available. Entrepreneurs, however, have the option to refuse to honor the withdrawal requests. If investors can withdraw funds, they either consume or store them.

At the beginning of $t = 2$ individual entrepreneurs learn whether they value collateral or not and project returns are realized. Entrepreneurs decide whether to repay lenders and consumption takes place.
2.3 The Contracting Environment

We compare optimal allocations for three “banking” environments. For each environment, the optimal allocation that is feasible and individual rational exhibits both a demandable debt (deposits) and a debt contract feature. The deposit contract feature arises from the fact that depositors can withdraw funds upon demand either to meet personal liquidity needs or to liquidate investment on the island in response to information about negative asset shocks. The financing of projects resembles collateralized debt, as loans are explicitly backed by borrowers collateral. Since demandable deposits and debt contracts are commonly offered as the defining features of a bank, we interpret the optimal allocation as a banking arrangement or bank.

The first banking environment we consider corresponds to banks which are restricted to operate within a specific region (island), and cannot trade with banks located in other regions. We use this set-up as a building block for the analysis of the two environments that we are primarily interested in. The first of these two environments allows regional banks to interact via a competitive market for borrowing and lending between banks. The third environment features “broad banks” which operate branches (or subsidiaries) across regions. We assume that funds can be transferred across locations within an internal capital market. However, these broad banks take into account the threat of entry by regionally focused banks when deciding upon their allocation of funds across locations. In the broad bank environments, this threat of entry leads to the banks lending and deposit contracts to depend directly on the overall distribution of asset and liquidity shocks.

To formalize these different banking environments, we analyze an optimal contracting problem under three different restrictions on how resources can be transferred across islands. We begin by assuming that goods cannot be transferred between islands. Hence, allocations depend only on the liquidity and asset shock of the individual island and every location essentially solves an optimal contracting (or planning problem) in isolation. We interpret this solution as a regional bank. Then, we look at a competitive market for borrowing and lending funds across locations. Again, each island – or regional bank – solves a planning problem.

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Calomiris and Kahn (1991) argue that demandable debt is the crucial feature characterizing banks.
problem, but takes into account that it has access to a lending market at \( t = 1 \). Interest rates are driven by supply and demand for funds by regional banks which in turn depend on the \textit{individual} realizations of asset and liquidity shocks. Borrowing, however, is limited by the ability of a bank to repay loans. Finally, we compare this equilibrium with a central planner – or a broad bank operating across locations – that decides directly on the allocation of funds across islands. Funds can be transferred across locations depending directly on the \textit{overall} distribution of asset and liquidity shocks. Transfers, however, are constrained since each island has the option to exit the broad bank contract and to choose the autarky contract \textit{given} the realization of its individual shocks.

Formally, we look at allocations associated with each of three different restrictions on interactions between the \( N \) islands. An \textit{allocation} specifies investment \(( x_i^0 \) and \( x_i^1(s) \)), non-negative consumption levels for entrepreneurs \(( c_{V,B,i}(s) \) and \( c_{0,B,i}(s) \)) and investors \(( c_{L,i}(s) \) and \( c_{2,L,i}(s) \)) and assigns ownership of collateral \(( \delta_i(s) \in \{0,1\} \)) for all states \( s \) and for all islands \( i \). The last object specifies whether the entrepreneur loses \(( \delta_i(s) = 0 \) or retains \(( \delta_i(s) = 1 \) his collateral, whenever it has value.

Regardless of the interaction between islands, an allocation has to be \textit{individually rational} for entrepreneurs and investors. Entrepreneurs always have the option of not repaying the initial investment. Hence, in order to borrow they must pledge their collateral good. We require that – when pledging collateral – they must always have an incentive to repay investors. At \( t = 1 \), the expected value of honoring withdrawal requests at \( t = 1 \) must be larger than continuing with the investment at the original level, or

\[
\pi_i[c_{V,B,i}(s) + \delta_i(s)V] + (1 - \pi_i)c_{0,B,i}(s) \geq Rx_i^0. \tag{1}
\]

At \( t = 2 \), entrepreneurs compare the value of not repaying and defaulting with the consumption level they obtain when paying back the loan. If entrepreneurs have worthless collateral, they always default and consume the entire project return. They then need an incentive to repay whenever collateral has value. Therefore, we require

\[
c_{0,B,i}(s) \geq Rx_i^1(s) \tag{2}
\]
\[
c_{V,B,i}(s) + \delta_i(s)V \geq Rx_i^1(s). \tag{3}
\]

Similarly, investor always have the option of not investing at \( t = 1 \) and \( t = 2 \). Hence,
their consumption level has to be at least as high as their outside option of storing goods, or equivalently,

\[ c_{L,i}^1(s) \geq 1 \]
\[ c_{L,i}^2(s) \geq 1. \]

for all \( s \). We assume that for all values of \( \tilde{\pi}_i \), the expected value of collateral is greater than the project value of the entrepreneur, or that

\[ \min_{\pi_i \in \Pi} \pi_i V \geq R. \]

This implies that all investors can withdraw their funds at \( t = 1 \) against the threat of seizing collateral from entrepreneurs.

3 Liquidity Provision through Competitive Markets

This section examines the optimal allocation with banks which are restricted in their scope of operation. We begin by considering the case where there is no interaction between individual islands. We call the optimal allocation in this case a regional bank. In subsection 3.2, we analyze the effect of allowing regional banks to trade with one another at period \( t = 1 \) after the island specific collateral and liquidity shocks have been realized.

3.1 Regional Banks

We begin with the case of banks which are prohibited from trading with banks in different locations. Formally, we assume that goods cannot flow between islands, so that consumption and investment allocation on island \( i \) only depends upon the realized shocks on \( i \). This yields three restrictions on the feasibility of an allocation.

\[ 0 \leq x^1_i(s) \leq x^0_i \]
\[ \theta_i c_{L,i}^1(s) \leq (1 - x^1_i(s)) \]
\[ \left[ \pi_i c_{B,i}^V(s) + (1 - \pi_i)c_{B,i}^0(s) \right] + (1 - \theta_i)c_{L,i}^2(s) \leq R x^1_i(s) + \left[ (1 - x^1_i(s)) - \theta_i c_{L,i}^1(s) \right] \]
The first restriction is that investment at \( t = 1 \) cannot exceed the initial size of the project. Equation 8 specifies that withdrawals in period \( t = 1 \) are limited by the initial investment and consumption of lenders that want to consume early have to be financed from such withdrawals. This captures the fact that there is no possibility for islands to provide liquidity through transfers or loans. Finally, in period \( t = 2 \), consumption for all entrepreneurs and the remaining investors must be provided through investment returns and goods that have been stored. Note that the feasibility restrictions for each island imply that there is no interactions between islands.

The optimal allocation for each island satisfies individual rationality and is feasible. Due to linearity of the utility functions, we can simply maximize total surplus at \( t = 0 \) on each island and assume that all surplus accrues to borrowers,\(^9\)

\[
E \left[ \pi_i (c_{B,i}^V(s) + \delta_i^V(s)V) + (1 - \pi_i) c_{B,i}^0(s) \right].
\] (10)

Assumption (6) guarantees that investors decide to advance all their funds initially \( (x_i^0 = 1) \). Withdrawals at \( t = 1 \) depend on the state \( s \), i.e., how large liquidity needs are and on how severe the default problem is. Since there are no markets for liquidity, each location has to withdraw a fraction \( \theta_i \) of investment to satisfy liquidity demands, i.e., to ensure that lenders that want to consume early receive at least one unit of the consumption good.

Whether the remaining funds stay invested depends on the average returns on loans on the island at \( t = 1 \). By assumption, only entrepreneurs value collateral and use it to secure repayment of loans. Whenever the realized value of collateral is 0 they will not repay the initial investment. So long as the realized value of their collateral exceeds the project return \( (V \geq R) \), they prefer to repay the loan rather than losing the collateral. Since investors can receive a return only from entrepreneurs with valuable collateral, the average return on loans is \( \pi_i R \).

If the collateral shock \( \pi_i \) is greater or equal than \( 1/R \) at \( t = 1 \), investors can receive a return greater or equal than the return on storage which has been normalized to 1. Hence, they will keep the remaining funds \( (1 - \theta_i) \) invested. Otherwise, the effective return on

\(^9\)One interpretation of this is that free entry of regional banks drives profits to zero, so that borrowers retain all of the surplus.
investment is less than one, since too many entrepreneurs default at \( t = 2 \). In this case, all funds are withdrawn at \( t = 1 \) so that investors receive at least 1. This can be summarized in the following proposition.

**Proposition 3.1.** Suppose funds cannot be transferred across islands. On islands with average returns \( \pi_i R \) greater than 1, as many projects as possible are financed after providing investors with liquidity. On all other islands, all investments are terminated at \( t = 1 \).

It is useful to briefly discuss our assumptions concerning the valuation of collateral. Our assumptions guarantee that there is a default problem. The assumption that the lowest value of the collateral is zero is not essential, as a positive, but sufficiently low value of collateral (\( V < R \)) would suffice to provide an entrepreneur with an incentive to default. Furthermore, since investors do not value collateral, they always prefer repayment of the loan. If investors valued collateral, they would compare the total expected return of their investment – including their value for the collateral – with the fixed return on storage. As long as the valuation of collateral by investors is low enough (and less than the entrepreneur’s valuation), it will be optimal to have borrowers repay the loan, and to only seize the collateral after an entrepreneur defaults. The optimal allocation would thus remain qualitatively the same.

This default problem, together with the need for liquidity, cause the optimal allocation to be inefficient. The *social* return on investing is fixed at \( R \) and corresponds to the total surplus from investing. The *average* return for investors, however, is lower and given by the total amount that can be recovered from entrepreneurs (\( \pi_i R \) on average). Total surplus is maximized when all projects are financed, but both liquidity needs and limited enforcement prevent investors from doing so. The next two banking arrangements partially alleviate these problems.

### 3.2 Interbank Markets

We now assume that goods at \( t = 1 \) can be traded on a competitive market in exchange for goods at \( t = 2 \) in order to meet liquidity needs. The initial investments \( x_0^i \) continue to be island specific. Since the possibility to trade funds does not influence the basic features of financing
investments (demandable and collateralized debt), we interpret this set-up as regional banks interacting through interbank markets with banks \( i \) being identified with island \( i \).

Every island can be seen as a regional bank represented by a local planner. The regional bank maximizes surplus for the island taking as given the presence of a competitive interbank market at \( t = 1 \). There are now two options. At \( t = 1 \), each island knows its liquidity needs (fraction \( \theta_i \) of investors would like to consume) and its overall default problem (fraction \( \pi_i \) of loans will not perform). Hence, given a gross interest rate \( R_{IB} \), the regional bank can borrow funds in order to satisfy withdrawal demands so as to avoid early liquidation of projects. Alternatively, it can terminate projects and supply goods on the interbank market. The first option corresponds to demanding liquidity on the market, while the latter supplies liquidity on the market that can be used by other regional banks to satisfy withdrawal requests by investors.

Let \( z_i \) denote the funds supplied on the market by the regional bank of island \( i \) where \( z_i \geq 0 \) (\( z_i \leq 0 \)) denotes lending (borrowing). The bank will maximize the surplus available for consumption at \( t = 2 \) after having satisfied all withdrawal requests by its investors at \( t = 1 \). Taking into account withdrawals by early investors, total resources at \( t = 2 \) available for consumption at \( i \) are given by

\[
Rx^1_i + R_{IB}z_i + [(1 - \theta_i) - z_i - x^1_i],
\]

where the choice for a regional bank is either to fund its own projects, be active on the interbank markets or to store goods. The surplus arises from three sources. The first two are the social returns from continuing investment on the island \( Rx^1_i \) and the returns from interbank loans \( R_{IB}z_i \). The final source corresponds to the amount of goods stored. We require of course that total investment in projects on island \( i \) has to be financed at \( t = 1 \) either by goods that haven’t been withdrawn or by goods obtained through interbank lending:

\[
x^1_i \leq (1 - \theta_i) - z_i.
\]

Each regional bank faces a borrowing (solvency) constraint on the interbank market. Specifically, it can borrow only up to an amount where the value of the assets at \( t = 2 \) after paying lenders in its own location is greater than the costs of repaying the loan. The
The borrowing constraint is thus given by

$$-z_i R_{IB} \leq \pi_i Rx_i^1 + [(1 - \theta_i) - z_i - x_i^1] - (1 - \theta_i).$$

The regional bank cannot pay back more than its net assets at \(t = 2\). These are given by the returns on performing loans (a fraction \(\pi_i\) of loans to entrepreneurs) and the amount of goods stored less the pay-outs to investors that have not withdrawn their goods at \(t = 1\). The total costs of fulfilling the obligation from borrowing is given by the principal plus interest. Note that this constraint is endogenous in the sense that it depends on the interest rate \(R_{IB}\). Furthermore, this constraint is common knowledge at \(t = 1\), as both the liquidity shock \(\theta_i\) and the asset shock \(\pi_i\) are publicly known.\(^{10}\)

What is crucial here is that the borrowing constraint for every island depends only on the individual characteristics of the island, but does not directly take into account the entire distribution of shocks across islands. Instead, the distribution of shocks only works through (as we will see, imperfectly) the interest rate \(R_{IB}\), which is pinned down by the decentralized exchange of goods among regional banks.

### 3.2.1 Equilibrium

Our definition of equilibrium is standard. An interbank market equilibrium at \(t = 1\) is an interest rate \(\hat{R}_{IB}\) and investment levels \(\{\hat{x}_i^1, \hat{z}_i\}_{i=1}^{N}\) such that (i) given \(\hat{R}_{IB}\), the planner for every location \(i\) chooses \(\hat{(x}_i^1, \hat{z}_i)\) optimally and (ii) the market for borrowing and lending clears \((\sum_{i=1}^{N} \hat{z}_i = 0)\). We make, however, for the rest of the analysis an assumption on the size of liquidity needs relative to the aggregate size of the default problem.\(^{11}\)

**Assumption 3.2.** Let \(\pi_1 < \cdots < \pi_N\) and let \(\Gamma\) be the smallest integer such that \(\sum_{\{i| i > \Gamma\}}^{N} (\pi_i R - 1) \geq 0\). We assume that \(\Gamma \geq \sum_{i=1}^{N} \theta_i\).

\(^{10}\)Such a constraint is required for the problem to be well-defined. We chose our particular formulation for two reasons. First, imposing a tighter constraint allows for less trade. Second, our set-up corresponds to the seniority of claims in interbank lending. A lending bank can seize all assets after claims of depositors have been satisfied by a defaulting bank. Of course, we take into account that only a fraction of a bank’s assets are performing.

\(^{11}\)For a complete characterization of the case when liquidity needs are high, see Koepl and MacGee (2001).
This assumption restricts the demand for liquidity relative to the overall default problem in the economy. The total demand for liquidity in the economy is given by $\sum_{i=1}^{N} \theta_i$. By cross-subsidizing projects across islands, one can at most finance so many projects that $\sum_{\{i: i > \Gamma\}}(\pi_i R - 1) \geq 0$ without having a lower return than storing goods. Hence, under Assumption 3.2 one can use the goods on some islands to finance all liquidity needs without having to terminate any projects on islands that could be profitably financed through some form of cross-subsidization across islands. In other words, liquidity needs are not so large that they dwarf the total default problem in the economy. This implies immediately that liquidity needs cannot reach a level where investment on some island with an average return on projects $\pi_i R > 1$ must be terminated.

We first determine the amount that a regional bank can borrow. When making its investment decision at $t = 1$, each regional bank compares the rate of return for projects at its own location with the return on interbank loans. Any interbank rate above $R$ or below 1 can never be an equilibrium, as either the demand or the supply would be zero and the market would not clear. For any other interest rate $R_{IB} \in [1, R]$, the social return $R$ exceeds the interest rate. A regional bank would then like to borrow on the market to satisfy all its liquidity needs, i.e. $-z_i = \theta_i$, but is restricted by its borrowing constraint.

How much a regional bank can borrow, depends on the average return on projects on its island at $t = 1$, which is equal to the fraction of performing loans, $\pi_i R$. We distinguish several cases. First, if $\pi_i R \geq R_{IB} \geq 1$, the average return for the bank exceeds the interest rate. The bank can then pay back the principal and interest of a loan that finances all liquidity needs directly from the additional projects that are financed: If it is not constrained, it will borrow up to $-z_i = \theta_i$ and finance all projects, $x^1_i = 1$.

Whenever $R_{IB} > \pi_i R$, however, a bank can be constrained in its borrowing. This follows directly from the borrowing constraint (13) that we can rewrite, if one takes into account that a regional bank can either store goods at a rate of 1 or invest them with return $\pi_i R$

$$\theta_i (R_{IB} - 1) \leq \max\{\pi_i R - 1, 0\}. \quad (14)$$

The left-hand side is the net interest cost of a loan that finances all liquidity needs ($-z_i = \theta_i$), while the right-hand side expresses the average return on investing one unit of goods.
\[\pi R \geq 1,\] the average return exceeds the storage technology, and the bank \(i\) can use returns from all its projects to finance some borrowing. How much it can borrow depends on its liquidity needs. If the liquidity needs are too high or, equivalently, if \(\theta_i \geq \frac{\pi_i R - 1}{R_{IB} - 1}\), a bank can only borrow up to

\[-z_i = \frac{\pi_i R - 1}{R_{IB} - \pi_i R}\]  \hspace{1cm} (15)

which is less than \(\theta_i\).

If \(1 > \pi_i R\), a regional bank cannot borrow at all irrespective of its liquidity needs. The average return on projects is below the interest rate on interbank loans. Hence, the bank will always supply a positive amount of funds \((z_i \geq 0)\) to the interbank market. Equation (13) is then a solvency constraint for each location. A regional bank that lends funds would like to use its returns from interbank lending to compensate for losses on some of the islands’ projects which all have a social return of \(R > R_{IB}\). Hence, investment on every island is positive at \(t = 1\) whenever \(R_{IB} > 1\). Furthermore, total investment is given by \(x_i^1 + z_i = 1 - \theta_i\) for every island, since the storage technology is strictly dominated. This implies the following result.

**Proposition 3.3.** Suppose there is a competitive interbank market between regional banks at \(t = 1\). In equilibrium, all projects on islands with average returns on projects \(\pi_i R\) greater than 1 get financed independent of liquidity needs. On all other islands, investments are terminated at \(t = 1\).

The unique equilibrium is given by the interest rate \(R_{IB} = 1\), so that only islands with an average return below 1 prefer to supply funds on the interbank market, but all other islands are not borrowing constrained. Hence, all projects with an average return of at least 1 are financed at \(t = 1\). This is possible as the interbank market allows for allocating liquidity across regional banks. The ability to trade liquidity across locations improves welfare since more projects are financed. Markets work well for providing liquidity where it is needed despite the default problem. All locations which are solvent \((\pi_i R \geq 1)\) obtain enough loans through the market to satisfy their liquidity needs. In fact, none of the borrowing constraints (13) bind in equilibrium. However, there is no cross-subsidization of projects across regional banks.

To see that this is the unique equilibrium, note first that by Assumption 3.2, there are
enough funds for satisfying the demand for liquidity from islands that have average return \( \pi_i R < 1 \). At \( R_{IB} = 1 \), such islands are indifferent between supplying goods on the interbank market or storing them. For higher interest rates \((R_{IB} > 1)\), one can show that there is always excess liquidity in the market.\(^{12}\) Liquidity is therefore never so scarce as to drive up the interest rate on the interbank market. But this also implies that the interest rate – and, hence, the volume of borrowing and lending – does not depend on the joint distribution of shocks.

### 4 A Market Failure: Lack of Cross-subsidization

We now look at the market outcome associated with a broad bank that operates branches or subsidiaries in each location. These branches intermediate local lending to entrepreneurs which still exhibit the crucial features of a bank: demandable, but collateralized debt. In addition, a broad bank can also run an internal “market” for investment funds which can allocate goods across locations so as to maximize total surplus. However, the extent to which a broad bank can effectively reallocate cross-subsidize projects across regions is limited by the threat of entry by local banks.\(^{13}\) We show that this implies that while the internal market of broad banks can lead to higher levels of investment than the interbank market equilibrium, it can also generate endogenous aggregate uncertainty.

The remainder of this section is organized as follows. The next subsection sets out the contracting problem for the broad bank environment and characterizes the equilibrium. Subsection 4.2 compares the broad bank allocation with the interbank market, and highlights the key role played by internal capital markets. The final subsection presents several numerical examples to illustrate the main results.

\(^{12}\)When the interest rate increases, the supply of funds actually falls. Banks that supply funds obtain more funds to subsidize its projects. In other words, an increase in \( R_{IB} \) relaxes the solvency constraint (13) for such banks and reduces the supply of funds. The supply of funds, however, always decreases less than the demand when interest rates rise.

\(^{13}\)An example of what we have in mind is competition from intermediaries such as credit unions which have a regional focus and tend not to be significant participants in the interbank market.
4.1 Broad Banks and Cross-subsidization

We formalize the operation of broad banks with two key assumptions. First, we assume that goods can be moved directly across all locations at \( t = 1 \) and \( t = 2 \). Second, we assume that banks can commit to contracts which allow them to reallocate resources \textit{internally} across locations. However, when deciding upon these transfers the bank must take into account the potential competitive threat of entry from regional banks. This effectively limits the extent of cross-subsidization of regions that is possible within a broad bank.

The planning problem objective function, where one maximizes overall surplus, remains unchanged from the previous environments.\textsuperscript{14}

\[
E \left[ \sum_{i=1}^{N} \pi_i \left[ c^V_{B,i}(s) + \delta^V_i(s)V \right] + (1 - \pi_i)c^0_{B,i}(s) \right] \tag{16}
\]

To satisfy withdrawal request from investors that want to consume early, one can stop initial investments on \textit{any} island,

\[
\sum_{i=1}^{N} \theta_ic^1_{L,i}(s) \leq \sum_{i=1}^{N} (1 - x^1_i(s)). \tag{17}
\]

Similarly, total consumption at \( t = 2 \) for entrepreneurs and the remaining investors are financed by project returns from all islands and storage

\[
\sum_{i=1}^{N} \left[ \pi_i c^V_{B,i}(s) + (1 - \pi_i)c^0_{B,i}(s) + (1 - \theta_i)c^2_{L,i}(s) \right] \leq \sum_{i=1}^{N} \left[ Rx^1_i(s) + (1 - x^1_i(s)) - \theta_ic^1_{L,i}(s) \right]. \tag{18}
\]

Note that these are now \textit{aggregate} resource constraints at \( t = 1 \) and \( t = 2 \) which reflect the fact that goods can be freely allocated across islands.

Allocating goods directly across locations allows the broad bank to cross-subsidize projects. Since the social return \( R \) on any project exceeds the return on storage, it is always optimal to finance as many projects as possible across islands. A broad bank can use its internal capital market to transfer funds from locations with high returns on projects \((\pi_i R > 1)\) to other locations with low returns \((\pi_i R < 1)\). Once investors there receive a transfer that guarantees them a private return of at least 1, they will continue to finance projects. It is

\textsuperscript{14}Note that in the single island case, since there are no interactions between the island maximizing the overall surplus on each island is the same a maximizing the surplus on all islands.
then optimal to obtain liquidity from islands with the lowest return for investors and to cross-
subsidize projects across all remaining islands to finance the largest number of projects. This
allows more projects to be financed across islands, which clearly improves welfare compared
to interbank markets.

The extent of cross-subsidization that broad banks can engage in is limited by the threat
of competition from regional banks. Cross-subsidizing projects means that at \( t = 1 \) some
branches are requested to make transfers to others at \( t = 2 \). This suggests that the entry
of a regionally focused bank could lead to the withdrawal of investors and entrepreneurs in a
location at \( t = 1 \) from the broad bank contract. This would allow the inhabitants of region \( i \)
to retain the internal transfer of funds of the broad banks. However, the cost of leaving the
broad bank is that the regionally focused bank lacks access to markets for short term liquidity
borrowing, which means that it cannot provide full investment in local projects.

We formally model this as an outside option, which we interpret as the possibility of
intermediating investment on any islands through a regional bank at \( t = 1 \). Such a bank
simply matches the deposits of patient investors with the claims on projects of entrepreneurs
according to the optimal contract in Proposition 3.1. This captures the potential of a regional
bank to offer local entrepreneurs better borrowing conditions than a broad bank which uses
funds from lending in that region to cross-subsidize projects in other regions.

First, transfers are limited by the average return on projects on any particular island. An
investment of 1 unit of the good yields \( \pi_i R \) for investors. Hence, transfers can be at most
\( \pi_i R - 1 \) after paying back investors at least their initial investment. This implies that banks
with high average returns of projects \( \pi_i R > 1 \) will make positive transfers, but all other
islands will always receive a transfer.

Second, transfers are also restricted to leave every island with at least the surplus it could
obtain from financing projects as a regional bank. Regional banks on islands with low average
return on projects \( (\pi_i R < 1) \) at \( t = 1 \) cannot finance any projects on the island and the surplus
is zero. As pointed out before, such islands must receive a positive transfer \( (T_i(s) \geq 0) \) and,
hence, have always a positive benefit from a broad bank. In other words, depending on the
asset shock, they are net beneficiaries of a broad bank. For all other islands, using Proposition
3.1, the surplus from a regional bank is given by

\[ R(1 - \theta_i) + \theta_i + V\pi_i. \]  

(19)

A regional bank has to stop a fraction \( \theta_i \) of projects in order to satisfy its liquidity needs itself. This allows then for the remainder to be invested and yield a social return of \( R(1 - \theta_i) \). The last term is the value of collateral retained by the entrepreneurs.

The surplus of an island (or branch) within a broad bank is given by total surplus from investment less transfers

\[ Rx_i^1(s) + (1 - x_i^1(s)) + \pi_i V + T_i(s) \]

(20)

where \( T_i(s) \) is the transfer of the island. The key difference is that with a broad bank the island does not have to satisfy its liquidity needs by itself. Instead, against paying a transfer it can keep more funds \( x_i^1(s) > \theta_i \) invested at \( t = 1 \). Comparing both surpluses, a broad bank cannot require transfers to be higher than \( \theta_i(1 - R) + (R - 1)(1 - x_i^1(s)) \). In conclusion, transfers from any island are feasible if and only if

\[ T_i(s) \geq \max\{\theta_i(1 - R) + (R - 1)(1 - x_i^1(s)), (1 - \pi_i R)x_i^1(s)\}. \]  

(21)

Since the social return \( R \) exceeds storage, it is optimal to finance as many projects across islands at \( t = 1 \) as possible. A broad bank will therefore make the maximum feasible transfer between islands. Liquidity needs are entirely financed by terminating all projects on locations with the lowest rate of return on loans, as these islands require the highest transfers to realize the minimum average return of 1 required by investors. Since liquidity needs are low enough by Assumption 3.2, all projects on islands with \( \pi_i R > 1 \) can be fully financed and used to cross-subsidize projects on other islands. Transfers for these islands are given by \( \bar{T}_i(s) = (-1) \min\{\theta_i(R - 1), (\pi_i R - 1)\} \). These transfers go to the islands with the highest average returns on loans less than 1 (i.e., the highest \( \pi_i \)'s such that \( \pi_i R < 1 \)). This maximizes the number of projects running at \( t = 1 \) while guaranteeing investors a return of at least 1 on all islands.

**Proposition 4.1.** Suppose there is a broad bank allocating goods directly across islands at \( t = 1 \). The broad bank will finance all projects on islands with average returns on projects
\( \pi_i R \) greater than 1 and some projects on other islands. The total amount of projects financed depends on both, the distribution of liquidity needs and asset returns across islands.

Unlike in the market equilibrium, the allocation of resources within a broad bank is determined by the joint distribution of asset and liquidity shocks at \( t = 1 \). Liquidity needs are essential in making participating in a broad bank valuable even when the broad bank imposes a negative transfers on a branch. As a regional bank, an island would have to provide for its own liquidity. With the broad bank, it obtains its liquidity in exchange for making the transfer. In the absence of liquidity shocks (\( \theta_i = 0 \) for all \( i \)), an island would never have an incentive to accept a negative transfer at \( t = 2 \).

Asset shocks still determine which branches keep their funds invested. The size of the transfer a location accepts depends, however, on the relative size of the asset and liquidity shocks, \( \pi_i \) and \( \theta_i \). The higher the liquidity shock for a given asset shock \( \pi_i > \frac{1}{R} \), the higher the value for the island of being a branch or subsidiary and the higher the transfer the island accepts. Hence, transfers, investment and overall welfare depend on the joint distribution of shocks across islands.

### 4.2 Inefficiency of Market Equilibrium

We now discuss why a broad bank always dominates regional banks that interact through markets. If a broad bank could simply impose its transfer scheme on the different islands, then the result would be immediate. However, in an interbank market equilibrium, regional banks voluntarily choose to participate in trade across banks taking the market interest rate as given. Our argument for the superiority of broad banks does not rely upon forcing islands into transfers. By giving individual islands the option to organize as a regional bank, we require that transfers have to give an incentive to each island to be a branch or subsidiary of the broad bank. Hence, both arrangements face restrictions on transferring funds across locations – regional banks face borrowing constraints, broad banks participation constraints – so what matters is their interaction with both types of shocks.\(^{15}\)

\(^{15}\)The first-best allocation in this economy would feature even more investment, since transfers are not restricted to satisfy the participation constraint (21) and are restricted only by the excess return on performing
While markets can efficiently allocate liquidity across islands, they are unable to transfer any surplus between locations. Locations with low average returns on projects ($\pi_i R < 1$) always supply enough liquidity keeping the equilibrium interest rate at 1 independent of the joint distribution of shocks. This allows profitable regional banks to borrow cheaply in order to cover their liquidity needs, but yields no profits for lenders to finance additional projects.

To the contrary, a broad bank can optimally take advantage of individual liquidity needs in order to transfer surplus across island. This cross-subsidization allows the bank to compensate investors for losses on non-performing loans and, hence, increase investment further. In fact, the optimal allocation of funds can be seen as a peculiar way of conditioning on the joint distribution of shocks. The broad bank charges differential positive interest rates to branches that obtain liquidity. For a given asset shock, the interest rate increases with the demand for liquidity, thereby extracting more surplus from the individual branch. Hence, we have the following result.

**Corollary 4.2.** A broad bank achieves higher investment and welfare than regional banks that interact through interbank markets at $t = 1$ independent of the realized joint distribution of liquidity and asset shocks. At $t = 0$, broad banks are then preferred to interbank markets.

Finally, note that a broad bank creates aggregate uncertainty. Islands are exposed only to idiosyncratic shocks. Hence, taken separately, the distribution of liquidity needs and the asset shock do not vary in our set-up and are constant in the aggregate. However, with a broad bank transfers are a function of the joint distribution at $t = 1$. Transfers, investment levels and, thus, welfare will differ across different states.

Interestingly, a negative correlation between both shocks helps to increase investment and, hence, welfare, at $t = 1$. This is due to the fact that the higher the liquidity needs on islands with good collateral shocks, the more transfers are feasible. Provided liquidity needs are high enough on all islands with $\pi_i R \geq 1$ – more specifically, $\theta_i \geq \frac{\pi_i R - 1}{R - 1}$ – the participation constraint is not binding for any location. The allocation is then first-best.

loans, i.e., $T_i(s) \leq (\pi_i R - 1)x^*_1(s)$.  

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4.3 A Numerical Example

The theoretical analysis developed two key results: Liquidity needs are crucial to support ex post insurance transfers and the joint distribution of shocks determines the exact structure of the optimal contract. To illustrate the importance of liquidity needs further we examine an example where all islands receive the same liquidity shock. Later, we will modify the example and allow for island-specific differences in the liquidity shock to study the importance of the joint distribution.

We attempt to highlight the forces that drive these results using the simplest possible examples. Hence, we look at an economy comprised of three regions \((N = 3)\). Since the gross return on entrepreneurs projects only scales the size of the total surplus, we set it at \(R = 1.4\) and keep it fixed throughout this section. Similarly, we hold the support of the collateral shock fixed, with values given by \(\Pi = \{0.8, 0.65, 0.5\}\). Without loss of generality, we assign these shocks to island \(1, 2, 3\), respectively. This implies that the maximum amount that can recovered by lenders on island \(1, 2, 3\) is \(\Pi R = \{1.12, 0.91, 0.7\}\). As a result, a regional bank on island 1 without markets will feature full investment of all funds left after any liquidity withdrawls are met, while regional banks on the other two islands would liquidate of all entrepreneurial projects at \(t = 1\).

We first consider a situation where all islands receive the same realization of the liquidity shock. Formally, we set \(\Theta = \{a, a, a\}\), and compare three different values of the liquidity shock: low \((a = 0)\), medium \((a = 0.1)\), and high \((a = 0.3)\). These parameters are chosen such that the aggregate liquidity are low enough so that they do not interfere with the investment of funds by investors. Table 1 summarizes the resulting investment allocations for the three different environments we consider.

The advantage of interbank markets compared to the case with regional banks is the ability of islands with good collateral shocks to borrow in order to meet liquidity needs. Island 1 – where the collateral shock is sufficiently high \((\pi_1R > 1)\) to guarantee repayment – can borrow an amount \(a\) to meet liquidity demands by early withdrawers and repay this interbank borrowing. As a result, interbank markets lead to a higher level of total investment than a world of regional banks which are prohibited from trading with one another.
The interbank market equilibrium does not vary with the size of the liquidity shock, as the only interest rate that clears the market features \( R_{IB} = 1 \). Thus, islands 2 and 3 do not earn returns above their opportunity cost of funds which can be used to subsidize investment on their islands. Hence, for this example, aggregate investment remains the same for all three levels of the liquidity shock considered.

<table>
<thead>
<tr>
<th>Regional Banks</th>
<th>( x_1^1 = 1 - a )</th>
<th>( x_2^1 = 0 )</th>
<th>( x_3^1 = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interbank Market</td>
<td>( x_1^1 = 1 )</td>
<td>( x_2^1 = 0 )</td>
<td>( x_3^1 = 0 )</td>
</tr>
<tr>
<td>Broad Banks</td>
<td>( x_1^1 = 1 )</td>
<td>( x_2^1 = 0 )</td>
<td>( x_3^1 = 0 )</td>
</tr>
<tr>
<td>( \Theta = {0, 0, 0} )</td>
<td>( T_1 = 0 )</td>
<td>( T_2 = 0 )</td>
<td>( T_3 = 0 )</td>
</tr>
<tr>
<td>( \Theta = {0.1, 0.1, 0.1} )</td>
<td>( x_1^1 = 1 )</td>
<td>( x_2^1 = 0.4444 )</td>
<td>( x_3^1 = 0 )</td>
</tr>
<tr>
<td>( T_1 = -0.04 )</td>
<td>( T_2 = 0.04 )</td>
<td>( T_3 = 0 )</td>
<td></td>
</tr>
</tbody>
</table>

In contrast, the allocation with broad banks varies with the level of liquidity shocks as summarized by Table 1. In the absence of liquidity shocks \((a = 0)\), there is no need to lend across branches and transfers are zero. As a result, the allocation without liquidity shocks is the same for all three banking environments. However, for positive values of liquidity shocks the broad bank is able to achieve higher levels of investment. The key reason is that the broad bank is able to run an internal capital market to provide liquidity at branch specific interest rates. The level of this interest rate is determined by the alternative of financing projects on an island through a regional bank. To see this, consider the case where liquidity needs are positive, but low \((\Theta = \{0.1, 0.1, 0.1\})\). The benefit for island 1 of having a branch is that they

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\[16\] This arises from the assumption that liquidity shocks are small enough, so that there is always an excess supply of funds (see Assumption 3.2).
get access to a liquidity loan in the amount of $a = 0.1$. Since the social return on lending is $R = 1.4$, the “surplus” from this liquidity loan is $0.1 \times (1.4 - 1) = 0.04$. The alternative of running a regional bank would mean that total investment could be only $1 - a = 0.9$ resulting in a loss of $0.04$ units of the consumption good.

To extract this surplus, the broad bank charges an internal interest rate for liquidity lending to its branch on island 1 of 40%. The bank uses the total repayment of the loan (0.14) to pay-off lenders that financed the loan (0.1) and to subsidize the continued operation of some projects on other islands (the net transfer of 0.04). The subsidy is used best in a branch with the highest collateral shock, but an average return on projects for investors below 1 (or $\pi R < 1$). With interbank markets, lending banks can never make a profit that would allow them to subsidizing projects on their island as the interest rate is zero.

With $a = 0.1$, this subsidy allows only partial investment on island 2. To break even on this island, the broad bank can finance at most $x^2_1 = (1 - 0.91)/0.04 = 0.4444$. The numerator expresses the loss for investors per project financed, while the denominator specifies the total subsidy available to compensate investors for these losses. Larger liquidity shocks increases the total amount that can be extracted from island 1 enabling more cross-subsidization of projects on islands 2 and 3. While the volume of loans rises, the interest rate on these loans remains the same and is solely driven by the social net return of $R - 1$. For $a = 0.3$, a broad bank reaches the first-best allocation of investing all funds after satisfying all withdrawal request.

The above example can be easily adapted to illustrate the key role the realized joint distribution plays. We hold all of the parameter values unchanged and consider values of $\Theta = \{0, 0.1, 0.2\}$. In this case there are $3! = 6$ possible realizations of the joint distribution of asset and liquidity shocks. Note that for each of these cases the aggregate liquidity shock is equal to that of the example in Table 1 with $\Theta = \{0.1, 0.1, 0.1\}$.

The allocations and transfers for each of these possible realizations in the broad bank environment are given in Table 2. Since the distribution of liquidity shocks does not influence the total level of investment in the interbank market, the interbank allocation is the same as that in Table 1. In this example, total transfers – and, hence, total investment – depends
critically upon the size of the liquidity shock for the branch with the good collateral shock which is island 1. As a result, we have only three different realizations of total investments: one for each possible realization of the liquidity shock on island 1.

Table 2: Investment and Transfers for Different Joint Distributions

<table>
<thead>
<tr>
<th>Case 1: $\theta_1 = 0$</th>
<th>$\pi R = 1.12$</th>
<th>$\pi_2 R = 0.91$</th>
<th>$\pi_3 R = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\theta_2 = 0.1, \theta_3 = 0.2)$ or $(\theta_2 = 0.2, \theta_3 = 0.1)$</td>
<td>$x_1^1 = 1$</td>
<td>$x_2^1 = 0$</td>
<td>$x_3^1 = 0$</td>
</tr>
<tr>
<td></td>
<td>$T_1 = 0$</td>
<td>$T_2 = 0$</td>
<td>$T_3 = 0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 2: $\theta_1 = 0.1$</th>
<th>$\pi R = 1.12$</th>
<th>$\pi_2 R = 0.91$</th>
<th>$\pi_3 R = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\theta_2 = 0, \theta_3 = 0.2)$ or $(\theta_2 = 0.2, \theta_3 = 0)$</td>
<td>$x_1^2 = 1$</td>
<td>$x_2^2 = 0.4444$</td>
<td>$x_3^2 = 0$</td>
</tr>
<tr>
<td></td>
<td>$T_1 = -0.04$</td>
<td>$T_2 = 0.04$</td>
<td>$T_3 = 0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 3: $\theta_1 = 0.2$</th>
<th>$\pi R = 1.12$</th>
<th>$\pi_2 R = 0.91$</th>
<th>$\pi_3 R = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\theta_2 = 0.1, \theta_3 = 0)$ or $(\theta_2 = 0, \theta_3 = 0.1)$</td>
<td>$x_1^3 = 1$</td>
<td>$x_2^3 = 0.8889$</td>
<td>$x_3^3 = 0$</td>
</tr>
<tr>
<td></td>
<td>$T_1 = -0.08$</td>
<td>$T_2 = 0.08$</td>
<td>$T_3 = 0$</td>
</tr>
</tbody>
</table>

When $\theta_1 = 0$, there is no need for liquidity. The threat of entry through a regional bank prevents the broad bank from extracting resources from branch 1 to subsidize projects in other branches. The amount that can be extracted from branch 1 increases with the realized value of the liquidity shock. The lower liquidity needs are on island 1, the lower is the threat of being excluded from lending within a broad bank and, hence, the lower is the quantity of transfers that can be extracted from branch 1. Since this determines the extent of resources available to subsidize investment on islands with bad collateral shocks, the aggregate level of investment depends upon the realized joint distribution of collateral and liquidity shocks. Higher levels of aggregate investment can be achieved when islands with good collateral shocks also have high needs for liquidity, since this allows the broad bank to extract more resources to support good projects in other locations. This implies that broad banks induce endogenous aggregate uncertainty, since total investment depends upon the realized joint distribution. However, this example also illustrates that the benefit of the endogenous aggregate uncertainty is a higher average level of investment and, hence, output.
5 Discussion

Our theoretical results show that restrictions on the scope of banking matter even in the presence of well-functioning interbank markets. In this section, we review the empirical literature on whether and how internal capital markets in banks matter. We focus particular attention on two implications of our model. First, broad bank lending within a region should respond less to regional asset shocks than that of regional banks. Second, the removal of barriers to broad banks should lead to a reduction in idiosyncratic output and investment fluctuations across regions.

There is substantial empirical evidence suggesting that internal capital markets matter, as “broad banks” respond differently to shocks than narrow banks. Several papers have compared the lending behavior of U.S. banks which are part of a bank holding company operating in multiple states with unaffiliated banks that only operate at a more local (state) level. Houston, James and Marcus (1997) find that loan growth at subsidiary banks of bank holding companies is more sensitive to the capital position of its parent holding company than to its own cash flow and capital. They argue that this suggests that banks operate internal capital markets which reallocate funds across subsidiary banks. Houston and James (1998) compare the lending patterns of banks that are part of a bank holding company which operates in multiple states with unaffiliated banks. They also find evidence consistent with the existence of internal capital markets, as loan growth at affiliated banks is less dependent on cash flow and capital than loan growth at unaffiliated banks. More recently, Campello (2002) and Ashcraft (2006) also find that banks belonging to holding companies are less responsive to monetary shocks than are non-affiliated banks.

There is also evidence that subsidiaries of banks operating in different countries are less responsive to local shocks than local banks. Clarke et al (2003) review several studies comparing the lending behavior of subsidiaries of foreign banks with domestic institutions during financial crises in emerging market economies. Overall, the evidence suggests that foreign banks lending does not decline very much during crisis episodes, while domestic bank lending falls substantially. Recent work by Haas and Lelyveld (2006) compares the behavior of foreign versus domestic banks in Central and Eastern Europe during business cycles and banking
crises and arrives at also a similar conclusion. This suggests that banks operating across countries use internal capital markets to support lending in regions hit by adverse shocks, which nationally (or regionally) focused banks cannot replicate using interbank markets.

An implication of our model is that a removal of barriers to the creation and operation of broad banks should be followed by a reduction in idiosyncratic output volatility. This implication is in line with the empirical findings on the effects of the recent U.S. experience with the lowering of barriers on inter-state banking. This process, which began in the early 1980’s, culminated with the passage of the Reigle-Neals act in 1994, which effectively removed most of the remaining barriers (Kane (2006)). Several papers have exploited cross-state variation in the timing of the relaxation of these restrictions to identify possible changes in the extent to which state-level idiosyncratic shocks are smoothed. Morgan, Rime and Strahan (2004) find that the reduction in these barriers was accompanied by an increase in the extent of interstate banking as well as a reduction in idiosyncratic state output fluctuations.17 Demyanyk, Ostergaard and Sorensen (2007) draw similar conclusion for an increase in income smoothing across states following U.S. banking deregulation. They link this phenomenon to the role of banks in financing small businesses.18 Interestingly, this channel also drives our theoretical result that broad banks lead to better inter-regional risk sharing among bank-financed entrepreneurs than regional banks with interbank markets.

Our results may also shed some light on examples of regionally operating banks entering into informal (and sometimes formal) arrangements to provide mutual insurance against shocks affecting their assets and liabilities. These interbank relationships typically feature some form of limited enforcement as banks are free to exit. For example, the arrangements between cooperative banks in Germany appear to match the story of the model. Independent, regionally operating cooperative banks and savings banks in Germany are arranged in groups centered around a head organization.19 This organization coordinates liquidity prov-

17 Jayaratne and Strahan (1996) and Clarke (2004) argue that the removal of interstate banking restrictions was also accompanied by an increase in the growth rate of output in the respective states.

18 Bank loans play an important role in financing small and medium sized businesses (see for example Degryse and Ongena (2005)).

19 See Ehrmann, Gambacorta, Martinez-Pages, Sevestre and Worms (2001) for an exposition on this feature of the German banking sector.
sion among the smaller individual banks. In addition, these clubs explicitly agree to provide insurance against asset shocks in financial crises to one another. Another historical example is provided by Gorton (1985), who documents that during periods of high liquidity needs, American clearinghouses in the National Banking era would provide guarantees of members assets. In both examples, there is active liquidity provision between banks and “insurance” against asset shocks. We conclude that the commitment to provide insurance is credible in these example, since leaving the arrangement could considerably aggravate access to liquidity.

Finally, we briefly turn to the question of what are the implications for future cross-border financial sector integration? Currently, financial market regulators in Europe are grappling with the question of how to structure cross-national financial markets and institutions to deepen financial integration after the introduction of the Euro. Our findings suggest that establishing pan-European interbank markets might not be enough. Instead, the optimal policy may entail a significant reduction in barriers to the cross-border operation of intermediaries. The potential benefits of such moves are a reduction in the variation fluctuation of GDP of member EU states, as per the recent U.S. experience.

6 Conclusions

Restrictions on the scope of banking matter even in the presence of well-functioning interbank markets. This is somewhat surprising, as one might think that interbank markets are sufficient for regional banks to deal with restrictions on their geographical or sectoral scope. We show, however, that broad banks can achieve higher level of investments than regional banks that are linked through an interbank market. Their advantage stems from the internal allocation of capital that leads to cross-subsidizing investment projects across regions. As a result,

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20 See also Williamson (1989) for a historical comparison of Canadian and U.S. banking arrangements.

21 Recently, the push to open up borders has resulted in several significant cross-border bank mergers (Dermine (2006)). However, overall, between 1997 and 2003, the market share of foreign banks in the EU 15 declined.

22 One caveat is that we abstract from informational and managerial frictions that figure prominently in the literature associated with internal capital markets in firms and conglomerates (see for example Stein (1997) and Maksimovic and Phillips (2002)).
idiosyncratic fluctuations in regional output are decreasing with the extent of cross-regional banking in our model.

Our analysis also implies that broad banks which allocate capital internally can endogenously generate aggregate uncertainty across regions. In our environment, the level of transfers between branches – and hence the level of investment – depends upon the realized joint distribution of asset and liquidity shocks. This feature which generates endogenous aggregate uncertainty leading to higher aggregate output and more aggregate volatility with broad banks than with regional banks and interbank markets.\textsuperscript{23}

Some brief remarks about the robustness of our results are in order. A key assumption is that the return on projects is independent of the value of collateral. This assumption is reasonable for many projects where entrepreneurs post assets as collateral for projects which do not directly use the collateral. In such a world, shocks to the value of collateral lead to a reduction in loans by local banks but do not directly affect the return on the projects. Our results are robust to relaxing this assumption by allowing for stochastic project returns. What is essential for our results, however, is that fluctuations in project returns are not perfectly positively correlated with the value of collateral. For such cases, fluctuations in the value of collateral will lead to ex-post variations in the wedges on investment returns for lenders and entrepreneurs across regions.

A second key assumption is that all the surplus from financing projects goes to entrepreneurs. Our results, however, survive as long as entrepreneurs receive some surplus from investment. In other words, what is required is that lenders do not receive all of the surplus from lending. Lenders would always agree to finance projects with a negative average return, as long as this translated into higher payouts to them. But entrepreneurs need an incentive to commit ex-ante to loan contracts that incorporate the cross-subsidization of projects. As long as they get some surplus from running their projects, they would prefer a broad bank over an interbank arrangement as this ensures a higher probability of getting

\textsuperscript{23}The result on endogenous aggregate uncertainty generalizes to any setting with multi-dimensional risk sharing. This result arises as long as there are two or more distinct sources of uncertainty where the threat of exclusion from trades that mitigate one source of risks can be used as “leverage” to induce or increase risk sharing along the second dimension.
financed. This also clarifies that a broad bank must be able to commit not to recall loans to loss making entrepreneurs as long as the bank’s overall solvency is maintained.

7 Appendix

Construction of the Probability Space

We construct the underlying probability space according to an urn experiment. Consider an urn containing \( N \) balls each labeled with a different value of \( \pi \in (0, 1) \). A second urn contains \( N \) balls, each labeled with a different value of \( \theta \in (0, 1) \). Let \( \Theta = \{\theta^1, \ldots, \theta^j, \ldots, \theta^N\} \) and \( \Pi = \{\pi^1, \ldots, \pi^j, \ldots, \pi^N\} \) such that \( \theta^j \neq \theta^k \) and \( \pi^j \neq \pi^k \) for all \( j \neq k \). For each island, one ball is drawn independently from each urn without replacement. A realization of the two shocks across islands is a specific assignment of draws from the two urns to the \( N \) islands.

Let \( S = \Pi \times \Theta \) be the set of states of nature. It consists of all possible assignments of \( N \) elements of \( \Theta \) to the \( N \) elements in \( \Pi \). A state \( s \in S \) is an \( N \)-vector of ordered pairs \((\pi^i, \theta^i)\), where the \( i \)th component represents the realization of \( \tilde{\theta}_i \) and \( \tilde{\pi}_i \) for island \( i \). A state \( s \) for the economy is thus given by a particular realization of both shocks across islands. There are \( N! \) elements in \( S \) and we define a probability measure \( U \) over \( S \) by putting equal mass \( 1/N! \) on each element of \( S \). Then, the probability space \( \{S, \mathcal{F}, U\} \) represents the uncertainty for the economy, where \( \mathcal{F} \) is the \( \sigma \)-algebra consisting of all subsets of \( S \). There is then also no aggregate uncertainty \textit{ex ante} in the sense that there is always exactly one island having a particular value \( \theta_j \) and exactly one (possibly different) island having the value \( \pi_j \).

Proof of Proposition 3.1

\textit{Proof.} It is optimal to set \( \delta_i^V(s) = 1 \), since collateral is only valued by entrepreneurs. By Assumption (6), this implies that the entrepreneur’s individual rationality constraints (1) and (3) do not bind for any investment \( x_0^i \) and \( x_1^i(s) \). Also, investors are able to withdraw their funds at stage \( t = 1 \) after they observe the realization of \( \tilde{\pi}_i \) and \( \tilde{\theta}_i \). Then \( x_0^i = 1 \) initiates the maximum number of projects and is, thus, optimal.

Since all other individual rationality constraints are binding, liquidity needs at \( t = 1 \) imply
that $x_i^1(s) \leq 1 - \theta_i$. The feasibility constraint (9) can be rewritten as

$$\pi_i c_{B,i}^V(s) \leq (\pi_i R - 1)x_i^1(s).$$

Suppose that $\pi_i R < 1$. Any strictly positive $x_i^1(s) \in (0, 1 - \theta_i]$ violates the non-negativity of $c_{B,i}^V(s)$. Hence, only $x_i^1(s) = 0$ is feasible.

If $\pi_i R \geq 1$, $c_{B,i}^V(s) \geq 0$. Since the objective function is strictly increasing in $c_{B,i}^V(s)$ and $c_{B,i}^0(s)$, which are increasing in $x_i^1(s)$, it is optimal to set $x_i^1(s) = 1 - \theta_i$. Investment levels are hence, given by

$$x_i^1(s) = \begin{cases} 
1 - \theta_i & \text{if } \pi_i R \geq 1 \\
0 & \text{otherwise.}
\end{cases}$$

**Proof of Proposition 3.3**

*Proof.* The regional bank at each location takes the interest rate $R_{IB}$ as given and chooses an incentive feasible allocation for its location that maximizes the entrepreneurs’ utility. Clearly, $x_i^0 = 1$ is still optimal and the same participation constraints bind as in the optimal allocation on the location with autarky. At $t = 1$ after the state $s$ has been realized, the bank chooses its investment level $x_i^1(s)$ and borrowing level $z_i(s)$ to maximize total surplus subject to the borrowing constraint. This problem is given by

$$\max_{(x_i^1, z_i)} \pi_i V + (R - 1)x_i^1(s) + (R_{IB} - 1)z_i(s)$$

subject to

$$x_i^1(s) \leq (1 - \theta_i) - z_i(s)$$

$$-z_i(s)R_{IB} \leq \pi_i R x_i^1(s) + [(1 - \theta_i) - z_i(s) - x_i^1(s)] - (1 - \theta_i)$$

$$x_i^1(s) \in [0, 1].$$

Let $R_{IB} = 1$. For all island with $\pi_i R < 1$, due to the borrowing constraint (13), the only feasible choice is $x_i^1 = 0$. The island is then indifferent between any supplying any positive level of $z_i(s) \leq \theta_i$ and storage. For all island with $\pi_i R \geq 1$, the borrowing constraint (13) is not binding. It is then optimal for the regional bank to set $x_i^1(s) = 1$ and to borrow $z_i(s) = -\theta_i$. 

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For market clearing it is then sufficient to show that the maximum supply of funds exceeds the demand, or that

\[ \sum_{\{i|\pi_i R < 1\}} (1 - \theta_i) \geq \sum_{\{i|\pi_i R > 1\}} \theta_i. \]

Suppose not. Then,

\[ \sum_{i=1}^{N} \theta_i > \sum_{\{i|\pi_i R < 1\}} 1. \]

This implies that liquidity needs are so large, that investment on some island with \( \pi_i R > 1 \) must be terminated. A contradiction of the definition of \( \Gamma \) in Assumption 3.2. Hence, \( R_{IB} = 1 \) is an equilibrium with investment levels equal to

\[ x_i^1(s) = \begin{cases} 1 & \text{if } \pi_i R \geq 1 \\ 0 & \text{otherwise.} \end{cases} \]

Next, we show uniqueness. Let \( R_{IB} \in (1, R] \) and suppose there exists an equilibrium. In equilibrium the return on investment of funds is then strictly greater than the storage technology. Hence, taking the interest rate \( R_{IB} \) as given all regional banks choose \( x_i^1(s) + z_i(s) = (1 - \theta_i) \) for all \( i \). By market clearing, we have

\[ \sum_{i=1}^{N} x_i^1 = \sum_{i=1}^{N} x_i^1(s) + \sum_{i=1}^{N} z_i(s) = \sum_{i=1}^{N} (x_i^1(s) + z_i(s)) = N - \sum_{i=1}^{N} \theta_i. \]

Hence, all funds are invested after liquidity needs have been satisfied. However, by the definition of \( \Gamma \) in Assumption 3.2, at most \( \sum_{i=1}^{N} x_i^1(s) = N - \Gamma < N - \sum_{i=1}^{N} \theta_i \) projects can be financed to satisfy a return of 1 for the \( N - \sum_{i=1}^{N} \theta_i \) remaining investors at \( t = 2 \). Hence, \( \sum_{i=1}^{N} z_i(s) > 0 \), a contradiction.

Proof of Proposition 4.1

Proof. The objective function (16) is strictly increasing in the number of projects run across islands at \( t = 1 \), \( \sum_{i=1}^{N} x_i^1(s) \). Hence, a broad bank will make the maximum amount of transfers across islands in order to finance as many projects as possible. By Assumption 3.2, liquidity needs are low enough so that all projects on islands with \( \pi_i R > 1 \) can be financed. The maximum transfer islands with \( \pi_i R > 1 \) can make given their participation constraint (21) and full investment \( x_i^1(s) = 1 \) is

\[ \bar{T}_i(s) = \max\{\theta_i(1 - R), (1 - \pi_i R)\}. \]
Suppose transfers from islands with $\pi_i R \geq 1$ are given by $\bar{T}_i(s)$ and set transfers for all other islands equal to $\pi_i R - 1$ in a descending order starting from the largest $\pi_i$ such that $\pi_i < 1/R$ until overall net transfers are zero. This allows the following investment levels

$$x^1_i(s) = \begin{cases} 
1 & \text{if } \pi_i > \tilde{\pi} \\
\tilde{x} & \text{if } \pi_i = \tilde{\pi} \\
0 & \text{if } \pi_i < \tilde{\pi}
\end{cases},$$

where $1/R \geq \tilde{\pi}$ and $\tilde{x} \in [0,1]$ satisfies $\sum_{\{i|\pi_i R < 1\}} \left[ (\pi_i R - 1)x^1_i(s) \right] = \sum_{\{i|\pi_i R \geq 1\}} \bar{T}_i(s)$. These investment levels are feasible, since by Assumption 3.2 there are enough islands with $\pi_i < \tilde{\pi}$ to provide the necessary liquidity $\sum_{i=1}^{N} \theta_i$ at $t = 1$. Also, the rate of return for investors on all islands is equal to 1 as required.

To prove that such investment levels and transfers are optimal, note that any lower amount of transfers reduces $\sum_{i=1}^{N} x^1_i(s)$ and, hence, cannot be optimal. Let $\epsilon > 0$ and consider the following, feasible reallocation of investment

$$\hat{x}^1_i(s) = \begin{cases} 
1 - \epsilon & \text{for some } n \text{ s.th. } \pi_n \geq \tilde{\pi} \\
\epsilon & \text{for some } m \text{ s.th. } \pi_m < \tilde{\pi} \\
x^1_i(s) & \text{otherwise.}
\end{cases}$$

This leads to a rate of return net of investment for investors across all islands at $t = 1$ equal to

$$(1 - \epsilon)(\pi_n R - 1) + \epsilon(\pi_m R - 1) + \sum_{i,i\neq n,m}^{N} x^1_i(s)(\pi_i R - 1) < 0.$$  

Therefore, to satisfy the individual rationality constraints (5) investment has to be reduced further for some island $i$ with $1/R > \pi_i > \pi_\Gamma$. This reduces overall investment and, hence, cannot be optimal.
References


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