STOCK MARKET MODEL

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Introduction

The stock market is a complicated system which itself is a part of an even more complicated system, the economy. Because of the value of a successful result, a lot of effort has been devoted to finding ways of predicting the future behavior of the stock market. Modern portfolio theory is a major part of this effort, and perhaps the most relevant part of MPT in this regard is quantitative portfolio management which appears to be based in large part on factor models (e.g. Grinold & Kahn [1]). Factor models assume that there is a linear relation between the return of an asset and a number of factors relating to the market, the economy, the industry, etc., and sometimes factors which have no clear interpretation. I want to suggest a different approach.

Practitioners often think of the market as an all powerful being over which they have no control. Benjamin Graham talks about Mr. Market, and he suggests investors wait until Mr. Market foolishly sends prices to bargain levels before buying. In fact, the market consists of just three groups of people whose actions can directly influence stock prices, one group (call them managers) being portfolio managers and individuals who make decisions to buy and sell. The other two groups are those (call them investors) who decide when to commit to or withdraw funds from the portfolio managers, and public companies which issue new shares or purchase and retire existing shares. Stock prices are set entirely by the actions of these three groups, although of course the thinking of the groups is influenced by a vast array of external information, including the past behavior of the stock prices themselves.

Thus, more precisely, it should be possible to calculate how stock prices have varied or will vary over a period of time if we know the following information:

- Share prices and the contents of the managers' portfolios at the start of the period in question;
- The rules followed by portfolio managers in deciding when and which shares to buy and sell;
- The rate of flow of investors' cash to or from the portfolios; and
- The rate at which companies by sale or purchase change the number of their shares outstanding.

My initial aim is to set up a model which can make the above calculation given the required information. A simple version of such a model is described below. To an extent even the simple model can be tested by using real world past information and seeing if it predicts something like the observed price behavior. It can also be used to perform experiments to determine how prices and portfolio values would change for certain choices of input information. This should provide help in understanding of how the market works.

The groups, portfolio managers, investors and companies, who directly influence the market, base their decisions on all sorts of exogenous information about the economy, individual companies, government policy, etc., and also perhaps on past behavior of the market. We could predict market behavior if we could predict how the groups adjust the input information in response to this exogenous information and prior market behavior. For example, factor model analysis of the input information coupled with the model calculation would provide a distinct alternative to current methods of forecasting the market. The results would certainly not in general be linear in the factors.

We begin by discussing a number of questions on which the model may be able to shed some light. We next derive the equations which characterize the simple version of the model and then present the results of some sample calculations relating to the questions based on this version of the model as presented below. There is scope for generalizing the model to a very considerable extent. At the end we comment on more general questions concerning the model.

<u>Questions for the Model</u>

Interesting questions from financial economics, some of which are related to each other, include:

<u>The long term trend of increasing US stock prices</u> It is often claimed that the prices of stocks have for many decades increased at a greater rate than those of other financial instruments, and perhaps also at a rate greater than fundamental factors such as productivity growth would warrant (I don't have adequate data on this point, but surely this is true for the last 15 years). It is the main article of faith of the professional investment community that this behavior will continue indefinitely. The usual aim of an institutional portfolio manager appears to be to outperform an appropriate benchmark without worrying about how the benchmark will perform. I think that the most important question for an investment strategist to examine, and one which does not seem to be discussed in any textbook I have seen, is whether the faith in endless, strong growth in stock prices is justified or even plausible.

In the last fifty years participation in the stock market has increased enormously. Pension funds and other institutions have increased the percentage of their assets in equities at the same time as the total assets managed have ballooned because of a continuing infusion of cash. Individuals have dramatically increased their purchases of mutual funds and perhaps also of stocks directly. It would be very interesting to insert the appropriate input information, which should include these effects, into the model to check that it explains the observed major trend in stock prices. If it does this adequately, then we can examine to what extent the above driving forces can and will continue. For instance, we can ask how near to saturation are individual participation levels, and what will happen to stock prices as saturation is reached? What will happen when baby boomers start withdrawing funds to finance retirement? What will happen to prices if the institutions further adjust their asset mix percentages? Using the model to answer such questions might help us understand whether the historical long term stock price growth trend is likely to continue.

<u>The equity risk premium</u> The long term growth rate of stock prices mentioned above is sometimes explained in terms of risk premium to take account of the risk that the company or the economy will collapse. Does this explanation make sense? Is this collapse eventually inevitable, and if so when is it likely to occur?

<u>The immediate cause of rapid short term price changes</u> There have been occasions when stock prices have changed in a short time by a much larger percentage than the percentage of total market value traded in that time (e.g. October 19, 1987). Do we know the structure of the input information for such a period and does the model predict the observed behavior when that input is used? If we don't know all the input information, can we construct a plausible set of input data that would produce the observed behavior when fed into the model? It seems likely that the flow of cash and shares into the system would be too small to explain much of the price variation in

short term significant corrections or rises, and that the main effect must have been due to changes in manager preferences.

Is market capitalization real value? The price of shares can change by a significant amount on very low volume, as above. Does it make sense to value the entire market on the basis of such light trading? We can look into this question with the model by asking how much the stock price would fall if investors decided to sell enough stock to raise cash amounting to a sizable proportion of the cash in the system.

Company share purchases If a company buys back its shares at the market price, is the reduction in the market value of the shares equal to the money spent by the company? The model can address this question.

Change in asset mix percentage If one or a group of institutions increases their portfolio equity percentage, what is the effect on its total portfolio value and on that of other institutions which keep their asset mixes unchanged? The model has an answer to this question.

For example, Canadian Press reported that, on the morning of January 7, 1999, the NYSE fell 1% on the news that market analyst Abby Joseph Cohen had reduced the percentage of stocks in her own portfolio. When it became clear that the reduction was "only modest", from 72% to 70%, the market recovered. It seems that the market does not realize that, if every investor made such a reduction, the market average would fall about 14%, according to the model described below. An example of cognitive dissonance? Not really - investors have not been told about the results of the model.

Volatility Some argue that stock prices exhibit more volatility than can be explained by changes in the fundamental value of the companies involved, thus contradicting one version of the EMH. It seems to me that thousands of managers, even if acting on the same basic information, will arrive at their decisions to trade at different times. If it has not been done already, it might be interesting to calculate the effects of time randomness of this sort in a model that simulates the trading of many managers.

The model may also be used to examine other similar questions.

Model

In the model the market system consists of a number of managers each of whom are responsible for a portfolio of different assets, which they trade amongst themselves. The market system includes all managers whose portfolios contain the assets in question, and, with the exception of the external flows described below, managers can trade assets only with other managers in the system.

In this simple version of the model we restrict attention to the case in which there are just two assets, which we call cash and shares. There is only one type of share which can be thought of as a market index, such as the S&P 500, while cash is a proxy for a combination of bonds, money market funds, etc.

As time passes there will be a flow of cash (possibly negative at times) from the investors into each portfolio (e.g. a pension fund receives cash contributions from its members and pays out funds to retirees). These net cash flows to each manager are regarded as given in the model.

Shares might also be redeemed on payment of cash to the managers whose portfolios contain them by the company that issued them, and similarly new shares might be issued for cash, all based on the current market price of the share. The net flow of shares into the system we take as given.

To complete the model we need to specify the preferences of each manager, which can change with time. Each manager will trade in such a way as to make sure that his portfolio satisfies his preferences. In Economics preferences are usually described by means of a utility function whose value depends on the quantities of the various assets in his portfolio. The manager will aim to maximise his utility. If he receives new cash, he will buy some shares in order to maximise utility under the new conditions, and similarly if the form of his utility function changes.

The dominant portfolio managers in today's market are pension funds and other institutional investors. Many institutions use a top-down investing approach, which begins with a choice of the percentages of funds to be allocated to the various asset classes, in the present model just cash and shares [2]. Thus it seems reasonable for us to describe the preferences of each investor in terms of the fraction of the total portfolio value that is in shares. We call this the Asset Allocation Ratio (AA ratio) [3]. The AA ratios will change with time, and will in general be different for each manager. In the model the time dependence of each AA ratio is regarded as being given. It turns out that this description of manager behavior is equivalent to the choice of utility function of a particular, simple form. We could extend the model by using other forms of utility function.

In practice the above ratios and the external flows of cash and shares, the information that determines the behavior of the market in the model, might be discontinuous functions of time, which will lead to erratic changes in share prices. However, we are generally interested in long term fluctuations only, so we assume that the externally determined functions have a sufficiently smooth dependence on time. The number of shares traded in a given, small time interval will be proportional to the length of the interval. In that interval an manager who trades will exchange small amounts of cash and shares with another manager on the basis of the current market price for the shares.

With the smoothness assumption, the model is characterised by a set of coupled, first order, nonlinear, ordinary differential equations with time as the independent variable. The equations are derived in the next section. I have written a FORTRAN program which will solve these equations numerically with adequate accuracy.

<u>Equations.</u> Let us suppose that there are M managers labelled from 1 to M, and that manager J has an amount C(J) of cash and a number N(J) of shares. If the price of a share is P, we denote the value his portfolio by V(J), <note that * means multiply>

$$V(J) = C(J) + P*N(J).$$

For manager J the AA ratio G(J), the fraction of the value of his portfolio in shares, is defined by

$$G(J) = P*N(J)/V(J), \quad \text{i.e.} \quad P*N(J) = G(J)*V(J) \qquad (1)$$

In the model trading begins at time T = 0 when all the above functions have specified values. We assume that cash from outside sources, or interest and dividends on the portfolio, is added to portfolio J in such a manner that a net amount of cash R(J) has flowed into the portfolio up to time T. We also assume that a net amount of S new shares have been bought from the company up to time T by all the managers.

All the above functions depend on time T. The functions G, R and S are specified externally, while C, N, V and P are obtained by solving the differential equations derived below, given their initial values.

We use the symbol G' to represent the time derivative of G. From the notion that an manager will preserve value when trading we obtain for each J

$$C(J)' = R(J)' - P*N(J)' \qquad (2)$$

This equation is equivalent to the statement that, for a small interval of time, the increase in the portfolio's cash balance is equal to the amount of cash flowing in from external sources minus the amount spent to buy additional shares.

Differentiating the definition of V(J) we find

$$V(J)' = C(J)' + P*N(J)' + P'*N(J)$$

which, with (2), gives for J = 1, M

$$V(J)' = R(J)' + P'*N(J) \qquad (3)$$

Differentiating the definition of G(J) leads to

$$N(J)*P' + N(J)'*P = G(J)*V(J)' + V(J)*G(J)'$$

Substituting (3) and rearranging produces for J = 1, M

$$N(J)' = \{V(J)*G(J)' + G(J)*R(J)' + P'*N(J)*\{G(J) - 1\}/P \qquad (4)$$

The set of equations will be completed with an expression for P'. This can be obtained by summing (1) over J, so that

$$\begin{aligned} SUM\{G(J)*V(J)\} &= P*SUM\{N(J)\} \\ &= P*\{SNINIT + S\}, \end{aligned}$$

where SNINIT is the initial total number of shares outstanding.

Differentiating gives

$$\begin{aligned} P'*\{SNINIT + S\} + P*S' &= SUM\{G(J)'*V(J) + G(J)*V(J)'\} \\ &= SUM\{G(J)'*V(J) + G(J)*\{ R(J)' + P'*N(J)\}\} \end{aligned}$$

from (3). Rearranging we have

$$\begin{aligned} P' = \{SUM\{G(J)'*V(J)\} - P*S' + SUM\{G(J)*R(J)'\}\} \\ /\{SNINIT + S - SUM\{G(J)*N(J)\}\} \end{aligned} \qquad (5)$$

The sets of equations (3) and (4) for J = 1, M, and the equation (5) constitute the required set of 2M + 1 coupled differential equations for N, V and P. Once these are solved, C is found from the definition of V.

<u>One Manager.</u> The simplest case to consider is when all managers use the same AA ratio function G and the flow of cash into each manager is in proportion to his initial cash holding. In this situation there will be no trading between managers and we might as well assume that there is only one manager [4]. We may assume that G, the number of shares N, and the amount of external cash added R are given functions of time. The dependence of cash C and price P on time has to be determined.

After using (1) equations (4) and (5) both reduce to

$$P'*(1 - G) = P*(G'/G - N'/N) + GR'/N \qquad (6)$$

a linear first order differential equation for P which can be solved by quadrature.

This simple equation might provide an approximation to the behavior of the average values of the quantities in a situation with several managers. It is interesting to consider some special cases.

Case 1: Assume that the number of shares N is constant. In this case no shares are being issued or redeemed and at any time

$$C = CI + R$$

where CI is the initial cash holding. We don't need to bother with the differential equations. We have at all times

$$P*N = G*V = G*(C + P*N)$$

so that

$$P*N*(1 - G) = G*C$$

and thus, with N constant,

$$PF/PI = [\{GF/(1 - GF)\}/\{GI/(1 - GI)\}]*[CF/CI] \qquad (7)$$

where I, F refer to the initial, final values of the variables.

Case 2: This time we assume that the AA ratio G is constant in time and that the amount of external cash flow R is zero. We allow the number of shares N to change in a predetermined manner. Equation (6) reduces to

$$P'/P = N'/(N(G - 1))$$

which may be solved to give

$$PF/PI = (NF/NI)**ALPHA \qquad (8)$$

where ALPHA = 1/(G - 1).

[cash flow can be calculated]

From these formulae we can describe how the share price in the case of one manager reacts to a change of the three primary factors one at a time, with the other two factors held fixed.

- *Cash from external sources* If the net amount of cash in the market is multiplied by a factor then the stock price increases by the same factor, assuming that the number of shares and the AA ratio remain fixed. [There is nothing surprising about this result. An influx of funds from Baby Boomers can drive up stock prices beyond a level provided by 'intrinsic value'.]

- *Shares issued or redeemed* If the number of shares outstanding is multiplied by a factor, with no external cash inflow and AA ratio fixed, then the share price varies according to equation (8). The table below shows the value of PF/PI for a number of choices of NF/NI and the AA ratio G.

	NF/NI					
AA ratio	**3**	**2**	**1.5**	**2/3**	**1/2**	**1/3**
20%	0.253	0.420	0.602	1.66	2.38	3.95
30%	0.208	0.371	0.560	1.785	2.69	4.80
40%	0.160	0.315	0.509	1.97	3.18	6.24
50%	0.111	0.250	0.444	2.25	4.00	9.00
60%	0.0642	0.177	0.363	2.76	5.66	15.6
70%	0.0257	0.0992	0.259	3.86	10.1	38.9
80%	0.00412	0.0313	0.132	7.59	32.0	243.

  It appears that the issue or redemption of a significant number shares can have a major effect on share prices, particularly if the average AA ratio is high.
- *AA ratio changed* If the proportion of stocks in the portfolio is increased while the number of shares remains fixed and there is no cash inflow then the stock price will increase. The amount of the increase will depend on the initial ratio. The table below gives the percentage increase in the stock price when the AA ratio increases by 10 percentage points from the indicated initial levels. There are corresponding decreases in share price if the AA ratio is decreased.

Initial AA ratio	**Increase in share price**
20%	71%
30%	56%
40%	50%
50%	50%
60%	56%
70%	71%
80%	125%

Note that the effect of a change in AA ratio is least when cash and stocks are held in equal proportions, but even at that point the change in share price is five times the percentage point change in AA ratio. The effect near the extremes in AA ratio gets much larger. Given the propensity for many managers to favor stocks, and therefore increase their AA ratio, after a period of gains in share prices, it is easy to see how a bubble can develop in the stock market.

Numerical Results We present a few numerical results for the cases of two and three managers. It is simple to generate many more. We have taken the external parameters, AA ratios, cash flows and share flow, to vary linearly with time. In every case the initial price of a share is unity. The symbols in the Tables have the following meaning:

M: Manager
CI, CF: Initial, final cash holding. NI, NF: Initial, final number of shares.
VI, VF: Initial, final portfolio value. GI, GF: Initial, final AA ratio (= %value in shares).
R: Cash in, not including that due to company share repurchase.
S: Total number of new shares issued to all managers.
PF: Final share price.

Case 1: Increase G1 from 50% to 60%; VI1 = VI2.

M	CI	CF	NI	NF	VI	VF	GI	GF	R	S	PF
1	0.500	0.447	0.500	0.548	1.000	1.117	0.500	0.600	0.000	0.000	1.223
2	0.500	0.553	0.500	0.452	1.000	1.106	0.500	0.500	0.000		

Case 2: Increase G1 from 50% to 70%, VI1 = VI2.

M	CI	CF	NI	NF	VI	VF	GI	GF	R	S	PF
1	0.500	0.385	0.500	0.594	1.000	1.283	0.500	0.700	0.000	0.000	1.513
2	0.500	0.615	0.500	0.406	1.000	1.230	0.500	0.500	0.000		

Case 3: Increase G1 from 50% to 70% ; VI1 >> VI2.

M	CI	CF	NI	NF	VI	VF	GI	GF	R	S	PF
1	0.900	0.854	0.900	0.932	1.800	2.846	0.500	0.700	0.000	0.000	2.138
2	0.100	0.146	0.100	0.068	0.200	0.292	0.500	0.500	0.000		

Case 4: Increase G1 from 50% to 60%, decrease G2 fro 50% to 40%; VI1 =VI2.

M	CI	CF	NI	NF	VI	VF	GI	GF	R	S	PF
1	0.500	0.400	0.500	0.600	1.000	1.000	0.500	0.600	0.000	0.000	1.000
2	0.500	0.600	0.500	0.400	1.000	1.000	0.500	0.400	0.000		

Case 5: Add cash to M1, Gs unchanged; VI1 = VI2.

M	CI	CF	NI	NF	VI	VF	GI	GF	R	S	PF
1	0.500	0.576	0.500	0.523	1.000	1.151	0.500	0.500	0.100	0.000	1.100
2	0.500	0.524	0.500	0.477	1.000	1.049	0.500	0.500	0.000		

Case 6: Add cash to M1, M2; VI1 = VI2 but different Gs from above; increase Gs by 20%.

M	CI	CF	NI	NF	VI	VF	GI	GF	R	S	PF
1	0.700	1.246	0.300	0.279	1.000	2.491	0.300	0.500	0.500	0.000	4.467
2	0.600	1.254	0.400	0.421	1.000	3.136	0.400	0.600	0.700		

Case 7: As Case 6 but add new shares.

M	CI	CF	NI	NF	VI	VF	GI	GF	R	S	PF
1	0.700	1.069	0.300	0.370	1.000	2.138	0.300	0.500	0.500	0.210	2.892
2	0.600	1.042	0.400	0.540	1.000	2.605	0.400	0.600	0.700		

Case 8: As Case 6 but redeem shares.

M	CI	CF	NI	NF	VI	VF	GI	GF	R	S	PF
1	0.700	1.567	0.300	0.189	1.000	3.134.	0.300	0.500	0.500	-0.21	8.281
2	0.600	1.660	0.400	0.301	1.000	4.151	0.400	0.600	0.700		

Case 9: As Case 6 but add third manager with G = 100%; add cash to M3.

M	CI	CF	NI	NF	VI	VF	GI	GF	R	S	PF
1	0.700	1.467	0.300	0.197	1.000	2.933	0.300	0.500	0.500	0.000	7.437
2	0.600	1.533	0.400	0.309	1.000	3.833	0.400	0.600	0.700		
3	0.000	0.000	1.000	1.194	1.000	8.876	1.000	1.000	0.500		

Case 10: As Case 9 but M3 starts with fewer shares.

M	CI	CF	NI	NF	VI	VF	GI	GF	R	S	PF
1	0.700	1.467	0.300	0.197	1.000	2.933	0.300	0.500	0.500	0.000	7.437
2	0.600	1.533	0.400	0.309	1.000	3.833	0.400	0.600	0.700		
3	0.000	0.000	0.500	0.694	0.500	5.157	1.000	1.000	0.500		

Commentary

Here we attempt to place our ideas in a broader perspective. We make some remarks which can perhaps later be made more coherent.

Investment Science   This is the title of a recent standard MBA style text by Luenberger [5] written from a slightly more mathematical viewpoint that many others. For one who has spent a career in Theoretical Physics, the first quantitative science, this title is a little hard to digest. The academic financial economics community, backed up by the investment practitioners, would have us believe that Modern Portfolio Theory has in the last 50 years provided a scientific basis for investment management comparable to the theories of Modern Physics (Three Nobel prizewinners can't be wrong - Bank of Montreal).

I dispute this view. Our understanding of Physics is at a number of levels, which we can liken to an onion. The outer layer is based on the results of measurements and often experiments on the systems of interest. Vast quantities of data can be collected and the first step is to find regularities and structure in the data. A modern way to do this is with the help of the mathematical discipline called Approximation Theory (which I have worked in for a long time). It is assumed that the variables measured to give the data are functions of certain independent variables, and Approximation Theory investigates ways in which these functional relations can be described approximately by simple forms.

In investing we are at this first level. We have enormous data banks of historical stock prices and other company and economic information. The principal achievement with practical significance to investing in the stock market seems to be the introduction of multi-linear factor models which, it is hoped, will approximate the relation of stock prices to other variables. These are purely empirical models chosen, I suspect, mainly for their simplicity and they are not derived from any more fundamental theory.

In relation to the motion of the planets Physics was at this stage several hundred years ago. The second layer of the onion was opened by Newton who suggested that all the data telling where the planets had been at any time could be explained in terms of the solution of some equations which could be written in a few lines, although the precise calculation in cases of practical interest could be done only with computers. The solution of the equations led to accurate predictions of the location of the planets at future times.

It is believed that Newtonian Physics describes many phenomena in the natural world with high precision. Even though accurate calculations for complicated systems are still impossible, scientists believe, but can never prove, that the motion of the systems follows certain conceptually simple laws.

In Physics there are several more layers to the onion. When it comes to atoms, Newtonian Physics must be replaced by Quantum Mechanics, another elegant theory which can involve calculations that are still beyond our computing capacity in many cases, but everyone is confident that the basic laws are known at this level. There are more layers beneath this one.

In the financial economics of the stock market, there is no sign of an underlying fundamental theory comparable to Newton's laws. Neither do I know of any reason to expect that there will be such a theory when what transpires in the market depends on the psychology of the people involved. The success of Newtonian and Quantum Mechanics is a truly remarkable phenomenon. Nobody knows why the basic laws governing nature have such a simple form, but the requirement of symmetry and elegance enabled Dirac to derive his equation combining Quantum Mechanics and relativity that implied the existence of anti-particles, later found in nature.

<u>Cause and Effect</u> The aim of science is to look behind an effect to determine a cause. It is not merely to find an ad hoc way to describe what happened or even predict what will happen in the future. Certainly a wide variety of factors will have an effect of stock prices, but we are asserting that all the information about these factors flows into stock prices through the minds of the three groups mentioned above, managers, investors and companies. The last link in the chain of cause and effect that ends in stock prices is the mechanism described by our model which translates AA preferences and decisions on the exogenous flow of cash and shares into stock prices.

Since we understand this last link it seems sensible to incorporate this mechanism into the process of predicting stock prices. Thus, instead of trying to predict stock prices directly as in MPT, we are suggesting that we make an effort to predict what will be the future AA choices of the managers and the exogenous flows of cash and shares. I believe that there is historical data on these numbers so that techniques similar to those used at present on stock prices could be used to approximate and extrapolate the input data needed for the model. Thus, for example, data for the past AA ratios of pension funds and mutual funds could be related via a linear factor model to economic data and the AA recommendations of brokerage houses (which perhaps now become public fast enough for them to have some predictive power of fund AA ratios; if this turns out to be the case these ideas should be kept secret).

Expectations We do not believe the validity of the assumptions of eminent economists such as Arrow (General equilibrium theory) and Sharpe (Capital asset pricing model) that economic agents, such as the managers in our model, have complete knowledge of all relevant aspects of the economy, including the preferences of all other agents, and neither do we believe that they have the ability to accurately compute how the economy will perform in the future given this information. We are more inclined to accept the view of Dreman [6] that even large organizations are incapable of making full use of the vast quantity of information available to them. We agree with Kurz [7] that rational well trained people can come to different conclusions based on the same, inevitably incomplete, set of information.

Thus the model can incorporate a number of different managers with a spread of opinions and choices of AA ratios, who can react in different ways to new information. Moreover, we believe that the expectations and decisions of the managers will be influenced by psychological factors to depart from what might be regarded as rational. We would like to take into account the above points when predicting the managers'AA ratios. We think that an advantage of the model compared to the standard approach is that managers' expectations and decisions are made explicit rather than being hidden as before.

A manager wishing to follow the approach of our model would devote resources to understanding how her competitors operated. Frankfurter and McGoun [8] call this grounded field theory. The research is not quantitative, but in order to use it we would have turn it into numerical predictions of their choice of AA ratio in various circumstances. There has already been some research in this direction, such as O'Barr and Conley [9]. We could say that the aim would be to produce an expert system to mimic the actions of a manager - actually we need several different versions to represent the diversity of managers. I note that Kurzweil [10] intends to create an artificially intelligent financial analyst that will outperform humans. My goal is a little less ambitious. We just need to know how the existing humans work and then we should be able to outperform them.

Perhaps we are preaching to the converted. A recent article in The Economist stated "Now, however, financial economics is in some disrepute and disarray. ... Andrew Lo of MIT, thinks that eventually the models will include the human and institutional factors that recent events have shown to be so crucial. However, another branch of economics known as "behavioural finance", which tries to use the insights of psychology to explain apparent market inefficiencies, has so far failed to produce a convincing theoretical model."

More Practical Considerations To determine if the model has any relevance to the real world requires data and effort. It would be interesting to find out whether the behavior of the stock market over the past few years can be described by the model. Thus we might proceed as follows:

System: The market could be all publicly traded stocks in the US, or whichever collection of stocks is most convenient with regard to the data available. We need to take account of all holders of these stocks. The pension funds, insurance companies, etc. should fit into our framework with little trouble, and these institutions could be handled by a few representative managers. We could deal with mutual funds in the same way by considering all their holdings, including bonds.

Alternatively, we could omit bond funds but include balanced funds as before. Equity funds and individual equity investors, including foreigners, could be represented by a single manager with an AA ratio of 1. The crucial information for this manager would be the flow of cash to be invested by that manager.

Some method would have to found to aggregate all the relevant information to fit the model, but no doubt this a problem often encountered by economists.

Data: I have seen indications in various places that a lot of the historical data needed for the model may be publicly available. For example the AA ratios of pension funds, the flow of funds into and out of pension funds, the flow of cash into mutual funds and its destination (equity or bond), individual investor and foreigner net purchases of equities, dividends and interest earned by manager assets, the issue and redemption of shares by companies, etc.

Calculation: The equations can be solved numerically for whatever period the external data is available. Data given only at discrete points in time will have to be interpolated as in the version above.

Results: The principal output for comparison with actual data is the behavior of the stock price index over time.

Notes and References

1. Richard C. Grinold and Ronald N. Kahn, "Active Portfolio Management", Probus, 1995.
2. Even if the manager does not consciously aim at a particular AA ratio, what ever process she does use will have the effect of managing to achieve the observed ratio, and this is how we describe her preferences. (But if the AA ratio chosen in a way that is dependent on the price, then the equations would have to be changed.)
3. Our use of the concept of asset allocation bears no relation to some views that are widely, but incorrectly, held in the investment community. See J. A. Nuttall and J. Nuttall, "Asset Allocation Claims - Truth or Fiction?", unpublished report.
4. We should perhaps expand on this point. I feel uncomfortable about a price which is not set by means of trading. There will be no trading between two managers identical in all respects. However, there would be trading if they differed by very small amount in some respect. The nature of the equations shows (rigorously, I presume; it does numerically) that the solutions are smooth functions of the external parameters. We just have to imagine that the two identical managers differ very slightly from one another. We can use the equations for identical managers, or equivalently the equation for one manager, to calculate a good approximation to the actual solution.
5. David G. Luenberger, "Investment Science", OUP, 1998.
6. David N. Dreman, "Contrarian Investment Strategies : The Next Generation", Simon & Schuster, 1998.
7. Mordecai Kurz, editor, "Endogenous Economic Fluctuations", Springer, 1997.
8. George M. Frankfurter and Elton G. McGoun, "Toward Finance with Meaning", JAI, 1996.
9. William O'Barr and John Conley, "Fortune and Folly", Irwin, 1992.
10. Ray Kurzweil, in Forbes, November 30, 1998, p. 182.