Ecological Computations Series (ECS): Vol. 3

ENTROPY AND INFORMATION

László Orlóci

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Ecological Computations Series (ECS: Vol 3)

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Contents

Preface

Information theoretical tools are described which help the user to quantify such structural properties as diversity, mutuality[1] and equivocation. Rényi's generalized entropy and information are the basic physical quantities. Unlike the familiar Shannon, Brillouin, Kullback measures (SBK), Rényi's entropy and information have "order". This is a potent and desired quality when a goal is to achieve structural descriptions of generality and flexibility.

The conventional SBK measures supply point descriptions of community and population structure. These contrast with the Rényi measures which allow viewing community and population structures under conditions of changing order. Changing order generates a scale process in which magnification and sharpness interplay to discriminate between cases. But the choice of order is arbitrary and

[1] Interaction, association.

some may not be prepared to make this choice. Instead, they may opt for vector descriptions or curves as shown in the 2nd chapter. On these curves the SBK quantities are points. The Shannon entropy, for instance, as a 1st order measure is a point in the vicinity of an infinitesimal break on the curve. On one side lies the Simpson point, a 2nd order entropy. The log state (species) richness index, the 0 order entropy measure, is an extreme point on the opposite side. It is well to remember that the structural magnification at these lower order entropy measures is rather poor, being worst at order zero. It should also be noted that any point on the entropy curve at orders greater than one can serve as a diversity measure with more discriminating power than the Shannon or Brillouin index. In a similar vain, the idea that association or interaction can have different orders will render the Kullback statistic (MDIS) or the Pearson's χ^2, which are 1st order measures, not so attractive for the ecologist.

The successive sections will clarify these propositions and also offer guidance to programs in CANAPACK which compute entropy graphs, entropy estimates and information estimates under process sampling[2]. The programs are conversational and completely self-contained. They run on any Macintosh in good order with a reasonably large RAM and disk memory. The minimum required RAM size will depend on the size of the code and on the size of the arrays. Code sizes are obtainable for individual applications from the disk directory. The arrays depend on the data to be analyzed. Since the arrays are dynamically defined, the

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[2] Orlóci and De Patta Pillar 1989. -- The proposition in process sampling is reminiscent of Poore's (1955,1956) successive approximation approach and the flexible analysis of Wildi and Orló ci (1987). The term "process" conjures a view of sampling in which step-by-step expansions are intricately tide to a monitoring of the evolution of the sample structures and structural connections in concurrent data analysis, based on which stability is judged. When structural stability is detected the sampling stops. Juhász-Nagy and Podani (1983), Podani (1984), Orlóci (1988), Kenkel, Juhász-Nagy and Podani (1989) and other works, to which they refer, are relevant references.

computer's memory use is the memory needed to accommodate the data. Insufficient memory will stop the processing with or without an error message.

Flexibility in output handling is built into the programs. Minimum results are stored in a PRINTDA file. Intermediate results are retained only if so requested during the start-up dialogue. The runs do not require immediate access to a printer. The PRINTDA files are editable and printable in the public domain program EDIT or in other word processing applications. The PRINTDA file has to be opened from the application. The graphs drawn on screen are stored automatically as picture files which can be edited and printed in a PAINT program.

The book contents are arranged in four parts. The first part contains a discussion of concepts and definitions. The second is on entropy graphs, the third on entropy estimation, and the forth on information estimation. Data type, data entree and the mechanics of running the programs on Macintosh are discussed. The program package INFOPACK and sample data files are offered on one high-capacity diskette.

In this project as in all the others before it, I had Márta's support. Elmond-hatatlanul szerencsés vagyok.

László Orlóci

Gorizia, September 1990

1
Concepts

The idea that entropy expresses disorder is central in science. Interestingly one of the most fundamental physical laws, Boltzman's 2nd law of thermodynamics, is about entropy and disorder. The relationship is such that entropy is low in orderly systems and high in disorderly systems. In translation, increasing entropy is the ally of stability, the omega state in the direction of which natural systems are pointed as they march through their evolution. By the same token, increasing entropy is also the ally of unpredictability in which lies the paradox that predictability is not a trait of natural stability.

In the most general case the tendency of increased entropy is a property of an expanding universe[3]. With this said but not explained, entropy is regarded a key

[3] Increasing entropy gives time its "arrow", says A. Eddington, and makes us remember the past but not to know the future, remarks S. W. Hawking.

physical attribute of the bioenvironmental system. Through the measurement of entropy, some believes[4] the very essence of the bioenvironmental process can be captured.

It is customary to measure entropy as a logarithm of proportions,

$$H = - K \sum_{i=1}^{n} p_i \; log \; p_i \qquad Eq. \; 1.1$$

This is Shannon's fundamental equation for the description of the symbol structure in signals that carry the message from source to destination. Anything that can distort the symbol structure and cause a discrepancy between the messages sent *(x)* and the message received *(y)* is called "noise"[5]. This discrepancy is a source of uncertainty at the time of interpretations. To measure it, Shannon uses

$$H_x(y) = - K \sum_{j=1}^{n} p_{ij} \; log \; p_i(j) \qquad Eq. \; 1.2$$

which he termed *equivocation information*[6]. Others refer to $H_x(y)$ as *specific entropy* unique to *x* in comparison to entropy in *y*. $H_x(y)$ is high when the structural distortion in the received message is high. Obviously $H_x(y)$ need not be the same as $H_y(x.)$

[4] R. Margalef asserts that as the system evolves, entropy "grows" about itself.
[5] This is irrespective of the substance of the message.

[6] $p_i(j)$ is the conditional probability of symbol j in y for given symbol i in x, defined by

$$p_i(j) = \frac{p_{ij}}{p_i} = \frac{f_{ij}}{f_i}$$

In this f_{ij} is the joint frequency of the ith and jth symbols.

Shannon's entropy function is well connected to the most general information theoretical functions that Ecology has used to measure community and population level diversity. For instance, $\frac{H}{ln\,2}$ is an approximation to

$$I = log_2 \frac{f_.!}{f_1!\,f_2!...f_n!} \qquad Eq.\ 1.3$$

which Brillouin termed "information"[7]. Note that $f_.$ is a sum of frequencies $(f_1 + f_2 + ... + f_n)$ which, in Shannon's terms happens to be the total length of the message. The message contains n δ $f_.$ different symbols with frequencies $f_1, f_2, ..., f_n$. Another connection is to generalized entropy,

$$H_\alpha = \frac{1}{1-\alpha} ln \sum_{i=1}^{n} p_i^a \qquad Eq.\ 1.4$$

---

[7] log_2 is logarithm to base 2, and "bit" identifies the unit in I. The unit "1 bit" is equal to

$$log_2\,2$$

and *n* bits of information is conveyed in

$$I = log_2\,2^n\,.$$

In general the distribution $[f_1\,f_2\,...\,f_s]$ conveys I bits of information, or $\frac{I}{f_.}$ on average per observation. The quantity I is what Brillouin defines as "information" and $\frac{I}{f_.}$ is what he calls entropy, or average information. Brillouin shows that when the f_i are large, say 100 or greater, $\frac{I}{f_.\,ln\,2}$ will come close in value to Shannon's entropy function.

which Rényi derived as "entropy of order α". In Rényi's terms, Shannon's entropy is 1st order entropy[8].

The term "information" means different things to different authors. What Brillouin describes as information *(Eq. 1.3)* is a multiple of the quantity that others describe as entropy in a single distribution. Brillouin's information is not the same as Rényi's which is a divergence measure on two distributions

$$\boldsymbol{P} = (p_1\ p_2\ \ldots\ p_n) \text{ and } \boldsymbol{Q} = (q_1\ q_2\ \ldots\ q_n)$$

and which has order (α):

$$I_\alpha = \frac{1}{1-\alpha} ln \sum_{i=1}^{n} \frac{q_i^a}{p_i^{a-1}} \qquad Eq.1.5$$

When α approaches 1, i is information in Kullback's terms

$$2I = 2f.\sum_{i=1}^{n} q_i \ ln \frac{q_i}{p_i} \qquad Eq.1.6$$

The elements in $\boldsymbol{P}$ and $\boldsymbol{Q}$ are uniquely paired so that every q_i has a corresponding p_i. No restrictions need be applied regarding the distribution totals or the level of summation in *Eq.1.5*, excepting the Kullback's manipulations in which case the two distributions must have equal totals and the summations must run through all n terms. The latter is desired when analytical and probabilistic connection are sought between "information" and Pearson's chi-squared.

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[8] Eq. 1 is not defined for $\alpha = 1$. The derivation is not simple and for details the l'Hospital's rule of calculus should be revisited.

The applications of information theoretical notions in Biology have derived their conceptual basis and also to some extent their methodology from the classics: C. E. Shannon, L. Brillouin, S. Kullback, A. Rényi[9]. It is clear that information theory offers the advantage of universal identifiability when describing population and community structures and structural connections. The specific methodologies that are centered around the cases of *Eq.1.1* and *Eq.1.6* include systems modelling, diversity estimation, and statistical data analysis:

ENTROPY BASED BIOLOGICAL MODELLING. Very much in fashion in the 1950's, the early efforts ended in disillusionments -- according to the story narrated in Yockey, Platzman and Quastler (1958.) The early models apparently produced no new biological insight. This should of course not reflect negatively on information theory, but rather it should point up the inadequacy of knowledge at that time about the systems that they tried to model.

DIVERSITY ESTIMATION. The idea of uncertainty being at its maximum when entropy is at its highest, has made the entropy concept a foundation of diversity theory. In fact reasoning from entropy appears early in work on community structure[10], but the applications that followed were straight jacketed by the authors not recognizing that diversity can have different orders. In this respect the full impact of A. Rényi's mathematical work has yet to materialize in diversity studies[11].

STATISTICAL DATA ANALYSIS. The analysis of community and population level relationships is among the more developed fields in the biological applications of

[9] See the Bibliography for key references.
[10] See R. Margalef's and E. C. Pielou's seminal works on the topic (1958, 1975).
[11] O. H. Hill's (1973) work which my comments on his early manuscript triggered is exceptional, but has apparently not been followed.

information theory, but these too are largely limited to 1st order information divergence measures[12] which largely owe their familiarity to S. Kullback's seminal work on an information based statistical methodology. A. Rényi's umbrella theory should spear further developments in these and other realms of data analysis not yet charted by applied work.

[12] Kullback (1959, 1968) and also Rényi (1961), Rajski (1961).

2

Entropy graphs

The frequency distribution of a single variable is the basic data source and *Eq. 1.4* is the entropy function. The entropy graph is the graph of this function generated by the process of changing the value of α. Any value 0 and up is permitted except exactly 1.

2.1 The data

The states of a discrete variable X are involved. The *jth* state has frequency f_j. There are s states and the frequency distribution is

$$\boldsymbol{F} = (f_1 \ f_2 \ \dots \ f_s)$$

The distribution total is $f_.$[13]. The states may or may not have a unique order.

2.2 Descriptors of *F* and limits

The descriptors include the number of cells s and a graph of Rényi's entropy as a function of α. $\boldsymbol{F}$ has two limiting distributions,

$$\boldsymbol{F}_l = (f_. - s + 1 \;\; 1 \ldots 1) \quad Eq.\ 2.2$$

the least dispersed, and

$$\boldsymbol{F}_m = (\bar{f} \;\; \bar{f} \;\; \ldots \;\; \bar{f}) \quad Eq.\ 2.3$$

the most dispersed. In the latter $\bar{f} = .$ $\boldsymbol{F}_l$ and $\boldsymbol{F}_m$ are both s-valued and $f_.$-totalled. The entropy in $\boldsymbol{F}_l$ is defined by

$$H_a = \frac{1}{1-\alpha}\ln\left[\frac{(f_. - s)^\alpha}{f_.^{\ \alpha}} + (s-1)\frac{1}{f_.^{\ \alpha}}\right] \quad Eq.\ 2.4$$

This is the possible lowest entropy in an s-valued distribution with $f_.$ total and given α. The entropy in $\boldsymbol{F}_m$,

$$H_\alpha = \ln s \quad Eq.\ 2.5$$

is the possible highest entropy in an s-valued frequency distribution regardless of $f_.$ or α.

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[13] A dot in the subscript indicates summation over the subscript replaced by the dot. For example, $f_{1.} = f_{11} + f_{12} + \ldots + f_{1k}$.

2.3 **Application** EntropGraphs

This program computes entropy quantities of different order based frequency or density data. Any number of distributions are permitted and in each case H_α graphs are drawn for $\boldsymbol{F}_m$, $\boldsymbol{F}$, $\boldsymbol{F}_l$ in that order. The α axis is scaled from 0 to a specified upper α value in 1/1000 parts. Tick marks are places as requested. The drawing automatically adjusts to screen size on tested equipment.

2.4 Sample data

Raunkiaer's biological spectra of different locations given in Braun-Branquet[14] are analyzed:

Spectrum name	Life-form				
	F	Ch	H	G	Th
Normal	46	9	26	6	13
Spitzbergen	1	22	60	15	2
Death Valley	26	7	18	7	42
Seychelles	61	6	12	5	16
Connecticut	15	2	49	22	12
Paris basin	8	6	52	25	9

Life-forms are the survival types of plant individuals, such as phanerophyte (F), chamaephyte (Ch), hemicryptophyte (H), geophyte (G), and therophyte (Th). This system is particularly well suited for use in character based community studies.

2.5 Sample calculations

In the Raunkiaer data set s is uniformly 5 and $f_.$ is uniformly 100. Such a uniformity is not required as a rule. The maximum entropy (Eq. 2.5) is uniformly

[14] 1932, p. 298.

$$H_O = ln\ 5 = 1.60944 \text{ or } 2.32193 \text{ bits}$$

regardless of α. The minimum entropy (Eq. 2.4) is also the same for each of the spectra, but this depends on α.

Consider the normal spectrum

$$\mathbf{F} = (46\ \ 9\ \ 26\ \ 6\ \ 13)$$

and the case of $\alpha = 0.7$. For this,

$$f_. = 100$$

$$H_{0.7} = \frac{\ln(0.46^{0.7} + 0.09^{0.7} + 0.26^{0.7} + 0.06^{0.7} + 0.13^{0.7})}{1 - 0.7}$$

$$= 1.42796$$

$$\max H_{0.7} = 1.60944$$

$$\min H_{0.7} = \frac{\ln\ (0.96^{0.7} + 4 \times 0.01^{0.7})}{1 - 0.7} = 0.41055 \ .$$

Similar computations emit other H_α values for any α except $\alpha = 1$ in which case *Eq. 1.1* is appropriate:

$$H_1 = 0.46\ ln\ 0.46 + 0.09\ ln\ 0.09 + 0.26\ ln\ 0.26 +$$
$$+ 0.06\ ln\ 0.06 + 0.13\ ln\ 0.13 = 1.35819$$

$$\min H_1 = -(0.96\ ln\ 0.96 + 0.04\ ln\ 0.01) = 0.22340$$

2.6 Creating data file

Data are presented for analysis in an ASCII text file on disk. For the Raunkiaer spectra, this file is 30-valued:

46
9
26
6
13
1
22
60
15
2
26
7
18
7
42
61
 6
12
5
16
15
2
49
22
12
8
6
52
25
9

Note that the data file does not contain zeros (as a rule) and there are no blank lines in the file (also a rule.) The data file begins with a number (no leading blank lines), data entree is by distribution, and each number entered is followed by an END-OF-

PARAGRAPH mark, except the last number where this is optional. The END-OF-PARAGRAPH mark is created by pressing the RETURN key after typing the number.

2.7 **Running** EntropGraphs

After the data file is created open a WORK folder and drag EntropGraphs and the data file (Raunkiaer.dat) ikons to this folder. Run EntropGraphs from the WORK folder by clicking twice on its ikon. With this, the start-up dialogue begins (*Fig. 2.7.1.*):

1. If application and data file are not in the same folder or outside any folder on disk and the run already started, respond on the first screen line by pressing key N. This will stop the run. Drag application and data ikons to same folder and try again. If instead of N key Y is pressed, the run will continue and new specifications are requested.

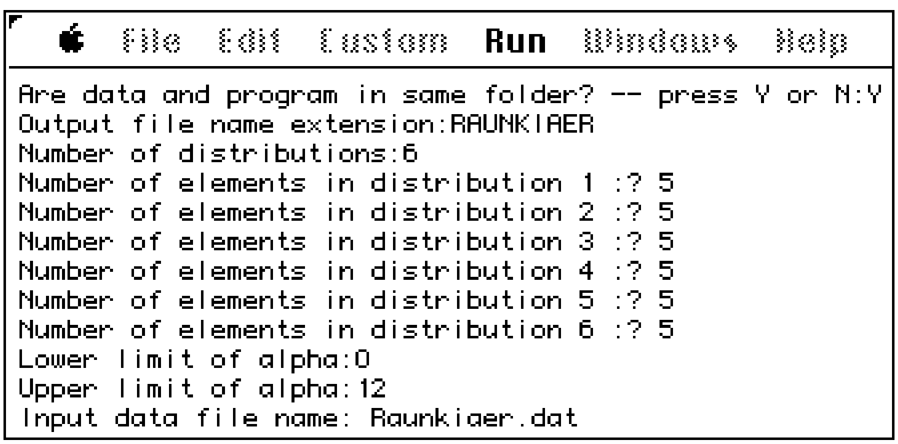


Fig. 2.7.1 First screen showing the start-up dialogue as the EntropGraphs run gets under way.

2. The output file name extension identifies the PRINTDA file where the PRINT output and run information are stored, and the PICT file(s) in which the entropy graphs are stored. For example, if RANUNKIAER (lower or uppercase) is typed as the output file name extension, the print file will have full name PRINTDA.RAUNKIAER and the PICT files will have full names PICT.RAUNKIAER/1, PICT.RAUNKIAER/2, etc. There will be as many PICT files created in the run as there are distributions specified on screen line 3.

3. The number of elements (screen lines 4 to 9) may differ depending on the distribution.

> If fewer numbers are found in the data file than the number specified in the dialogue, or if blank lines are present, the program will stop. Thorough checking is in order before the next attempt to run EntropGraphs.

4. The lower limit of α may be 0 or a value higher than zero, but not exactly 1. The upper limit should be any positive number, except exactly 1.

5. The data file name is the full name in the disk directory, such as Raunkiaer.dat.

Do not press the RETURN key after input when an option is specified. EntropGraphs assumes GET KEY input in such a case.

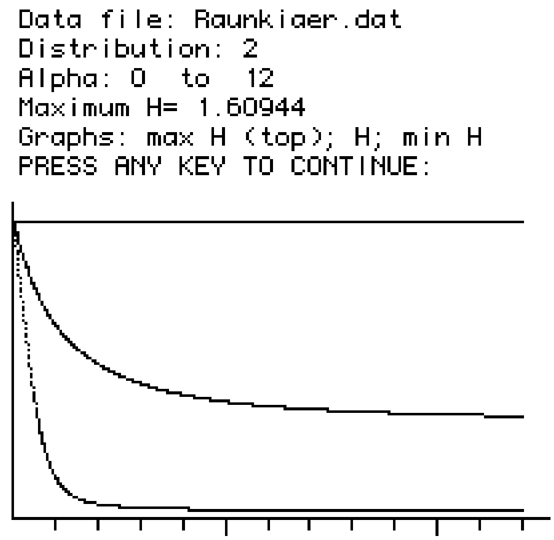


Fig. 2.7.2 An intermediate scree in the run of EntropGraphs. Information and graphs are shown for the Spitzbergen spectrum (Section 2.4.) The values of H_α are on the vertical axis. The α values are plotted on the horizontal axis from 0 to 12 in steps of 1.

After completion of the opening dialogue the progressive screens will display graphs and graph information. *Fig. 2.7.2* is an example. The program pauses after drawing the graph and the message PRESS ANY KEY TO CONTINUE appears on the screen. This halts the processing to give the user time to inspect the screen. Processing resumes when a key is pressed. The graph is automatically stored (PICT.RAUNKIAER/1, etc.) The final screen is shown in *Fig. 2.7.3*.

Fig. 2.7.3 Last screen in the run of application EntropGraphs.

2.8 Handling PRINTDA and PICT files

The run information and the results are accessed by opening the PRINTDA file from application EDIT. The contents of PRINTDA.RAUNKIAER are displayed in *Table 2.8.1.* The graphs and graph information from the PICT files are inserted in the same table. Since EDIT is not suitable for the latter operation, a paint program is needed to open the picture files for editing and a word processing program is needed which can accept files from the paint program.

Table 2.8.1 Contents of file PRINTDA.RAUNKIAER with graphs inserted from the PICT.RAUNKIAER files.

PROGRAM EntropGraphs

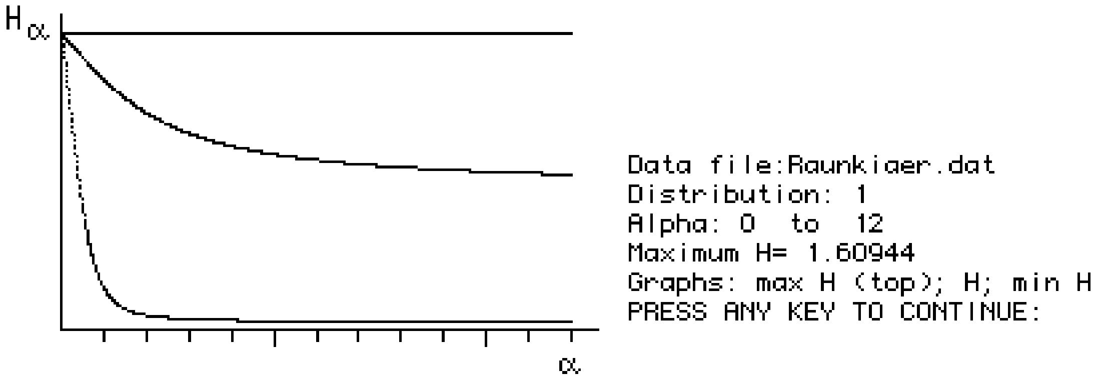
Entropy of order alpha is computed and entropy graphs drawn for F and the limiting F_m and F_l.

Lower limit alpha = 0
Upper limit alpha = 12
Input data file name: Raunkiaer.dat

DISTRIBUTION 1
PICT file: PICT.RAUNKIAER/1
Maximum entropy: 1.6094

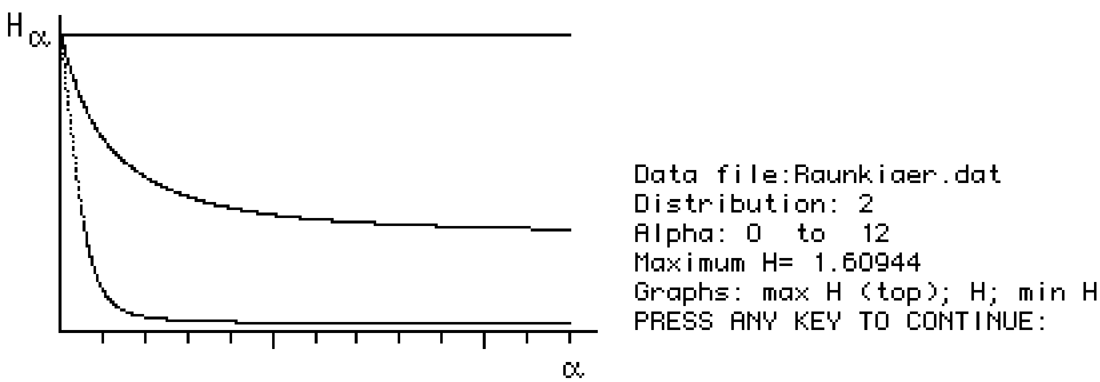
alpha	H alpha	minimum	evenness
.0000	1.6094	1.6094	1.0000
1.0000	1.3555	.2187	.8422
2.0000	1.1766	.0808	.7311
3.0000	1.0673	.0611	.6631
4.0000	.9999	.0544	.6213
5.0000	.9557	.0510	.5938
6.0000	.9250	.0490	.5747
7.0000	.9026	.0476	.5608
8.0000	.8858	.0466	.5504
9.0000	.8727	.0459	.5423
10.0000	.8623	.0454	.5358

11.0000	.8539	.0449	.5306
12.0000	.8469	.0445	.5262



DISTRIBUTION 2
PICT file: PICT.RAUNKIAER/2
Maximum entropy: 1.6094

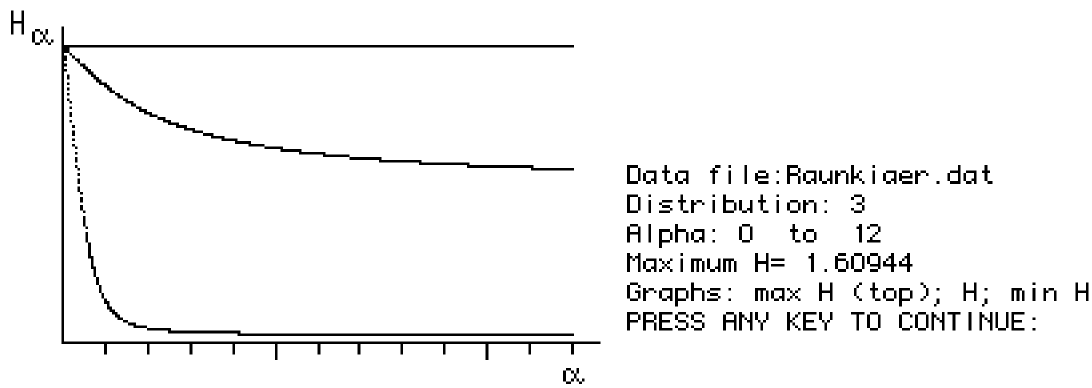
alpha	H alpha	minimum	evenness
.0000	1.6094	1.6094	1.0000
1.0000	1.0448	.2187	.6492
2.0000	.8390	.0808	.5213
3.0000	.7338	.0611	.4560
4.0000	.6733	.0544	.4183
5.0000	.6363	.0510	.3953
6.0000	.6122	.0490	.3804
7.0000	.5956	.0476	.3701
8.0000	.5836	.0466	.3626
9.0000	.5746	.0459	.3570
10.0000	.5675	.0454	.3526
11.0000	.5618	.0449	.3491
12.0000	.5572	.0445	.3462



DISTRIBUTION 3
PICT file: PICT.RAUNKIAER/3
Maximum entropy: 1.6094

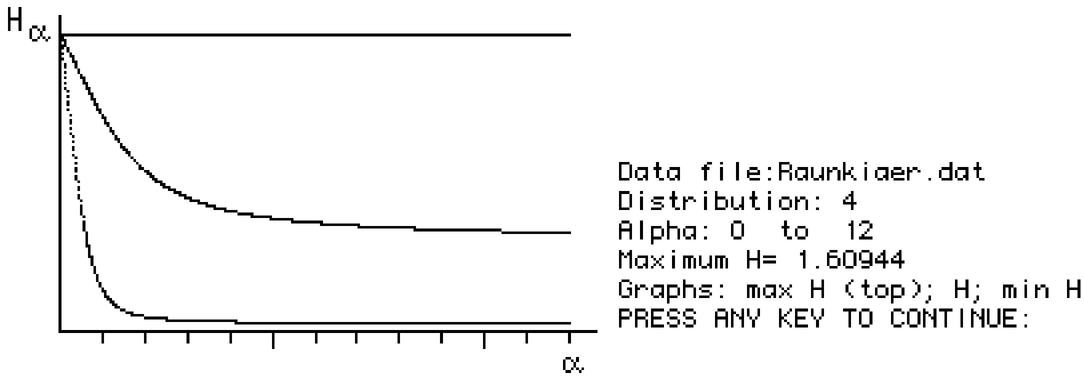
alpha	H alpha	minimum	evenness
.0000	1.6094	1.6094	1.0000

1.0000	1.3934	.2187	.8658
2.0000	1.2497	.0808	.7765
3.0000	1.1596	.0611	.7205
4.0000	1.1003	.0544	.6837
5.0000	1.0589	.0510	.6579
6.0000	1.0286	.0490	.6391
7.0000	1.0057	.0476	.6249
8.0000	.9880	.0466	.6139
9.0000	.9741	.0459	.6052
10.0000	.9628	.0454	.5982
11.0000	.9536	.0449	.5925
12.0000	.9460	.0445	.5878



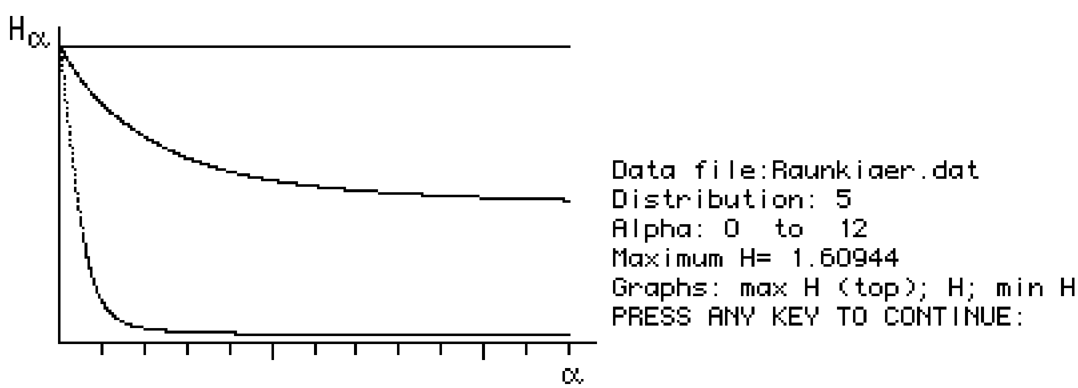
DISTRIBUTION 4
PICT file: PICT.RAUNKIAER/4
Maximum entropy: 1.6094

alpha	H alpha	minimum	evenness
.0000	1.6094	1.6094	1.0000
1.0000	1.1631	.2187	.7226
2.0000	.8694	.0808	.5402
3.0000	.7269	.0611	.4516
4.0000	.6563	.0544	.4078
5.0000	.6171	.0510	.3834
6.0000	.5928	.0490	.3684
7.0000	.5765	.0476	.3582
8.0000	.5648	.0466	.3509
9.0000	.5560	.0459	.3455
10.0000	.5491	.0454	.3412
11.0000	.5437	.0449	.3378
12.0000	.5392	.0445	.3350



DISTRIBUTION 5
PICT file: PICT.RAUNKIAER/5
Maximum entropy: 1.6094

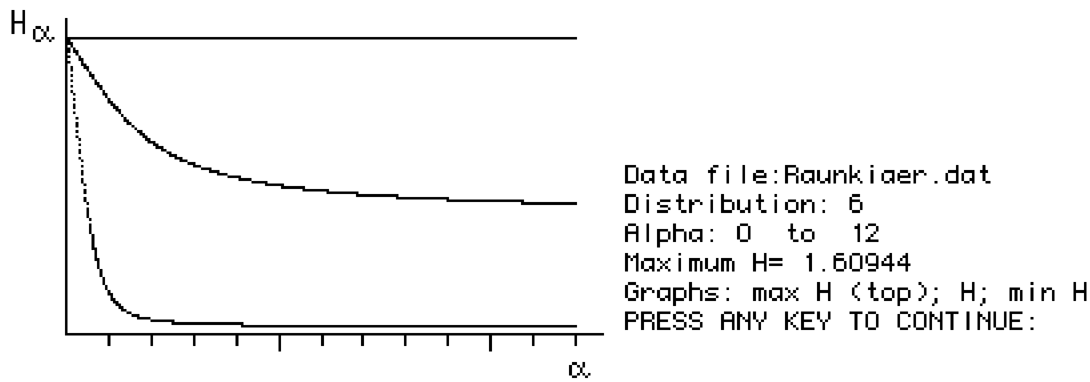
alpha	H alpha	minimum	evenness
.0000	1.6094	1.6094	1.0000
1.0000	1.2972	.2187	.8060
2.0000	1.1198	.0808	.6958
3.0000	1.0061	.0611	.6251
4.0000	.9332	.0544	.5798
5.0000	.8858	.0510	.5504
6.0000	.8539	.0490	.5305
7.0000	.8313	.0476	.5165
8.0000	.8148	.0466	.5063
9.0000	.8023	.0459	.4985
10.0000	.7925	.0454	.4924
11.0000	.7846	.0449	.4875
12.0000	.7781	.0445	.4835



DISTRIBUTION 6
PICT file: PICT.RAUNKIAER/6
Maximum entropy: 1.6094

alpha	H alpha	minimum	evenness

.0000	1.6094	1.6094	1.0000
1.0000	1.2707	.2187	.7895
2.0000	1.0450	.0808	.6493
3.0000	.9225	.0611	.5732
4.0000	.8534	.0544	.5302
5.0000	.8106	.0510	.5036
6.0000	.7820	.0490	.4859
7.0000	.7617	.0476	.4733
8.0000	.7468	.0466	.4640
9.0000	.7354	.0459	.4569
10.0000	.7264	.0454	.4513
11.0000	.7192	.0449	.4469
12.0000	.7133	.0445	.4432



2.9 Remarks

Each graph contains a straight horizontal line on top and two curves. These represent the entropy in distributions $\boldsymbol{F}_m$, $\boldsymbol{F}$ and $\boldsymbol{F}_l$. The following are useful to remember when attempting to interpret the results:

1. Entropy is a physical property. When entropy is maximal, the disorder is maximal, the predictability of specific states is minimal and diversity is maximal (case $\boldsymbol{F}_m$.) Conversely, when entropy is lowest, diversity is minimal (case $\boldsymbol{F}_l$.) Entropy and therefore diversity has order.

2. Entropy of any order is expressible in "evenness" terms,

$$E\alpha = \frac{H^{a}}{\ln s}$$

This depicts the relative closeness of $\boldsymbol{F}$ to $\boldsymbol{F}_m$. E_α should not be confused with I_α *(Eq. 1.5)* which expresses the divergence of $\boldsymbol{F}$ from $\boldsymbol{F}_m$.

3. Entropy of orders 0, 1 and 2 denotes cases that biologists have used as diversity indices:

$$H_0 = \ln s \qquad \text{-- state (species) richness index.}$$

$$H = -\sum_{j=1}^{s} p_j \ln p_j \qquad \text{-- Shannon index.}$$

$$H_2 = -\ln \sum_{j=1}^{S} p_j^2 \qquad \text{-- log Simpson index.}$$

The state richness index is the maximum value that the other two indices can possibly attain. The Shannon index is far the most popular, albeit a rather odd point on the entropy graph where the measured value may undergo a dramatic rise, or fall with even a relatively small change in α. It would be better to use another point on the entropy graph at an α where the curve begins leveling off. Alternatively, the entire graph extending from $\alpha = 0$ up to some chosen α value may be used.

4. Entropy comparisons may involve the H_α values directly between distributions of equal s. In other cases where s is not constant, the minimum value, maximum value and the evenness index should also be considered jointly.

5. Regarding Raunkiaer's biological spectra (Section 2.4) the contents of *Table 2.9.1* are relevant. Note that in the spectra s = 5 uniformly which makes the maximum entropy uniformly ln 5. The minimum entropy does not under go change, since the s value is constant and the spectral totals are constant. Note the use of entropy of order 12 in the table. High order entropy amplifies the

differences between the spectra. The ordering of the spectra by entropy is rather revealing. The vegetation of a hot semi-desert has the most diverse biological spectrum and diversity declines towards the extremes, such as the wet tropics and the tundra. Considering that the Raunkiaer spectrum reflects the survival characteristics of individual plants, it may even be argued that the vegetation in the hot semi-desert have greater stability than in the wet tropics or the tundra.

Table 2.9.1 The Raunkiaer spectra (Section 1.2) ordered according to high-order entropy.

Spectrum	Entropy			Evenness
	max H	H_{12}	min H_{12}	
Death Valley	1.6094	.9460	.0445	.5878
Normal spectrum		.8469		.5262
Connecticut		.7781		.4835
Paris basin		.7133		.4432
Spitzbergen		.5572		.3462
Seychelles		.5392		.3350

3

Entropy estimation

The data source is process sampling through k surges and the estimated quantity is Brillouin's entropy *(Eq. 1.3)* Rényi's entropy of order α *(Eq. 1.4.)* The averaging technique is Pielou's[15].

3.1 Choices

E. C. Pielou argues the question of choice and comes down in favour of the Brillouin equation. She is concerned with the potential of the Shannon entropy function *(Eq. 1.1)* being inaccurate in small populations. It is interesting, however, to note that her concern is not generally shared. In fact others, most notably C. E. Shannon, A. Rényi and S. Kullback approach information theory based on *Eq. 1.1*,

[15] 1975.

Eqs. 1.4, 1.5 and *Eq. 1.6,* not *Eq. 1.3.* In all of these cases the Brillouin information is not considered a benchmark value.

3.2 **Application** EntropEst

Program EntropEst has two versions, EntropEstB and EntropEstR. EntropEstB computes Brillouin's function based on the natural logarithm. EntropEstR computes Rényi's entropy of order α for legitimate values of α from zero up to a chosen limit in increments of 1.

3.3 Data type

Frequencies or density counts are involved. These are arranged in k s-valued distributions:

f_{11}	f_{12}	...	f_{1k}	$f_{1.}$
f_{21}	f_{22}	...	f_{2k}	$f_{2.}$
.	.	...	.	.
f_{s1}	f_{s2}	...	f_{sk}	$f_{s.}$
$f_{.1}$	$f_{.2}$	...	$f_{.k}$	$f_{..k}$

In a typical ecological example the data rows identify species and the distributions are composites of several relevés per sampling surge. It is assumed that sampling surge size (relevé number per sampling surge) is constant and the descriptor set is kept intact throughout the sampling.

3.4 Averaging entropy

After each sampling surge an entropy value is computed for each α and averaged with the previous entropy values. One may be tempted to use the weighted average in u = k sampling surges,

$$H'_{\alpha;\theta u} = \frac{f_{.1}H_{a1} + f_{.2}H_{a2} + ... + f_{.u}H_{au}}{f_{..u}}$$

which is equivalent to

$$H_{\alpha;\theta u} = \frac{1}{(1-a)f_{..u}} \sum_{j=1}^{u} f_{j} \ln \sum_{i=1}^{s} p_{ij}^{a} \quad \text{Eq. 3.4.1}$$

where

$$p_{ij} \frac{f_{ij}}{f_{.j}}$$

But $H'_{\alpha;\varnothing u}$ is not optimal, since it does not incorporate terms for shared information which links the u sampling surges into an entropy process. The following expression does include a term for shared information:

$$H^{*}_{\alpha;\varnothing u} = H'_{\alpha;\varnothing u} + H_{\alpha;\rightarrow u(descriptors;sampling)}$$

Considering entropy of order 1,

$$H^{*}_{1;\varnothing u} = -\frac{1}{f_{..u}}\left(\sum_{j=1}^{u}\sum_{i=1}^{s} f_{ij} \ln\frac{f_{ij}}{f_{.j}} + \sum_{j=1}^{u}\sum_{i=1}^{s} f_{ij} \ln\frac{f_{ij}f_{..u}}{f_{i.u}f_{.j}}\right)$$

$$= -\frac{1}{f_{..u}} \sum_{i=1}^{s} f_{i.u} \ln\frac{f_{i.u}}{f_{..u}} \qquad \text{Eq. 3.4.2}$$

which happens to be a multiple of Shannon's entropy. The corresponding Brillouin entropy is

$$H^{*}_{\varnothing u} = \frac{1}{f_{..u}} \ ln \ \frac{f_{..u}!}{f_{1.u}!\, f_{2.u}! \ldots f_{s.u}!} \ . \qquad Eq.\ 3.4.3$$

and the generalized entropy is

$$H^{*}_{\alpha\varnothing u} = \frac{1}{1 - \alpha} \ ln \sum_{i=1}^{s} p_{i.u}^{\alpha} \qquad Eq.\ 3.4.4$$

with proportions defined according to

$$p_{i.u} = \frac{f_{i.u}}{f_{..u}}$$

****3.5 Sample data

Consider density estimates for 3 species in quadrat samples of constant sampling fraction emitting from a 6-step sampling process:

Species	Distribution					
	1	2	3	4	5	6
1	100	93	43	87	97	97
2	1	26	42	65	100	86
3	27	50	11	17	21	19
Total	128	169	96	169	218	202

This data set is entered on file (Density.dat) by distribution (column):

100
1
27
93
26

50
43
42
11
87
65
17
97
100
21
97
86
19

The creation of this file follows principles which are discussed under Section 2.6.

3.6 Calculations

3.6.1 Brillouin's entropy (Eq. 3.4.3)

1. At the start, u is equal to 1 and the data vector consists of

$$f_{1.1} = 100$$

$$f_{2.1} = 1$$

$$f_{3.1} = 27$$

$$f_{..1} = 128$$

The Brillouin entropy is

$$H^*_{\varnothing 1} = \frac{1}{128} \ln \frac{128!}{100!\ 1!\ 27!} = 0.53210$$

or 0.53210/ln 2 = 0.76766 bits[16].

2. For u = 2,

$f_{1.2} = 193$

$f_{2.2} = 27$

$f_{3.2} = 77$

$f_{..2} = 297$

$$H^*_{\varnothing 2} = \frac{1}{297} \ln \frac{297!}{193!\ 27!\ 77!} = 0.82974 .$$

3. For other u:

$$H^*_{\varnothing 3} = 0.93166$$
$$H^*_{\varnothing 4} = 0.96229$$
$$H^*_{\varnothing 5} = 0.98045$$
$$H^*_{\varnothing 6} = 0.97858 .$$

4. If the 6 values were graphed, the graph segment from point 3 on may be taken as being "flat" and the entropy estimates will accord with

$$H_{\varnothing u} = \frac{\left| f_{..u} H^*_{\varnothing u} - f_{..u-1} H^*_{\varnothing u-1} \right|}{f_{..u} - f_{..u-1}} \quad ; \quad u = 1, ..., 6$$

Numerically,

$$H_{\varnothing 4} = \frac{562 \text{ x } 0.96229 - 393 \text{ x } 0.93166}{562 - 393} = 1.03352$$

---

[16] To minimize the rounding errors in long-hand computations, retain an ample number of digits in intermediate steps.

$H_{\varnothing 5} = 1.02726$

$H_{\varnothing 6} = 0.97134.$

The average of these is the entropy estimate sought[17]:

$$H = \frac{1}{k - IP} \sum_{u=IP+1}^{k} H_{\varnothing u} = \frac{1.03352 + 1.02726 + 0.97134}{6 - 3} = 1.01071$$

An estimate of the sampling variance of H is

$$S_H^2 = \frac{1}{(k - IP)(k - IP - 1)} \sum_{u=IP+1}^{k} (H_{\varnothing u} - H)^2$$

$$= \frac{(1.03352-1.01071)^2+(1.02726-1.01071)^2+(0.97134-1.01071)^2}{6}$$

$$= 0.00039070$$

3.6.2 Rényi's generalized entropy (Eq. 3.4.4)

The steps are similar as before:

1. For u=1,

$f_{1.1} = 100$

$f_{2.1} = 1$

$f_{3.1} = 27$

$f_{..1} = 128$

[17] See Pielou (1964).

$$H^{*}_{1;\varnothing 1} = \frac{100}{128} \ln \frac{100}{128} + \frac{1}{128} \ln \frac{1}{128} + \frac{27}{128} \ln \frac{27}{128} = 0.55903$$

$$H^{*}_{2;\varnothing 1} = \frac{1}{1-2} \ln\left(\frac{100^2}{128^2} + \frac{1^2}{128^2} + \frac{27^2}{128^2}\right) = 0.42326 .$$

$$H^{*}_{3;\varnothing 1} = \frac{1}{1-3} \ln\left(\frac{100^3}{128^3} + \frac{1^3}{128^3} + \frac{27^3}{128^3}\right) = 3.36056$$

$$H^{*}_{4;\varnothing 1} = \frac{1}{1-4} \ln\left(\frac{100^4}{128^4} + \frac{1^4}{128^4} + \frac{27^4}{128^4}\right) = 0.32738$$

Similar computations would yield any higher order entropy.

2. For u = 2,

$f_{1.2} = 193$

$f_{2.2} = 27$

$f_{3.2} = 77$

$f_{..2} = 297$

$$H^{*}_{1;\varnothing 2} = \frac{193}{297} \ln \frac{193}{297} + \frac{27}{297} \ln \frac{27}{297} + \frac{77}{297} \ln \frac{77}{297} = 0.84808$$

$$H^{*}_{2;\varnothing 2} = \frac{1}{1-2} \ln\left(\frac{193^2}{297^2} + \frac{27^2}{297^2} + \frac{77^2}{297^2}\right) = 0.69764$$

$$H^{*}_{3;\varnothing 2} = \frac{1}{1-3} \ln\left(\frac{193^3}{297^3} + \frac{27^3}{297^3} + \frac{77^3}{297^3}\right) = 0.61449$$

$$H^{*}_{4;\varnothing 2} = \frac{1}{1-4} \ln\left(\frac{193^4}{297^4} + \frac{27^4}{297^4} + \frac{77^4}{297^4}\right) = 0.56626$$

Other higher order entropy values are similarly computed.

3. In the following steps, similar computations are applied to obtain entropy values of different order. For example, for entropy of order 3,

$$H^{*}_{3;\varnothing 3} = 0.72795$$
$$H^{*}_{3;\varnothing 4} = 0.78051$$
$$H^{*}_{3;\varnothing 5} = 0.83742$$
$$H^{*}_{3;\varnothing 6} = 0.84709$$

4. Considering the $H^{*}_{3\varnothing u}$ graph and taking IP as being 3, the entropy estimate for each sampling surge accords with

$$H_{3;\varnothing u} = \frac{\left| f_{..u} H^{*}_{3;\varnothing u} - f_{..u-1} H^{*}_{3;\varnothing u-1} \right|}{f_{..u} - f_{..u-1}} \quad ; \quad u = 3, ..., 6$$

Numerically,

$$H_{3;\varnothing 4} = \frac{562 \text{ x } 0.78051 - 393 \text{ x } 0.72795}{562 - 393} = 0.90273$$

$$H_{3;\varnothing 5} = 0.98414$$

$$H_{3;\varnothing 6} = 0.88442$$

5. Based on the above the pooled entropy estimate of order 3 is

$$H_3 = \frac{1}{k - IP} \sum_{u=IP+1}^{k} H_{3;\varnothing u} = \frac{0.90273 + 0.98414 + 0.88442}{6 - 3} = 0.92376$$

and the variance of this mean is

$$S^2_{H_3} = \frac{1}{(k - IP)(k - IP - 1)} \sum_{u=IP+1}^{k} (H_{3;\varnothing u} - H_3)^2$$

$$= \frac{(0.90273-0.923763)^2+(0.98414-0.923763)^2+(0.88442-0.92376)^2}{6}$$

$$= 0.00093928$$

These values of the mean and the variance are specific to $\alpha = 3$. For other cases of α, similar computations would be performed.

3.7 **Running** EntropEstB

The data file is explained in Section 3.5. After having created the data file, open a WORK folder and drag the ikons of EntropEstB and the data file (Density.dat) to this folder. Start up EntropEstB by clicking twice on its ikon. As the run gets underway, respond to requests for information on the screen:

1. If application and data ikons are not in the same folder or outside any folder on disk and the run already started, stop the run by pressing key N (do not press the RETURN key after N) on the 1st screen line (see dialogue in Fig. 3.7.1.) Following this, file rearrangements can be made to meet the requirements of a new run. If the application and data ikons are in the same folder or are outside any folder on disk, press key Y (do not press the RETURN key after Y.) The run will continue and new specifications will be requested.

2. If the printing of intermediate results is required, respond on the 2nd screen line by pressing key Y (do not press the RETURN key after Y). The PRINTDA file will receive the intermediate results that would not be retained otherwise.

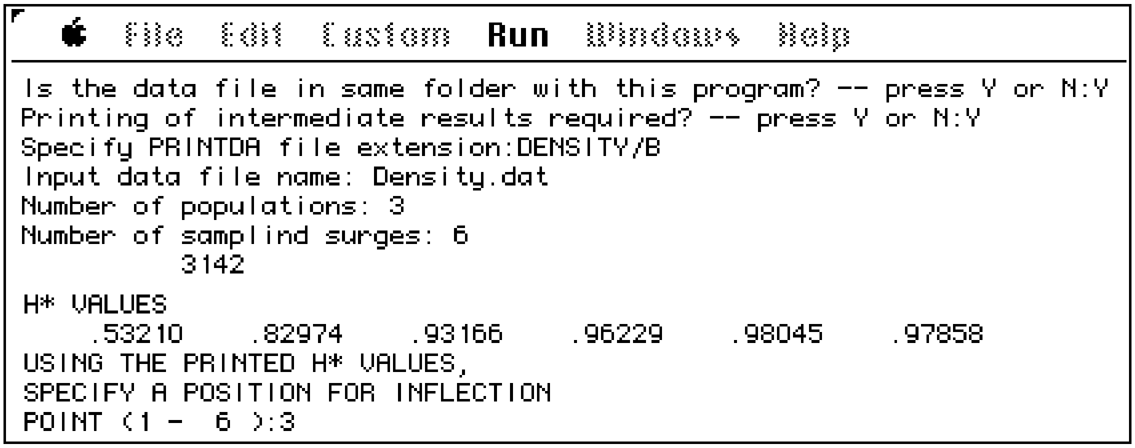


Fig. 3.7.1 First screen showing the start-up dialogue as the EntropEstB run gets under way.

3. The name extension requested on screen line 3 identifies the output file for storing results. For example, if DENSITY/B is typed, as it has been in the example, the print file will have full name PRINTDA.DENSITY/B. This file is stored in the same folder (if a folder is used) on disk where the application program and data file are stored.

4. The input data file name is the full name of the data file (Density.dat in the example.)

5. The number of populations is the number of relevé descriptors (rows in the data table.)

6. The number of sampling surges is the number of columns in the data table.

If fewer numbers are found on file then specified in the dialogue, or if blank lines are present, the application will stop.

7. The running number seen on the 7th screen line is a count which changes as the program computes factorials. This is just a reminder that the program is running.

8. The $H^*_{\varnothing u}$ values (as many in number as there are successive sampling surges) are printed on the screen and the user is requested to pick a position which he deems to be the "main" inflection point.

9. The run concludes with a screen message identifying the PRINTDA file (Fig. 3.7.2.) After making note of this, press a key and press again if necessary, to quit.

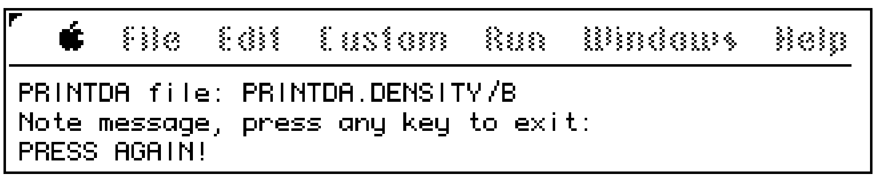


Fig. 3.7.2 Last screen message in the run of program EntropEstB.

After exiting the application, the PRINTDA file can be opened, edited and optionally printed from program EDIT. This has been done with file PRINTDA.DENSITY/B (Table 3.7.1.)

Table 3.7.1 Contents of file PRINTDA.DENSITY/B from a run of program EntropEstB. The raw data are given in Section 3.5.

```
Program: EntropEstB

INPUT DATA FILE: Density.dat
NUMBER OF POPULATIONS: 3
NUMBER OF SAMPLING UNITS: 6

Cumulative counts

      100       193       236       323       420       517
        1        27        69       134       234       320
```

```
       27       77       88      105      126      145

Cumulative sample totals

      128      297      393      562      780      982

H* values

    .53210    .82974    .93166    .96229    .98045    .97858

Inflection point chosen: 3


H estimates (on right side of inflection point)

   1.03352   1.02726    .97134


Mean H = 1.010706
Maximum H = 1.098612
Variance of H = .00117210
Variance of the mean H = .00039070
```

3.8 **Running** EntropEstR

The start-up dialogue (Fig. 3.8.1) is similar to that discussed in Section 3.7. There is a difference though on the 7th screen line which requests the user to specify the upper limit of α. The starting value is zero and the step size is 1. At each step, the program computes the $H^{*}_{\alpha;\,\varnothing u}$ values, as many in number as there are columns in the data table (Section 3.5). These values are printed on the screen and the user is asked to pick a position to serve as the main inflection point.

```
File  Edit  Custom  Run  Windows  Help
Are data and program in same folder? -- press Y or N:Y
Output file name extension:DENSITY/R
Printing of intermediate results required -- press Y or N:Y
Data file:Density.dat
Number of populations:3
Number of sampling surges:6
Upper limit for alpha:4

H-ASTERISK VALUES AT ALPHA= 0
 1.09861  1.09861  1.09861  1.09861  1.09861  1.09861
USING THE PRINTED H-ASTERISC VALUES, SPECIFY
POSITION OF PERCEIVED INFLECTION POINT (1 TO  6 )? 1


H-ASTERISK VALUES AT ALPHA= 1
 .559025  .848076  .946778  .973569  .989007  .985592
USING THE PRINTED H-ASTERISC VALUES, SPECIFY
POSITION OF PERCEIVED INFLECTION POINT (1 TO  6 )? 3


H-ASTERISK VALUES AT ALPHA= 2
 .423262  .697635  .817405  .862571  .901315  .903452
USING THE PRINTED H-ASTERISC VALUES, SPECIFY
POSITION OF PERCEIVED INFLECTION POINT (1 TO  6 )? 3


H-ASTERISK VALUES AT ALPHA= 3
 .360544  .614493  .727952  .780511  .837422  .84709
USING THE PRINTED H-ASTERISC VALUES, SPECIFY
POSITION OF PERCEIVED INFLECTION POINT (1 TO  6 )? 3


H-ASTERISK VALUES AT ALPHA= 4
 .32738  .566258  .671207  .72514  .792269  .807954
USING THE PRINTED H-ASTERISC VALUES, SPECIFY
POSITION OF PERCEIVED INFLECTION POINT (1 TO  6 )? 3
```

Fig. 3.8.1 First screen displaying the start-up dialogue in a run of program EntropEstR.

To exit the run press a key and repeat as needed after the last screen message (Fig. 3.8.2). The contents of the file PRINTDA.DENSITY/R are displayed in Table 3.8.1.

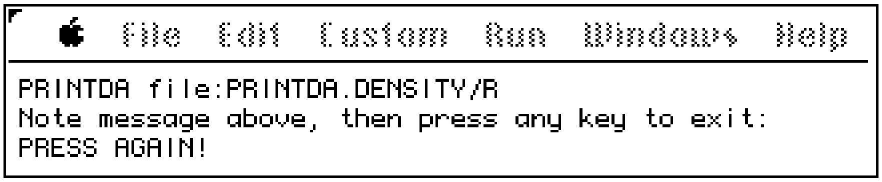


Fig. 3.8.2 Last screen in the run of application EntropEstR.

Table 3.8.1 The contents of PRINTDA.DENSITY/R written in the sample run of program EntropEstR

```
PROGRAM EntropEstR

Data file:Density.dat
Number of populations: 3
Number of sampling surges: 6

CUMULATIVE COUNTS

      100      193      236      323      420      517
        1       27       69      134      234      320
       27       77       88      105      126      145

CUMULATIVE COLUMN TOTALS

      128      297      393      562      780      982


H-ASTERISC VALUES AT ALPHA= 0

  1.09861  1.09861  1.09861  1.09861  1.09861  1.09861


Hu of order  0  in position  1  AND UP:

  1.09861  1.09861  1.09861  1.09861  1.09861  1.09861


H-ASTERISC VALUES AT ALPHA= 1

   .55903   .84808   .94678   .97357   .98901   .98559


Hu of order  1  in position  3  AND UP:

   .55903   .84808   .94678  1.03587  1.02881   .97240


H-ASTERISC VALUES AT ALPHA= 2

   .42326   .69764   .81740   .86257   .90131   .90345
```

```
Hu of order  2  in position  3  AND UP:

   .42326   .69764   .81740   .96760  1.00119   .91170


H-ASTERISC VALUES AT ALPHA= 3

   .36054   .61449   .72795   .78051   .83742   .84709


Hu of order  3  in position  3  AND UP:

   .36054   .61449   .72795   .90273   .98414   .88442


H-ASTERISC VALUES AT ALPHA= 4

   .32738   .56626   .67121   .72514   .79227   .80795


Hu of order  4  in position  3  AND UP:

   .32738   .56626   .67121   .85056   .96533   .86852


Alpha = 0
Inflection point selected: 1
Mean H = 1.09861
Variance of estimate H = -2.22045e-16
Sampling variance =-4.44089e-17
Maximum H: 1.09861
Evenness: 1

Alpha = 1
Inflection point selected: 3
Mean H = 1.01236
Variance of estimate H =  1.20981e-3
Sampling variance = 4.03269e-4
Maximum H: 1.09861
Evenness: .92149

Alpha = 2
Inflection point selected: 3
Mean H = .960167
Variance of estimate H =  2.04368e-3
Sampling variance = 6.81228e-4
Maximum H: 1.09861
Evenness: .873982

Alpha = 3
Inflection point selected: 3
Mean H = .923763
Variance of estimate H =  2.81783e-3
Sampling variance = 9.39277e-4
Maximum H: 1.09861
Evenness: .840846

Alpha = 4
Inflection point selected: 3
```

```
Mean H = .894802
Variance of estimate H =  3.81093e-3
Sampling variance = 1.27031e-3
Maximum H: 1.09861
Evenness: .814484


Mean entropy at alpha 0 to  4

   1.0986123   1.0123603    .9601669    .9237635    .8948022

Variance of entropy at alpha 0 to  4

-    .0000000    .0012098    .0020437    .0028178    .0038109

Sampling variances of mean entropy at alpha 0 to  4

-   .0000000   .0004033   .0006812   .0009393   .0012703

Evenness values at alpha 0 to  4

  1.000000   .921490   .873982   .840846   .814484
```

3.9 Remarks

Most of the general properties outlined in Section 2.9 will apply. It has to be emphasized that unlike in Section 2.4, the distributions should always be equal valued (3 in the example), but unlike in the example (Section 3.5) they need not equal totals.

Attention is drawn to the phenomenon which clearly manifests itself in Table 3.8.1. This is the decline of entropy with increasing order, but not of the variance which in fact increases with order. This behaviour of the variance poses a dilemma when selecting an entropy point to serve as a diversity index.

4

Information estimation

Process sampling in k surges generates the frequencies. The information is Rényi's (Eq. 1.5) and the averaging method is Pielou's.

4.1 The data

The frequencies are arranged in k r x t tables. In symbolic terms the hth of the tables is

Table h

	B_1	B_2	...	B_t	Total
A_1	f_{h11}	f_{h12}	...	f_{h1t}	$f_{h1.}$
A_2	f_{h21}	f_{h22}	...	f_{h2t}	$f_{h2.}$
.	.	.	...	.	.
A_r	f_{hr1}	f_{hr2}	...	f_{hrt}	$f_{hr.}$
Total	$f_{h.1}$	$f_{h.2}$	...	$f_{h.t}$	$f_{h..}$

The following definitions of symbols apply:

r - number of categories, classification A.

t - number of categories, classification B.

k - number of sampling surges, the sampling process (classification C.)

f_{hij} - joint frequency of category i, classification A and category j, classification B with category h, classification C.

$f_{hi.}$ - joint frequency of category i, classification A and category h, classification C.

$f_{h.j}$ - joint frequency of category j, classification B and category h, classification C.

$f_{h..}$ - frequency of category h, classification C.

4.2 Information

Information of order α is computed for two distributions

$$\mathbf{Q} = (q_1\ q_2\ ...\ q_s) \quad \text{and} \quad \mathbf{P} = (p_1\ p_2\ ...\ p_s)$$

Indirectly when the strength of association is measured there may be more than two sets of classificatory criteria and a multidimensional joint distribution from which the q and p quantities are derived. The exact definition of q and p will depend on the perceived type of mutuality. If q and p are formulated as in

$$q_{hij} = \frac{f_{hij}}{f_{...}} \quad \text{and} \quad p_{hij} = \frac{f_{h..}}{f_{...}}\frac{f_{.i.}}{f_{...}}\frac{f_{..j}}{f_{...}} \qquad \text{Eq. 4.2.1}$$

I_α of Eq. 1.5 will measure interaction information between classifications A,B,C. If this information is of order 1, $2f_{.}I_1$ will be Kullback's one-way information divergence. If on the other hand q and p are formulated as in

$$q_{hij} = \frac{f_{hij}}{f_{...}} \quad \text{and} \quad p_{hij} = \frac{f_{hi.}}{f_{h..}} \frac{f_{.ij}}{f_{.i.}} \frac{f_{h.j}}{f_{..j}} \quad \text{Eq. 4.2.2}$$

I_α will measure mutual information. The two types of mutuality are identified by the shaded areas in Fig. 4.2.1.[18]

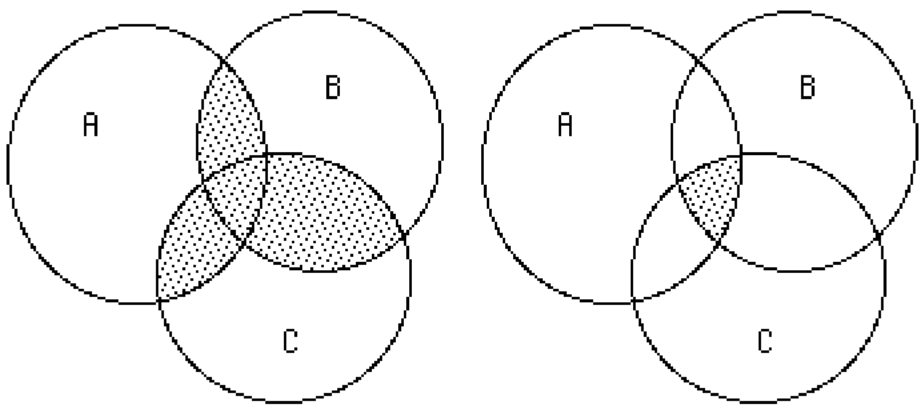


Fig. 4.2.1 Venn representation of interaction information (shaded area left) and mutual information (shaded area right) in a 3d frequency distribution. In terms of the example (Section 4.1) A,B are the row and column classifications and C the k-step sampling process.

4.3 Estimation

4.3.1 The averaging method

The data set is described in Section 4.1. There are k r x c tables and for each an information quantity $I_{\alpha hAB}$ is computed. Following the logic outlined in Section 3.4, one possibility is to average information according to

[18] See Abramson (1963).

$$I'_{\alpha;\,\varnothing uAB(...)} = \frac{f_{1..}I_{\alpha 1AB(...)} + f_{2..}I_{\alpha 2AB(...)} + ... + f_{u..}I_{\alpha uAB(...)}}{f_{...u}}$$

which is equivalent to

$$I'_{\alpha;\,\varnothing uAB(...)} = \frac{1}{(\alpha - 1)\, f_{..u}} \sum_{h=1}^{u} f_{h.j} \ln \sum_{i=1}^{r} \sum_{j=1}^{t} \frac{q_{hij}^{\alpha}}{p_{hij}^{\alpha-1}}$$

Since $I'_{\alpha;\,\varnothing uAB(...)}$ does not incorporate shared information, one should opt for the alternative quantity

$$I^{*}_{\alpha;\,\varnothing uAB(...)} = I'_{\alpha;\,\varnothing uAB(...)} + I_{\alpha;\,\rightarrow uAB(shared)}$$

The exact definition of $I'_{\alpha;\,\varnothing uAB(...)}$ depends on the definition of q and p which in turn depends on whether interaction or mutual information is wanted (see Section 4.2.)

4.3.1.1 Interaction information

The desired estimator of interaction information of order one, sampling surge u, is given by

$$I^{*}_{1;\,\varnothing uAB(inter)} = I'_{1;\,\varnothing uAB(inter)} - \frac{1}{f_{...u}} \sum_{h=1}^{u} \sum_{i=1}^{r} \sum_{j=1}^{t} f_{hij} \ln \frac{(f_{.i.u}/f_{...u})(f_{..ju}/f_{...u})}{(f_{hi.}/f_{h..})(f_{h.j}/f_{h..})}$$

$$= \frac{1}{f_{...u}} \sum_{h=1}^{u} \sum_{i=1}^{r} \sum_{j=1}^{t} f_{hij} \ln \frac{f_{hij}\, f^{2}_{...u}}{f_{h..} f_{.i.u} f_{..ju}}$$

In general terms,

$$I^{*}_{\alpha;\,\varnothing uAB(inter)} = \frac{1}{\alpha - 1} \ln \sum_{h=1}^{u} \sum_{i=1}^{r} \sum_{j=1}^{t} \frac{q^{\alpha}_{hiju}}{p^{\alpha-1}_{hiju}} \quad \text{Eq. 4.3.1}$$

This is equivalent to Eq. 1.5 with q and p defined according to Eq. 4.2.1. The maximum value of $I^{*}_{\alpha;\,\varnothing uAB(inter)}$ is ln m. This m represents the median value of u, r, t.

4.3.3 Mutual information

The estimator in this case is

$$I^{*}_{1;\,\varnothing uAB(mut)} = I'_{1;\,\varnothing uAB(mut)} - \frac{1}{f_{...u}} \sum_{h=1}^{u} \sum_{i=1}^{r} \sum_{j=1}^{t} f_{hij} \ln \frac{f_{.iju}\, f_{...u}}{f_{.i.u}\, f_{..ju}}$$

$$= \frac{1}{f_{...u}} \sum_{h=1}^{u} \sum_{i=1}^{r} \sum_{j=1}^{t} f_{hij} \ln \frac{f_{hij}/f_{...u}}{\frac{f_{hi.}}{f_{h..}} \frac{f_{.iju}}{f_{.i.u}} \frac{f_{h.j}}{f_{..ju}}}$$

The general form $I^{*}_{\alpha;\,\varnothing uAB(mut)}$ is similarly defined as in Eq. 4.3.1, but p and q accord with Eq. 4.2.2. The maximum value of $I^{*}_{\alpha;\,\varnothing uAB(mut)}$ is ln m where m is the smallest of u, r, t.

4.4 **Application** InfoEst

Keeping Eq. 4.2.1 and Eq. 4.2.2 as the basic definitions of q and p, and Eq. 4.3.1 as the definition of information, options of InfoEst compute estimates for interaction and mutual information of different orders. Regarding the data, there is no limit on surge size or the number of sampling surges, but it is assumed that surge size, table dimensions and classificatory criteria are kept intact as the sampling proceeds.

4.5 Sample calculations

4.5.1 Interaction information

The sample data set contains 3 tables with 2 rows and 2 columns in each:

Table 1

	B		
A	17	6	23
	13	14	27
	30	20	50

Table 2

15	11	26
3	4	7
18	15	33

Table 3

8	3	11
2	14	16
10	17	27

The cumulative frequencies are

32	17	49
16	18	34
48	35	83

for u = 2 and

40	20	60
18	32	50
58	52	110

for u=3. Recall that for interaction information (Eq. 4.3.1) q and p accord with Eq. 4.2.1 and proceed as follows:

1a. For u = 1,

$$q_{1111} = \frac{17}{50} \qquad p_{1111} = \frac{50}{50}\frac{23}{50}\frac{30}{50}$$

$$q_{1121} = \frac{6}{50} \qquad p_{1121} = \frac{50}{50}\frac{23}{50}\frac{20}{50}$$

$$q_{1211} = \frac{13}{50} \qquad p_{1211} = \frac{50}{50}\frac{27}{50}\frac{30}{50}$$

$$q_{1221} = \frac{14}{50} \qquad p_{1221} = \frac{50}{50}\frac{27}{50}\frac{20}{50}$$

$$I^{*}_{1;\,\varnothing 1AB(inter)} = 0.035059$$

(within computer rounding errors.)

1b. For u=2 ,

Table 1

$q_{1112} = \frac{17}{83}$ $p_{1112} = \frac{50}{83}\frac{49}{83}\frac{48}{83}$

$q_{1122} = \frac{6}{83}$ $p_{1112} = \frac{50}{83}\frac{49}{83}\frac{35}{83}$

$q_{1212} = \frac{13}{83}$ $p_{1212} = \frac{50}{83}\frac{34}{83}\frac{48}{83}$

$q_{1222} = \frac{14}{83}$ $p_{1222} = \frac{50}{83}\frac{34}{83}\frac{35}{83}$

Table 2

$q_{2112} = \frac{15}{83}$ $p_{2112} = \frac{33}{83}\frac{49}{83}\frac{48}{83}$

$q_{2122} = \frac{11}{83}$ $p_{2122} = \frac{33}{83}\frac{49}{83}\frac{35}{83}$

$q_{2212} = \frac{3}{83}$ $p_{2212} = \frac{33}{83}\frac{34}{83}\frac{48}{83}$

$q_{2222} = \frac{4}{83}$ $p_{2222} = \frac{33}{83}\frac{34}{83}\frac{35}{83}$

$I^{*}_{1;\varnothing 2AB(inter)} = 0.081160$

1c. For u=3,

Table 1

$q_{1113} = \frac{17}{110}$ $p_{1113} = \frac{50}{110}\frac{60}{110}\frac{58}{110}$

$q_{1123} = \frac{6}{110}$ $p_{1123} = \frac{50}{110}\frac{60}{110}\frac{52}{110}$

$q_{1213} = \frac{13}{110}$ $p_{1213} = \frac{50}{110}\frac{50}{110}\frac{58}{110}$

$q_{1223} = \frac{14}{110}$ $p_{1223} = \frac{50}{110}\frac{50}{110}\frac{52}{110}$

Table 2

$q_{2113} = \frac{15}{110}$ $p_{2113} = \frac{33}{110}\frac{60}{110}\frac{58}{110}$

$q_{2123} = \frac{11}{110}$ $p_{2123} = \frac{33}{110}\frac{60}{110}\frac{52}{110}$

$q_{2213} = \frac{3}{110}$ $p_{2213} = \frac{33}{110}\frac{50}{110}\frac{58}{110}$

$q_{2223} = \frac{4}{110}$ $p_{2223} = \frac{33}{110}\frac{50}{110}\frac{52}{110}$

Table 3

$q_{3113} = \frac{8}{110}$ $p_{3113} = \frac{27}{110}\frac{60}{110}\frac{58}{110}$

$q_{3123} = \frac{3}{110}$ $p_{3123} = \frac{27}{110}\frac{60}{110}\frac{52}{110}$

$q_{3213} = \frac{2}{110}$ $p_{3213} = \frac{27}{110}\frac{50}{110}\frac{58}{110}$

$q_{3223} = \frac{14}{110}$ $p_{3223} = \frac{27}{110}\frac{50}{110}\frac{52}{110}$

$I^{*}_{1;\,\varnothing 3AB(inter)} = 0.13827$

2. With inflection point at IP, the estimated interaction information of order 1 is computed according to

$$I_{1;\,\varnothing uAB(inter)} = \frac{\left| f_{...u}\, I^{*}_{1;\,\varnothing uAB(inter)} - f_{...u-1}\, I^{*}_{1;\,\varnothing u-1AB(inter)} \right|}{f_{...u} - f_{...u-1}}$$

u = IP+1, ..., k. For given IP = 1 and k = 3,

$$I_{1;\,\varnothing 2AB(inter)} = \frac{83 \text{ x } 0.081160 - 50 \text{ x } 0.035059}{83 - 50} = 0.15101$$

$$I_{1;\,\varnothing 3AB(inter)} = 0.31382$$

4. The average of the above is the estimated interaction information,

$$I_{1;AB(inter)} = \frac{1}{k - IP} \sum_{u=IP+1}^{k} I_{1;\,\varnothing uAB(inter)}$$

$$= \frac{0.15101 + 0.31382}{3 - 1}$$

$$= 0.23242$$

5. The variance of the mean is

$$S^{2}_{1;\,AB(inter)} = \frac{1}{(k - IP)(k - IP - 1)} \sum_{u=IP+1}^{k} (I_{1;\,\varnothing uAB(inter)} - I_{1;AB(inter)})^{2}$$

$$= \frac{(0.15101 - 0.23242)^{2} + (0.31382 - 0.23242)^{2}}{(3 - 1)(3 - 1 -1)}$$

$$= 0.0066268$$

Interaction information of any legitimate order is computed on the basis of the same q and p values as above. Application InfoEst, option I, does the computations automatically.

4.5.2 Mutual information

The cumulative frequencies are the same as before, but the q and p quantities are differently defined (Eq. 4.2.2.) Considering mutual information of order 1, the arithmetic is shown below:

1a. For u=1, the q_{hij1} and p_{hij1} values are the same as their counterparts at u=1 in Section 4.5.1. The mutual information is also the same,

$$I^{*}_{1;\,\varnothing 1AB(mut)} = 0.035039$$

2a. For u=2 ,

Table 1

$$q_{1112} = \frac{17}{83} \qquad p_{1112} = \frac{23}{50}\frac{32}{49}\frac{30}{48}$$

$$q_{1122} = \frac{6}{83} \qquad p_{1122} = \frac{23}{50}\frac{17}{49}\frac{20}{35}$$

$$q_{1212} = \frac{13}{83} \qquad p_{1212} = \frac{27}{50}\frac{16}{34}\frac{30}{48}$$

$$q_{1222} = \frac{14}{83} \qquad p_{1222} = \frac{27}{50}\frac{18}{34}\frac{20}{35}$$

Table 2

$q_{2112} = \frac{15}{83}$ $p_{2112} = \frac{26}{33}\frac{32}{49}\frac{18}{48}$

$q_{2122} = \frac{11}{83}$ $p_{2122} = \frac{26}{33}\frac{17}{49}\frac{15}{35}$

$q_{2212} = \frac{3}{83}$ $p_{2212} = \frac{7}{33}\frac{16}{34}\frac{18}{48}$

$q_{2222} = \frac{4}{83}$ $p_{2222} = \frac{7}{33}\frac{18}{34}\frac{15}{35}$

$I^{*}_{1;\varnothing 2AB(mut)} = 0.0075568$

2b. For u=3,

Table 1

$q_{1113} = \frac{17}{110}$ $p_{1113} = \frac{23}{50}\frac{40}{60}\frac{30}{58}$

$q_{1123} = \frac{6}{110}$ $p_{1123} = \frac{23}{50}\frac{20}{60}\frac{20}{52}$

$q_{1213} = \frac{13}{110}$ $p_{1213} = \frac{27}{50}\frac{18}{50}\frac{30}{58}$

$q_{1223} = \frac{14}{110}$ $p_{1223} = \frac{27}{50}\frac{32}{50}\frac{20}{52}$

Table 2

$q_{2113} = \frac{15}{110}$ $p_{2113} = \frac{26}{33}\frac{40}{60}\frac{18}{58}$

$q_{2123} = \frac{11}{110}$ $p_{2123} = \frac{26}{33}\frac{20}{60}\frac{15}{52}$

$q_{2213} = \frac{3}{110}$ $p_{2213} = \frac{7}{33}\frac{18}{50}\frac{18}{58}$

$q_{2223} = \frac{4}{110}$ $p_{2223} = \frac{7}{33}\frac{32}{50}\frac{15}{52}$

Table 3

$q_{3113} = \frac{8}{110}$ $p_{3113} = \frac{11}{27}\frac{40}{60}\frac{10}{58}$

$q_{3123} = \frac{3}{110}$ $p_{3123} = \frac{11}{27}\frac{20}{60}\frac{17}{52}$

$q_{3213} = \frac{2}{110}$ $p_{3213} = \frac{16}{27}\frac{18}{50}\frac{10}{58}$

$q_{3223} = \frac{14}{110}$ $p_{3223} = \frac{16}{27}\frac{32}{50}\frac{17}{52}$

$I^{*}_{1;\varnothing 3AB(mut)} = 0.019087$

3. With inflection point at IP, the information estimates accord with

$$I_{1;\varnothing uAB(mut)} = \frac{\left|f_{...u}\, I^{*}_{1;\varnothing uAB(mut)} - f_{...u-1}\, I^{*}_{1;\varnothing u-1AB(mut)}\right|}{f_{...u} - f_{...u-1}}$$

for u = IP+1, ..., k. For given IP = 1 and k = 3,

$$I_{1;\varnothing 2AB(mut)} = \frac{\left|83 \text{ x } 0.0075568 - 50 \text{ x } 0.035039\right|}{83 - 50} = 0.034113$$

$I_{1;\varnothing 3AB(mut)} = 0.054533$

4. The average of the above values is the mutual information estimate sought,

$$I_{1;\,AB(mut)} = \frac{1}{k - IP} \sum_{u=IP+1}^{k} I_{1;\,\varnothing uAB(mut)} = \frac{0.034113 + 0.054533}{3 - 1}$$

$$= 0.044323$$

5. The variance of the mean is

$$S^2_{1;\,AB(mut)} = \frac{1}{(k - IP)(k - IP - 1)} \sum_{u=IP+1}^{k} (I_{1;\,\varnothing uAB(mut)} - I_{1;\,AB(mut)})^2$$

$$= \frac{(0.034113 - 0.044323)^2 + (0.054533 - 0.044323)^2}{(3 - 1)(3 - 1 -1)}$$

$$= 0.00010424$$

Use program InfoEst, option M, to compute estimates for mutual information of any legitimate order.

4.6 More data

Sampling along 4 line transects across 3 elevation belts (500 - 1000 m, 1000 - 1500 m, 1500 - 2000 m) and 6 stratal groups (herb, fern, low shrub, high shrub, evergreen tree, deciduous tree) yielded the following data:

Table 1 - North transect

0	2	11	0	0	0
3	9	5	2	0	0
1	3	8	1	1	1

Table 2 - East transect

1	6	6	3	1	2

0	3	10	1	1	0
2	10	4	0	1	0

Table 3 - South transect

2	1	12	0	1	1
1	5	6	2	1	0
0	1	12	2	0	0

Table 4 - West transect

1	7	6	0	2	1
0	0	9	1	2	1
1	6	8	4	0	0

Average frequencies determined from line intercepts are recorded. These are entered on file (Frequency.dat) by row:

0
2
11
0
0
0
3
9
5
2
0
0
1
3
8
1
1
1
1
6

6
3
1
2
0
3
10
1
1
0
2
10
4
0
1
0
2
1
12
0
1
1
1
5
6
2
1
0
0
1
12
2
0
0
1
7
6
0
2

1
0
0
9
1
2
1
1
6
8
4
0
0

Recall that this type of file begins with a number and each number is followed by an END-OF-PARAGRAPH mark created by pressing the RETURN key. Zeros are legitimate in the data as long as at least one cell in any row or column of a table is a non- zero value. No blanks are permitted.

4.7 Running InfoEst

After the data file is created, open a WORK folder and drag the ikons of InfoEst and the data file (Frequency.dat) to this folder. Start up InfoEst from the WORK folder by clicking twice on its ikon. The start-up dialogue is shown in Fig. 4.7.1 (interaction information) and in Fig. 4.7.3 (mutual information.) Observe that:

1. If the application and data are not in same folder, respond on the 1st screen line by pressing N (do not hit the RETURN key after N), otherwise press Y. Key N stops the run while key Y allows it to continue.

2. Press Y (do not hit the RETURN key) on 2nd screen line for printing of intermediate results. If N is pressed, only some results will be retained in the PRINTDA file.

```
   File  Edit  Custom  Run  Windows  Help
Is data file and this program in same folder? - press Y or N:Y
Is printing of intermediate results required? - press Y or N:Y
Output file name extension:FREQUENCY/I
Data file:Frequency.dat
Number of tables (sampling surges):4
Does any of the tables have at least one blank
row or column?-- press Y or N:N
Number of rows per table:3
Number of columns per table:6
Upper limit for alpha:4

Type of information: mutual or interaction? -- press M or I:I
RUN UDERWAY!           !

I*u values; alpha= 1 :
 .195431  .205699  .220424  .231202
Inspect these values and specify the position of the
deemed inflection point (1 to  3 ):? 1

I*u values; alpha= 2 :
 .30054  .314973  .332058  .333904
Inspect these values and specify the position of the
deemed inflection point (1 to  3 ):? 1

I*u values; alpha= 3 :
 .387459  .414601  .429318  .427252
Inspect these values and specify the position of the
deemed inflection point (1 to  3 ):? 1

I*u values; alpha= 4 :
 .462978  .509895  .520777  .513792
Inspect these values and specify the position of the
deemed inflection point (1 to  3 ):? 1
```

Fig. 4.7.1 First screen with details of the run dialogue as InfoEst, option 1 gets under way.

3. The output file name extension identifies the current PRINTDA file. If FREQUENCY/I is typed, the PRINDA file created in the run will have full name PRINTDA.FREQUENCY/I.

4. The data file is identified by its full name on the 4th screen line.

5. The number of tables is not limited.

6. Respond with N on the seventh screen line to abort the run if a blank row or a blank column is present in a table (do not press the return key after pressing N.) If Y is pressed, the run continues.

7. The number of table rows is invariant.

8. The number of table columns is invariant.

> If fewer numbers are given in the data file then specified in the start-up dialogue, or if blank lines are present in the data file, the application will stop.

9. The upper limit for alpha (screen line 10) is freely chosen. The lower limit is always 1 and the step size is also 1.

10. The type of information is either interaction (option I) or mutual (option M.)

11. The $I^{*}_{\alpha;\,\varnothing uAB(inter)}$ or $I^{*}_{\alpha;\,\varnothing uAB(mutual)}$ values, as many in number as there are successive frequency tables, are printed on the screen for each value of α and the user is requested to pick a position which is deems to represent the main inflection point.

The run concludes with identification of the PRINTDA file on the screen (Fig. 4.7.2 and Fig. 4.7.4.) The contents of the PRINTDA files are shown in Tables 4.7.1 and 4.7.2.

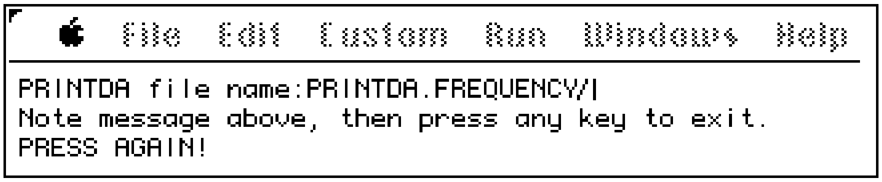


Fig. 3.7.2 Last screen in a run of application InfoEst.

Table 4.7.1 Contents of file PRINTDA.FREQUENCY/I created in a run of application InfoEst, option I.

```
PROGRAM InfoEst
Interaction information computed for
different alpha.



Input data file:Frequency.dat
Number of tables (sampling surges)= 4
Number of rows= 3
Number of columns= 6

DATA

TABLE 1
      0      2     11      0      0      0
      3      9      5      2      0      0
      1      3      8      1      1      1
TABLE 2
      1      6      6      3      1      2
      0      3     10      1      1      0
      2     10      4      0      1      0
TABLE 3
      2      1     12      0      1      1
      1      5      6      2      1      0
      0      1     12      2      0      0
TABLE 4
      1      7      6      0      2      1
      0      0      9      1      2      1
      1      6      8      4      0      0

Table totals

     47     51     47     49
Row totals

     66     62     66
Column totals

     12     53     97     16     10      6


I*u values; alpha= 1
 .195431  .205699  .220424  .231202



Relevant Iu values for alpha= 1 in positions 1 and up:
 .195431  .215162  .251129  .263095
```

```
I*u values; alpha= 2
 .30054  .314973  .332058  .333904


Relevant Iu values for alpha= 2 in positions 1 and up:
 .30054  .328273  .367682  .339367


I*u values; alpha= 3
 .387459  .414601  .429318  .427252


Relevant Iu values for alpha= 3 in positions 1 and up:
 .387459  .439615  .460002  .421138


I*u values; alpha= 4
 .462978  .509895  .520777  .513792


Relevant Iu values for alpha= 4 in positions 1 and up:
 .462978  .553132  .543468  .493123


alpha = 1
Inflexion point= 1
Mean I = .243128
Variance = 6.22405e-4
Sampling variance = 2.07468e-4

alpha = 2
Inflexion point= 1
Mean I = .345107
Variance = 4.12976e-4
Sampling variance = 1.37659e-4

alpha = 3
Inflexion point= 1
Mean I = .440252
Variance = 3.77915e-4
Sampling variance = 1.25972e-4

alpha = 4
Inflexion point= 1
Mean I = .529907
Variance = 1.03818e-3
Sampling variance = 3.46061e-4


Mean I for alpha=1 to 4
 .243128  .345107  .440252  .529907

Variance of I for alpha=1 to 4
 6.22405e-4  4.12976e-4  3.77915e-4  1.03818e-3

Sampling variance of I for alpha=1 to  4
 2.07468e-4  1.37659e-4  1.25972e-4  3.46061e-4


Maximum I= 1.38629

Relative I for alpha=1 to  4
```

```
.17538  .248942  .317575  .382247
```

```
   File  Edit  Custom  Run  Windows  Help
Is data file and this program in same folder? - press Y or N:Y
Is printing of intermediate results required? - press Y or N:Y
Output file name extension:FREQUENCY/M
Data file:Frequency.dat
Number of tables (sampling surges):4
Does any of the tables have at least one blank
row or column?-- press Y or N:N
Number of rows per table:3
Number of columns per table:6
Upper limit for alpha:4

Type of information: mutual or interaction? -- press M or I:M
RUN UDERWAY!           !

I*u values; alpha= 1 :
 .195431  .143359  .150524  .177986
Inspect these values and specify the position of the
deemed inflection point (1 to  3 ):? 1

I*u values; alpha= 2 :
 .30054  .220369  .236621  .283045
Inspect these values and specify the position of the
deemed inflection point (1 to  3 ):? 1

I*u values; alpha= 3 :
 .387459  .283179  .320309  .411055
Inspect these values and specify the position of the
deemed inflection point (1 to  3 ):? 1

I*u values; alpha= 4 :
 .462978  .337107  .40455  .573947
Inspect these values and specify the position of the
deemed inflection point (1 to  3 ):? 1
```

Fig. 4.7.3 First screen showing details of the run dialogue as InfoEst, option M gets under way.

```
   File  Edit  Custom  Run  Windows  Help
PRINTDA file name:PRINTDA.FREQUENCY/M
Note message above, then press any key to exit.
PRESS AGAIN!
```

Fig. 4.7.4 Last screen in the second run of application InfoEst.

Table 4.7.2 Contents of PRINTDA.FREQUENCY/M created in a run of application InfoEst, option M.

```
PROGRAM InfoEst
Mutual information computed for
different alpha.

Input data file:Frequency.dat
Number of tables (sampling surges)= 4
Number of rows= 3
Number of columns= 6


DATA

TABLE 1
      0      2     11      0      0      0
      3      9      5      2      0      0
      1      3      8      1      1      1
TABLE 2
      1      6      6      3      1      2
      0      3     10      1      1      0
      2     10      4      0      1      0
TABLE 3
      2      1     12      0      1      1
      1      5      6      2      1      0
      0      1     12      2      0      0
TABLE 4
      1      7      6      0      2      1
      0      0      9      1      2      1
      1      6      8      4      0      0

Table totals

     47     51     47     49
Row totals

     66     62     66

Column totals

     12     53     97     16     10      6


I*u values; alpha= 1
 .195431  .143359  .150524  .177986



Relevant Iu values for alpha= 1 in positions 1 and up:
 .195431  9.53716e-2  .165463  .259252



I*u values; alpha= 2
 .30054  .220369  .236621  .283045
```

```
Relevant Iu values for alpha= 2 in positions 1 and up:
 .30054  .146487  .270506  .420423


I*u values; alpha= 3
 .387459  .283179  .320309  .411055


Relevant Iu values for alpha= 3 in positions 1 and up:
 .387459  .187078  .397727  .679591


I*u values; alpha= 4
 .462978  .337107  .40455  .573947


Relevant Iu values for alpha= 4 in positions 1 and up:
 .462978  .221108  .545176  1.07522


alpha = 1
Inflexion point= 1
Mean I = .173362
Variance = 6.76098e-3
Sampling variance = 2.25366e-3

alpha = 2
Inflexion point= 1
Mean I = .279139
Variance = 1.88162e-2
Sampling variance = 6.27207e-3

alpha = 3
Inflexion point= 1
Mean I = .421466
Variance = 6.10648e-2
Sampling variance = 2.03549e-2

alpha = 4
Inflexion point= 1
Mean I = .613835
Variance = .185914
Sampling variance = 6.19713e-2


Mean I for alpha=1 to 4
 .173362  .279139  .421466  .613835

Variance of I for alpha=1 to 4
 6.76098e-3  1.88162e-2  6.10648e-2  .185914

Sampling variance of I for alpha=1 to  4
 2.25366e-3  6.27207e-3  2.03549e-2  6.19713e-2


Maximum I= 1.09861

Relative I for alpha=1 to  4
 .157801  .254083  .383635  .558737
```

4.8 Remarks

The properties outlined in Section 2.9 apply to the marginal distributions and the joint distribution. There are also new properties:

1. The mutual information is a more restrictive descriptor of relationships than the interaction information. The interaction information cannot be less than the mutual information.

2. Whereas entropy in the marginal and joint distributions has a descending trend with increasing α, information (mutual or interaction) has an ascending trend. In all cases the variance increases with increasing α.

3. Both mutual and interaction information of order 1 have statistical meaning under Kullback's definition of MDIS. InfoEst estimates information of different orders; this allows flexibility in the characterizations of the data.

4. To pass from Rényi's I of order 1 to Kullback's MDIS, multiply the former by twice the grand total of the tables. For example, a relevant estimate in Table 4.7.1 is I_1 = 0.243128 or in relative terms 0.243128/ln 4 = 0.175379 (also in Table 4.7.1.) The relevant grand total of the frequencies is 147 (last 3 tables in Table 4.7.1.) The corresponding MDIS quantity is 0.243128 x 2 x 147 = 71.480 which has 20 degrees of freedom. The possible maximum MDIS is 2 x ln 4 x 147 = 407.570 and the relative MDIS is the same as the relative I_1. The latter indicates a rather weak relationship.

Glossary

accuracy -- closeness to the true value.

ASCII -- a standard sorting order for characters; a coding system used in computer work, *e.g.*, ASCII code 77 identifies the capital letter M.

biological type -- the organism's strategy by which it survives the unfavourable season; also life-form.

bit -- the unit of entropy; $\log_2 2$ is one bit.

community -- here a plant assemblage structured by types and interactions.

diversity -- the number or richness of alternatives; usually expressed as a logarithm of proportions.

disorder -- a state of reduced predictability; diversity.

distribution -- an arrangement of events or objects between types.

entropy -- information per observation; the level of disorder; surprisal value.

equivocation -- the portion that is specific; the opposite of mutual; information in one distribution not repeated in another.

evenness -- the closeness to an equi-distribution.

information -- a multiple of entropy; a logarithmic measure of mutuality or equivocation; a divergence or equivocation.

interaction -- here an analytical property measurable as information.

MDIS -- Kullback's information theoretical measure on which his brand of statistics is based; a one-way divergence measured as the logarithm of the ratio of proportions.

mutual -- not specific; shared.

population -- a collection of events of the same generic type characterized by a frequency distribution; a collection of organisms characterized by common inheritance.

process sampling -- sampling in surges with intermittent analyses to monitor the evolution of specific internal sample properties and their environmental connections.

sample -- a subset of the population.

sampling -- the act of selecting units for measurement.

sampling surge -- a step in process sampling.

species richness -- the number of species in a community; the logarithm of this number; state richness.

state richness -- a property of distributions; the logarithm of the number of states; species richness.

surge size -- sample size per step in process sampling.

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Three main topics are covered: diversity graphs, entropy estimation, information estimation. Concepts are discussed, methods described and step-by-step examples presented. A synopsis of the program package INFOPACK is given and the run dialogue is explained. The presentations assume program implementation on a Macintosh. Book and programs are directed to users interested in diversity theory and research.