A Geometrical Approach Assessing Instability Trends for Galloping

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1 Introduction

The galloping of iced electrical conductors has been considered, since early in this century, by many researchers. Important results have been obtained, for example, by Den Hartog (1932), Simpson (1965), Chadha (1973), and Blevins and Iwan (1974). The main focus of these papers, however, was to find the requirements for the initiation of galloping as well as to determine the conditions for the instability of the vibrations. However, galloping is highly complex as a result particularly of the nonlinear aerodynamic forces and because many physical parameters are involved. Although several analytical expressions have been given for the criteria of instability (e.g., Blevins and Iwan, 1974), the direct application of these expressions to the design of appropriate control devices is still not possible. This serious deficiency suggests that it is necessary to investigate the stability trends with respect to the system's parameters. Moreover, exact solutions and instability conditions may not be meaningful in view of the approximations in the mathematical modeling and analysis and the errors in laboratory experiments and field trials. Instability trends, therefore, should provide more suitable and effective guidelines for a robust control strategy which makes a particular design more tolerant of uncertain parameters.

A geometrical approach will be introduced to consider the instability trends for the galloping of a two degrees-of-freedom model (Blevins and Iwan, 1974). The approach is based upon the instability conditions for the steady-state solutions associated with equilibrium, periodic, and quasi-periodic motions. Of particular interest here are the conditions for the initiation of galloping as well as the critical boundaries where a plunge or torsional motion, or even a mixed-mode motion, loses stability. A two-dimensional parameter space will be chosen to be compatible with the instability trends. Thus, instead of expressing the instability conditions in the whole (at least eight-dimensional) parameter space, a single parameter—the wind speed—will be chosen as the critical variable. Then the critical values of this variable, which correspond to the initiation of galloping and to the onset of instability of the periodic motions, will be determined. The influence of the remaining variables will be found in the two-dimensional parameter space by considering their individual effects on the critical values of the wind speed.

A brief derivation of the equations is given in the Appendix and the results obtained are presented in the following section. The geometrical approach and two practically important instability trends are discussed in Section 3. Finally, conclusions are drawn in Section 4.

2 Initiation of Galloping and Critical Boundaries for a Two Degree-of-Freedom System

The two degree-of-freedom model shown in Fig. 1 has mass $m$ and moment of inertia $I$. It represents a cross-section of an iced conductor where $y$ is the vertical (plunge) displacement and $\theta$ is the angle of rotation or twist. The $k_y$ and $k_\theta$ are the vertical and torsional stiffness, respectively, and the $c_y$ and $c_\theta$ are corresponding viscous dampers (which are not shown in the figure).

Fig. 1 Elastically supported two degrees-of-freedom model
Table 1  Steady-state solutions

<table>
<thead>
<tr>
<th>Solution</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
<td>I.E.S. $A_y = 0$, $A_0 = 0$</td>
</tr>
<tr>
<td>(II)</td>
<td>H.B.S. (P) $A_y^2 = -3B_1 B_3$, $A_0^2 = 0$</td>
</tr>
<tr>
<td>(III)</td>
<td>H.B.S. (T) $A_y^2 = 0$, $A_0^2 = -3B_2 B_4$</td>
</tr>
<tr>
<td>(IV)</td>
<td>2-D Torus $A_y^2 = B_2 B_3$, $A_0^2 = B_4 B_5$</td>
</tr>
</tbody>
</table>

Fig. 1). Furthermore, $s_x$ generally indicates the eccentricity so that $s_x/m$ is the lateral position of the iced conductor’s center of gravity (C.G.) measured from its center of rotation $O$. However, only the eccentric case corresponding to $s_x = 0$ will be considered here. The rate equations, describing the motion of the eccentric model, expressed in terms of the amplitudes ($A_y$, $A_0$) and phases ($\phi_y$, $\phi_0$) of $y$ and $\theta$, respectively, are obtained by setting $\dot{\eta}_y = 0$ ($\dot{\theta} = 0$) in equations (A12) and (A13) as follows:

$$
A_y = \eta_y A_y \left( \frac{1}{2} U_r a_1 - \frac{\xi_y}{\eta_y} \right) + \frac{3a_0}{8U_r} \left[ A_y^2 + 2(U_y^2 + R_1^2 \omega_1^2) A_0^2 \right],$$

$$
A_0 = \eta_0 A_0 \left( \frac{1}{2} U_r a_1 - \frac{\omega_0}{\eta_0} \right) + \frac{3a_0}{8U_r} \left[ A_y^2 + (U_y^2 + R_1^2 \omega_1^2) A_0^2 \right].$

and

$$
\phi_y = 1, \\
\phi_0 = \omega + \frac{U_r a_1}{2a} \left[ U_j a_1 + \frac{3a_0}{4U_r} \left( 2A_y^2 + (U_y^2 + R_1^2 \omega_1^2) A_0^2 \right) \right],$$

where

$$
\xi_y = \frac{c_y}{2 \rho U_r}, \quad \xi_0 = \frac{c_0}{2 \rho U_r}, \quad \eta_y = \frac{\rho d^2}{2m}, \quad \eta_0 = \frac{\rho d^2}{2m}, \quad \eta_y = \eta_0, \quad \eta = \eta_0,$$

and

$$
U_j = \frac{U}{\omega d}, \quad r_1 = \frac{r_1}{d}, \quad \omega_2 = \frac{k_2}{m}, \quad \omega = \frac{\omega_2}{\omega_1}, \quad \omega = \frac{\omega_2}{\omega_1}.$$

In equations (1) through (3), $\rho$ is the density of air, $U$ is the free-stream wind speed, $d$ is a characteristic length which is usually taken as the maximum width of the cross-section normal to the free stream, and $r_1$ is the characteristic radius of the section. The aerodynamic lift force and moment are approximated by the best fit cubic polynomials having coefficients $a_1$, $a_2$, $b_1$, and $b_2$ (Blevins and Iwan, 1974).

The steady-state solutions can be obtained readily from (1) by setting $A_y = A_0 = 0$. They are listed in Table 1, where the abbreviations I.E.S., H.B.S.(P), H.B.S.(T), and 2-D Torus represent the initial equilibrium solution, the Hopf bifurcation solution corresponding to a periodic plunge vibration, the Hopf bifurcation solution for a periodic torsional vibration, and a motion of a two-dimensional torus, respectively. In Table 1, the coefficients $B_i$ ($i = 1, 2, \ldots, 6$) are given by

$$
B_1 = \frac{1}{2} U_r a_1 - \frac{\xi_y}{\eta_y}, \quad B_2 = \frac{8U_r}{9a_1}.$$

Table 2  The stability conditions and critical boundaries for $s_x = 0$

<table>
<thead>
<tr>
<th>Solution</th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stability Condition</td>
<td>$B_1 &lt; 0$, $B_2 &lt; 0$</td>
<td>$B_1 &lt; 0$, $B_2 &lt; 0$</td>
<td>$B_1 &lt; 0$, $B_2 &lt; 0$</td>
<td>$B_1 &lt; 0$, $B_2 &lt; 0$</td>
</tr>
<tr>
<td>Frequency</td>
<td>$\omega$, $\frac{3\Omega_0}{7}$</td>
<td>$\omega$, $\frac{3\Omega_0}{7}$</td>
<td>$\omega$, $\frac{3\Omega_0}{7}$</td>
<td>$\omega$, $\frac{3\Omega_0}{7}$</td>
</tr>
<tr>
<td>Stability Condition</td>
<td>$B_1 &lt; 0$, $B_2 &lt; 0$</td>
<td>$B_1 &lt; 0$, $B_2 &lt; 0$</td>
<td>$B_1 &lt; 0$, $B_2 &lt; 0$</td>
<td>$B_1 &lt; 0$, $B_2 &lt; 0$</td>
</tr>
<tr>
<td>Critical Boundary</td>
<td>$C_1 \cdot B_1 = 0$</td>
<td>$C_2 \cdot B_2 = 0$</td>
<td>$C_3 \cdot B_3 = 0$</td>
<td>$C_4 \cdot B_4 = 0$</td>
</tr>
<tr>
<td>Bifurcation Solution</td>
<td>(I)</td>
<td>(II)</td>
<td>(III)</td>
<td>(IV)</td>
</tr>
<tr>
<td></td>
<td>3-D Torus</td>
<td>Chaos</td>
<td>Chaos</td>
<td></td>
</tr>
</tbody>
</table>

Note: The bifurcation solutions (I), (II), (IV) and 3-D torus or chaos given in the last row, denote the solutions bifurcating from (I), (II), (III) and (IV) along the critical boundaries $C_1$, $C_2$, $C_3$, and $C_4$, respectively.

$$
B_2 = \frac{1}{2} U_r b_1, \quad B_3 = B_2 - 2S_2 B_1, \quad \frac{3}{5} S_3 = B_2 - \frac{8U_1}{9a_1}, \quad B_3 = B_1 \cdot \frac{2}{S_3} B_2,$$

where

$$
S_3 = \frac{B_2}{B_1}.$$

Stability conditions for the steady-state solutions and the critical boundaries, where bifurcations occur, can be obtained from equation (I). The results are summarized in Table 2 and are illustrated graphically in Fig. 2.

Table 2 and Fig. 2 present an overview of the dynamical behavior of the system. Critical boundaries where bifurcations
occur are given explicitly. Bifurcations and secondary bifurcations into periodic and nonresonant quasi-periodic motions are also indicated. The aerodynamic coefficients $a_1$ and $b_1$ can be seen to play a very important role in determining the stability of these bifurcations. Indeed, depending upon $a_1$ and $b_1$, four distinct cases exist. They are categorized in Fig. 2 as case (a) $a_1 > 0$, $b_1 > 0$, for which no stable periodic solution exists. Case (b) $a_1 < 0$, $b_1 < 0$ where both the plunge and torsional motions can be stable but they cannot exist simultaneously. Case (c) $a_1 < 0$, $b_1 > 0$ for which the plunge motion is stable but the torsional motion is unstable. (Furthermore, subsequent bifurcations from the stable plunge motion may lead motion is unstable. (Furthermore, subsequent bifurcations from the stable plunge motion may lead to a family of two-dimensional or even three-dimensional tori.) Case (d) $a_1 > 0$, $b_1 < 0$ where the torsional motion is stable but the plunge motion is unstable. Then the same family of the two-dimensional or three-dimensional tori may bifurcate from the stable torsional motion.

3 Instability Trends

3.1 A Geometrical Approach. Although the stability conditions of the steady-state solutions have been given explicitly in the previous section, instability trends with respect to different parameter are still unknown. A geometrical approach will be presented next to give a clear intuitive view of how these instability trends change with variations in the system's parameters. The main thrust will be to first introduce a reference line (or curve) representing a given parameter in the two-dimensional, $B_1 - B_2$ parameter space employed, for example, in Fig. 2. Then critical values will be found, with respect to this reference, which correspond to the initiation of galloping and to the (dynamic) instability of the periodic as well as the quasi-periodic motions.

It has been reported that a change in a steady wind speed is an important factor in causing instability (Edwards and Madeyski, 1956). This observation suggests that the dimensionless wind speed, $U_j$, is a reasonable choice for the reference line. Indeed, it may be deduced, from the critical boundaries given in Table 2, that all the critical values can be expressed in terms of $U_j$ because every critical boundary is described by an homogeneous equation. The equation of the reference line can be obtained from equation (4) by eliminating $U_j$ in the following form:

$$U_j: B_2 = S_1 B_1 + \left( \frac{\xi_2}{\eta_2} \right) (S_1 - Q)$$  \hspace{1cm} (5)

where

$$S_1 = r_l \left( \frac{b_1}{a_1} \right) = r_l b$$

and

$$Q = \left( \frac{\omega}{\eta_2} \right) \left( \frac{\xi_2}{\eta_2} \right) = \frac{(\omega_{k_0}/\eta_2)}{\left( \frac{\xi_2}{\eta_2} \right)} = \frac{1}{b} \left( \frac{c_i}{c_i} \right) .$$  \hspace{1cm} (6)

Here, the ratio $(1/d^2)$ $(c_i/c_i)$ is defined as a new parameter $P$. In practice the natural frequency ratio $\omega = \omega_{k_0}/\omega_0$ is often manipulated to alleviate galloping by using dampers (Havard, 1988; Sasaki et al., 1986). Now, $Q$ is related directly to the natural frequency ratio because $c_i/c_i = (1/m)$ . Therefore, it is believed that $Q$ plays a significant role. Moreover, changing the ratio $\omega$ (e.g., by using dampers) may also simultaneously vary the inertia and mass of the system. It is more practically useful, therefore, to consider $Q$ instead of the simpler ratio, $\omega$, as a control parameter. Also, it may be noted from equations (5) and (6) that the slope of the line $U_j$ involves a factor $b = (b_1/a_1)$. The $b$ is an uncontrollable parameter because it is very sensitive to the uncertain geometric shape produced by the

Fig. 3. The critical boundaries weather-dependent ice accumulation. However, $b$ is very important in determining the stability of the conductor's initial configuration because the stability conditions of the initial equilibrium state depend upon both $a_1$ and $b_1$. The effects of the two parameters $Q$ and $b$, therefore, will be considered separately later.

It has been observed that $a_1$ and $b_1$ are both positive, whereas $a_2$ and $b_2$ are both negative in most practical situations (Novak, 1971). Thus, special attention will be given in the following analysis to the case: $a_1 > 0$, $b_1 > 0$, $a_2 < 0$, and $b_2 < 0$. These stipulations correspond to case (b) in Fig. 2. However, it is not difficult to extend the approach to the other cases presented in Fig.2.

The following two distinct cases can be found by comparing the slope of $U_j$, i.e., $S_1$, to the control parameter $Q$:

Case (P): $Q > S_1$, corresponds to the initiation of plunge vibration, and

Case (T): $Q < S_1$, corresponds to the initiation of torsional vibration.

These two distinct cases are illustrated in Fig. 3. The hatched lines in this figure indicate the stability boundaries of the initial equilibrium solution. The $U_1$ and $U_2$ are the initiation values for plunge and torsional vibrations, respectively. The $U_{f1}$ and $U_{f2}$, on the other hand, respectively represent the practically important critical values where the stability of the
plunge motion and the torsional motion are lost. All the initiation and critical values can be obtained from the critical boundaries listed in Table 2 and equation (4). They are given as follows:

\[
\begin{align*}
U_{(P1)} &= \frac{2\xi_y}{\eta_ya_1}, \\
U_{(T1)} &= \frac{2\omega_y}{\eta_yb_1}, \\
U_{(P2)} &= \left(\frac{2\xi_y}{\eta_ya_1}\right) \frac{(Q-2S_1)}{(S_1-2S_2)}, \quad \left(\frac{1}{Q} - \frac{2}{S_2}\right), \\
U_{(T2)} &= \left(\frac{2\xi_y}{\eta_ya_1}\right) \frac{(S_2-2Q)}{(S_2-Q)} \frac{(1/2)}{S_2}. \\
\end{align*}
\]

(7)

It can be seen from Fig.3(a) that if the slope of \( C_1 \), the critical boundary for the plunge motion defined in Table 2 is larger than that of \( U_y \) (i.e., \( 2S_1 > S_1 \)), there is no intersection of \( C_1 \) and \( U_y \) in the first quadrant. This implies \( U_{(P2)} = \infty \) so that the actual wind speed, \( U_y \), can never exceed \( U_{(P2)} \), and the plunge motion is always beneficently stable.

A similar conclusion can be drawn from Fig. 3(b) for the torsional vibration case, H.B.S.(T). The two sets of results are summarized, for convenience, in Table 3.

Table 3 suggests that both the plunge and the torsional motion are stable when \( 1/2 \leq S_1 \leq S_2 \), irrespective of the wind speed (i.e., \( U_{(P1)} = \infty \) and \( U_{(T1)} = \infty \)). This region, therefore, is of no practical interest. Consequently, it is assumed from the following analysis that

\[
S_1 > 2S_2 \text{ for H.B.S.(P), and} \\
S_1 < \frac{1}{2} S_2 \text{ for H.B.S.(T).} \\
\]

(8)

3.2 General Instability Trends. Consider the effects of variations in the system's parameters on the critical values \( U_{(P1)}, U_{(T1)}, U_{(P2)}, U_{(T2)} \). As the parameters of their combinations change, lines like \( U_y, C_y, C_t, \text{ etc.,} \), will move in the \( B_1-B_2 \) plane and new values will be obtained for \( U_{(P1)}, U_{(T1)}, U_{(P2)} \) and \( U_{(T2)} \). For illustration, examine the simple example of \( c_y \), which results in \( \xi_y/\eta_y = c_y/\omega_y \), where the arrow indicates an increase (decrease) in the given parameter. This example is illustrated in Fig. 4 where a superscript prime denotes a new value of a parameter whereas the corresponding unprimed quantity indicates the old values. An "x" in this figure represents the point to which the old \( U_{(P1)} \) or old \( U_{(T2)} \) will move when \( c_y \) increases. The movements are indicated by the dashed lines which are parallel to the line passing through the points marked \( U_y = 0 \) and \( U_y = 0 \). These latter points correspond to the old and new values of \( c_y \), respectively.

![Fig. 4 Instability trends for increasing \( c_y \) and \( Q > S_1 \).](image)

The following results can be observed from Fig. 4. When \( c_y \) increases, the point marked \( U_y = 0 \) will move to the left to a point marked \( U_y' = 0 \). Hence, line \( U_y', \text{ which is parallel to \( U_y \)} \), will be displaced to the left, too. Thus, the new value \( U_{(P1)} \) obtained from the moving up of \( U_{(P1)} \) parallel to axis \( B_1 \), is greater than the value of the corresponding point marked "x," which moves from \( U_{(P1)} \) parallel to axis \( B_1 \). Therefore, an increase in \( c_y \) causes \( U_{(P1)} \) to grow. Similarly, it can be concluded that an increase in \( c_y \) makes \( U_{(T2)} \) diminish. Moreover, it is seen that the stable region for the initial equilibrium solution is a rectangle in the third quadrant which is bounded by the \( B_1, B_2 \) axis, and the dashed lines passing through \( U_y = 0 \) or \( U_y = 0 \). So, a greater \( c_y \) will enlarge the stable region of the I.E.S. In summary,

\[
c_y \xrightarrow{\uparrow} U_{(P1)} \xrightarrow{\uparrow} U_{(P2)} \quad \text{and S.R. of I.E.S.} \\
\]

where S.R. denotes the stable region. The same conclusions may also be deduced straightforwardly from equation (7) for this simple example.

Although conclusion (9) was obtained on the basis of a particular point \( U_y = 0 \) and a small increase in \( c_y \), it can be
Table 6  Trends of the I.E.S. stability region (S.R.) and the key points w.r.t. Q in case (P)

<table>
<thead>
<tr>
<th>Case</th>
<th>Region</th>
<th>Definition</th>
<th>I.E.S. at</th>
<th>U_{(p_3)}</th>
<th>U_{(p_4)}</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>Q&lt;0</td>
<td>Q&lt;0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>Q&lt;0</td>
<td>Q&lt;0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td>Q&lt;0</td>
<td>Q&lt;0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(iv)</td>
<td>Q&lt;0</td>
<td>Q&lt;0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

observed from Fig. 4 that this conclusion is true for any arbitrary location of the point U_p = 0. Thus, conclusion (9) is true globally so that is robust.

Following the previous procedure, instability trends were derived for separate changes in several simple parameters. These trends are summarized in Tables 4 and 5. Except for the stability region (S.R.) of the I.E.S., the results can be obtained, alternatively, from equation (7). Later, however, two important cases will be studied in which it may not be feasible to use equation (7). Tables 4 and 5 suggest that the stable I.E.S. region and the the initiation points U_{(p_3)} and U_{(p_4)} generally grow beneficially with individual increases in c_p or c_0. On the other hand, such increases, when simultaneous, have disadvantageously counteracting influences on the dynamic stability points U_{(p_3)} and U_{(p_4)}. Consequently, the obvious control strategy of simple enlarging both c_p and c_0 does make the initiation of galloping harder, but the wind speed for the onset of a dynamic instability may or may not be affected. Increasing the aerodynamic coefficients a_1, b_1 or the ratio b_1/a_1 produces similar opposing trends for U_{(p_3)} and U_{(p_4)}, but the wind speed at the initiation of galloping is either unchanged or reduced disadvantageously. Therefore, the problem of controlling galloping may not be straightforward.

3.3 Two Important Practical Cases. Specific instability trends with respect to (w.r.t.) first Q (rather than c_p or c_0) and then D will be considered next. It is more appropriate, from a practical viewpoint, to consider the effects of c_p and c_0 simultaneously because, as seen in Section 3.1, Q is the key parameter which distinguishes the torsional from the plunge vibration. Further, Q plays a significant role in controlling the stability trends of galloping.

3.3.1 Instability Trends w.r.t. Parameter Q. First, consider case (P), in which Q > S_1, and, further, suppose that Q is increasing. Draw the line U_p and lines having the old slope, Q, and the new slope, Q', in the B_1 - B_2 plane as indicated in Fig. 5. To draw the line corresponding to a new U_p = 0, the origin U_p = 0 has to be chosen in the third quadrant on the line having slope U_p = 0, the origin U_p = 0 has to be chosen in the third quadrant on the line having slope Q'. This origin reflects the absolute values of c_p and c_0 and its location will

Fig. 6  Different characteristic regions which depend upon the origin of U_p in case (P)

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Table 7: Trends of the I.E.S. stability region (S.R) and the key points w.r.t. $Q$ in case (T)

<table>
<thead>
<tr>
<th>Case</th>
<th>Region</th>
<th>Definition</th>
<th>S.R. of I.E.S.</th>
<th>$U(r_1)$</th>
<th>$U(r_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>$q_y &lt; q_0$</td>
<td></td>
<td>$\downarrow$</td>
<td>$\downarrow$</td>
<td>$\downarrow$</td>
</tr>
<tr>
<td>(ii)</td>
<td>$q_y &lt; q_0 &lt; Q_0\gamma$</td>
<td>Indefinite</td>
<td>$\downarrow$</td>
<td>$\downarrow$</td>
<td>$\downarrow$</td>
</tr>
<tr>
<td>(iii)</td>
<td>$Q_0\gamma &lt; q_y &lt; Q_0\gamma^2$</td>
<td></td>
<td>$\downarrow$</td>
<td>$\downarrow$</td>
<td>$\downarrow$</td>
</tr>
<tr>
<td>(iv)</td>
<td>$Q_0\gamma^2 &lt; q_y$</td>
<td></td>
<td>$\downarrow$</td>
<td>$\downarrow$</td>
<td>$\downarrow$</td>
</tr>
</tbody>
</table>

Table 8: Trends of the critical boundaries w.r.t. $b$

<table>
<thead>
<tr>
<th>Case</th>
<th>Critical Boundary</th>
<th>Condition</th>
<th>$b$ (✓)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q &gt; S_1$</td>
<td>$U_{(P_1)}$</td>
<td>$\beta &lt; \frac{1}{2}$</td>
<td>✓</td>
</tr>
<tr>
<td>$Q &gt; S_1$</td>
<td>$U_{(P_2)}$</td>
<td>$\beta &lt; \beta_3$</td>
<td>✓</td>
</tr>
<tr>
<td>$Q &lt; S_1$</td>
<td>$U_{(T_1)}$</td>
<td>$\beta &gt; 0$</td>
<td>✓</td>
</tr>
<tr>
<td>$Q &lt; S_1$</td>
<td>$U_{(T_2)}$</td>
<td>$\beta &gt; \beta_4$</td>
<td>✓</td>
</tr>
</tbody>
</table>

Note: $\beta_3 = -\tan^{-1}(2S_3)$ and $\beta_4 = \pi - \cot^{-1}(2S_3)$ where $S_3$ is defined in equation (6).

Fig. 7: Different characteristic regions which depend upon the origin of $U'_y$ in case (T)

Fig. 8: Instability trends for increasing $b$ in case (P)

affect the results. Once chosen, the line $U'_y$ can be drawn parallel to the line $U_y$ and the new values of $U_{(P_1)}$ and $U_{(P_2)}$ can be obtained. Next, draw dashed lines through the points $U_{(P_1)}$ and $U_{(P_2)}$ which are parallel to the line from the point $U_y = 0$ to the point $U'_y = 0$. These dashed lines intersect the line $U_y$ at the point marked by "x" in Fig. 5. Thus, an increase or a decrease in $U_{(P_1)}$ and $U_{(P_2)}$ can be determined, as a result of $Q$ increasing, by comparing $U_{(P_1)}$ and $U_{(P_2)}$ with the corresponding points marked by "x." For the case depicted in Fig. 5, $U_{(P_1)}$ and $U_{(P_2)}$ when $Q$. However, it can be shown that the choice of the origin $U'_y = 0$ affects this conclusion. Four different regions can be distinguished depending
upon the location of \( U' = 0 \). They are indicated by (i), (ii), (iii), and (iv) in Fig. 6(a). A similar procedure can be followed for the case \( Q \) and the results are shown in Fig. 6(b). The associated instability trends for \( Q \) and \( Q' \) are listed in Table 6, where

\[
c^* = \frac{(S_i - Q)}{(S_i - Q')} c^*,
\]

(10)

It should be noted that, although the notation for the different regions given in Fig. 6 and Table 6 are identical, the definitions of these regions are very different. Also, it is interesting to note that, if compared in the reverse order of the rows in Table 6, the trends for \( Q' \) are opposite to those for \( Q \) because of antisymmetry.

The procedure described above can be applied similarly to case (T) where \( Q < S_i \). The results are shown in Fig. 7 and they are summarized in Table 7. Due to the symmetry between the equation describing the torsional motion and that giving the plunge motion (see equation (1)), the trends of the critical arrowed values listed in Table 7 for \( Q \) (\( Q' \), respectively) are identical to those given in Table 6 for \( Q' \) (\( Q \), respectively).

The need for four separate regions in Tables 6 and 7 suggests that the effects of changing \( Q \) are not described simply. However, it appears from these tables that a larger or a smaller \( Q \) generally produces conflicting trends in the initiation and the dynamic stability of either plunge or torsional galloping. Furthermore, these trends are usually opposite for the plunge and corresponding torsional situation. Therefore, a control strategy for galloping may have to be a careful compromise which depends upon individual circumstances.

3.3.2 Instability Trends w.r.t. Parameter \( b \). Parameter \( Q \) does not change for this case but the slope of the line \( U' \) does vary. First, consider case (P), \( Q > S_i \), which corresponds to the initiation of a plunge vibration. Suppose \( b = (+ b_1/a_1) \) is increasing as exemplified in Fig. 8.

In order to compare \( U(p_1) \) with \( U(p_2) \) and \( U(p_3) \) with \( U(p_4) \), the positions of the points \( U(p_1) \), \( U(p_2) \), and \( U(p_3) \) have to be found on line \( U' \). First, define a new parameter (angle) \( \beta \) given by

\[
\tan \beta = \frac{\Delta b_1}{b_1 - b_1} \frac{b_1 - b_1}{a_1 - a_1},
\]

(11)

where the range of \( \beta \) is found to be

\[
-\tan^{-1}(S_i) = \beta_1 \leq \beta \leq \beta_2 = \pi - \beta_1.
\]

(12)

Then it is straightforward, in terms of \( \beta \), to draw similar diagrams to Fig. 8 for the cases presented in Table 8. For example, consider the changing trends of \( U(p_1) \). It is easy to observe from Fig. 8 that \( \beta = \pi/2 \) is a critical value for which the new point \( U(p_1) \) will be located at the place marked by "x." So \( U(p_1) \) does not change when \( b \) varies such that \( \beta = \pi/2 \). Here, it should be noted that, for convenience, the direction of measuring angle \( \beta \) in Fig. 8 is always clockwise irrespective of whether parameter \( b \) increases or decreases.

The results shown in Table 8 suggest that the possibilities of the initiation as well as the dynamic instability of galloping are less if \( b \) is small, or even negative, when \( b \) grows. Of course, the reverse is true when \( b \) decreases.

4 Conclusions

A geometrical approach has been introduced to find the instability trends of galloping of an iccd, transmission lines when its parameters change. Based on the stability boundary derived from a two degrees-of-freedom model, critical wind speeds are obtained, with respect to (at least eight) system parameters, but in a single two-dimensional space. Besides considering the general instability trends, important practical cases are studied in detail. Tabulations are presented of the increases or decreases in the stability trends, with respect to various parameter ranges, for the initiation of galloping and periodic vibrations. Not only does the geometrical approach provide robust solutions, but it can also be generalized straightforwardly to accommodate eccentricities, and more than the same representative, but still only two degrees-of-freedom model employed here. Changing the parameters of an iccd conductor has been shown to often lead to contradictory instability trends. Therefore it is not surprising that the most practical way to best control galloping is still debated.

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References


APPENDIX

A brief outline of the derivation of equations (1) and (2) is given when \( s/m \neq 0 \). The equation describing the motion of a two degree-of-freedom model are (Blevins, 1974):

\[
\begin{align*}
\frac{m}{4} \ddot{y} + c_2 \dot{y} - s_x \dot{x} + k_2 y &= F_y, \\
\dot{x} + c_0 \dot{x} - s_y \dot{y} + k_0 x &= F_M,
\end{align*}
\]

(A1)

where \( m, I, s_x \), and \( s_y \) are defined with reference to the center of rotation, \( O \), in Fig. 1 as

\[
m = \int_A \rho dV, \quad I = \int_A (\xi^2 + \eta^2) \rho dV, \\
s_x = \int_A \xi \rho dV,
\]

(A2)

and \( \kappa \) is the mass density over cross-section \( A \). The \( F_y \) and \( F_M \) represent the vertical aerodynamic force and the aerodynamic moment, respectively. They are functions of the angle of attack \( \alpha \), and can be expressed by

\[
F_y = \frac{1}{2} \rho U^2 d C_y (\alpha),
\]

(A3)

\[
F_M = \frac{1}{2} \rho U^2 d C_M (\alpha).
\]

(A4)

The \( \alpha \) can be approximated by

\[
\alpha \approx -\dot{R}_1 \frac{1}{U} \dot{y}.
\]
Coefficients $C_m$ and $C_{p}$ are relatively smooth continuous functions of $\alpha$ and they may be expressed as experimentally determined polynomials in $\alpha$. The cubic polynomial approximation suggested by Blevins and Iwan (1974) for symmetric ice shapes is used here, i.e.,

$$C_m = C_{p} = a_1 \alpha + a_2 \alpha^3.$$  \hspace{1cm} (A10)

A combination of equations (A1), (A3), and (A5) produces the following set of dimensionless equations:

$$\begin{align*}
\tilde{Y} - \eta \tilde{\phi} + \omega^2 \tilde{Y} = -2 \xi_\eta \eta \tilde{U}_r \tilde{U}_r \tilde{U}_r \tilde{U}_r \tilde{A} \tilde{B} \tilde{C} \tilde{D} \tilde{E} \tilde{F} \tilde{G} \tilde{H} \tilde{I} \tilde{J} \tilde{K} (a_1, a_2, a_3, a_4) \\
\tilde{\theta} - \eta \tilde{\theta} + \omega^2 \tilde{\theta} = -2 \xi_\theta \theta - \xi_\theta \theta \tilde{U}_r \tilde{U}_r \tilde{A} \tilde{B} \tilde{C} \tilde{D} \tilde{E} \tilde{F} \tilde{G} \tilde{H} \tilde{I} \tilde{J} \tilde{K} (b_1, b_2, b_3, b_4)
\end{align*}$$  \hspace{1cm} (A6)

where $Y = y/d$, $\xi_\eta = s/m$, and the definitions of the other coefficients are given by equation (3). Here, dots indicate derivatives with respect to time, $t = \omega t$. The characteristic frequencies for the homogeneous system (A6) may be defined as

$$\omega_{1,2} = \frac{1}{2} T \left[ 1 + \omega^2 \pm \sqrt{(1 + \omega^2)^2 - 4 T^2} \right].$$  \hspace{1cm} (A7)

where $T = 1/(1 - \eta \omega^2)$. Next, the introduction of the linear transformation

$$\begin{bmatrix}
Y \\
\dot{Y} \\
\theta \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & K_2 & 0 \\
0 & \omega_{1e} & 0 & \omega_{2e} \\
K_1 & 0 & 1 & 0 \\
0 & \omega_{1e} & 0 & \omega_{2e}
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}$$  \hspace{1cm} (A8)

into (A6) results in the state variable equations

$$\dot{x} = \mathbf{L} x + \mathbf{N}$$  \hspace{1cm} (A9)

whose Jacobian matrix, evaluated on the initial equilibrium solution $x = 0$, is now in the standard form

$$J = \begin{bmatrix}
0 & \omega_{1e} & 0 & 0 \\
-\omega_{1e} & 0 & 0 & 0 \\
0 & 0 & 0 & \omega_{2e} \\
0 & 0 & -\omega_{2e} & 0
\end{bmatrix}.$$  \hspace{1cm} (A10)

The $\mathbf{L}$ and $\mathbf{N}$ in equation (A9) represent the linear and nonlinear parts of $\dot{x}$, respectively, and $K_1$ and $K_2$ in equation (8) are given by

$$K_1 = \frac{\omega_{1e}^2}{\omega_{1e}^2 - \omega_1^2} \eta \omega_{1e} \omega_{1e}$$  \hspace{1cm} (A11)

Finally, by applying classical methods (e.g., averaging, multiple scale, normal form theory), rate equations governing the local dynamic behavior, expressed in terms of the amplitudes ($A_1, A_2$) and phases ($\phi_1, \phi_2$) of the periodic vibrations having frequencies $\omega_1$ and $\omega_2$, respectively, are obtained as

$$\dot{A}_1 = E_1 U_r \omega_{1e} \eta \phi_1 [a_1 + E_2 (a_3 A_2^2 + 2 A_4 A_2)]$$  \hspace{1cm} (A12)

$$\dot{A}_2 = E_1 U_r \left( \frac{\omega_{2e}}{\omega_{1e}} \right) \left( \frac{\omega_{2e}}{\omega_{1e}} \right) \eta \phi_2 [a_2 + + E_3 (2 A_4 A_2^2 + A_4 A_2^2)]$$

Here,

$$A_1 = \frac{1}{2} U_r (1 + K_1 r_1) (a_1 + K_1 b_1) - \left[ \frac{\xi_{1e} + \omega_{1e}^2}{\eta_{1e}} \right]$$

$$A_2 = \frac{1}{2} U_r (K_2 + r_1) (K_2 a_1 + b_1) - \left[ K_2 \xi_{1e} + \omega_{1e}^2 \right]$$

$$A_3 = \frac{3}{8} U_r \left[ K_1^2 U_r^2 + (1 + K_1 r_1)^2 \omega_{1e}^2 \right] (\eta_{1e})$$  \hspace{1cm} (A13)

$$A_4 = \frac{3}{8} U_r \left[ U_r^2 + (K_2 + r_1)^2 \omega_{2e}^2 \right] (\eta_{2e})$$

$$E_1 = \frac{1}{(1 - K_1 K_2) U_r}$$

$$E_2 = (1 + K_1 r_1) (a_3 + K_1 b_3)$$

$$E_3 = (K_2 + r_1) (K_2 a_1 + b_3)$$