THEORETICAL NOTES

Dynamic Differentials of Stress and Coping

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Dynamic properties aligned with temporal interactions of stress, coping, and related variables are presented. Observations surrounding simple unidimensional systems are used to introduce more comprehensive systems of multidimensional interactions. Dimensions include collective levels of environmental stressors, levels of organismic stress arousal, coping-related cognitive efficiency, and engagement in selected coping activity. Changes in the relative impact of one dimension on another with the progression of time also are accommodated. Important prototypical features of dynamic systems pertinent to the present substantive domain are noted. Relations of dynamic-systems models to other formal treatments of stress-coping variables then are discussed. Finally, avenues and issues of empirical testing are presented. It is concluded that hurdles to crafting valid multidimensional dynamic systems are justified by the obtained explicitness of intervariable structures and specificity of variable trajectories over time.

Psychological stress, coping, and related constructs, such as "cognitive appraisal," have been characterized with terms such as dynamic, processlike, transactional, and systemic (e.g., Edwards, 1992; Endler, 1989; Lazarus & Folkman, 1984; Spielberger, 1972). Examples of reference to such properties are plentiful. For instance, Folkman and Lazarus (1985) have stated that "the essence of stress, coping, and adaptation is change... Therefore, unless we focus on change, we cannot learn how people come to manage stressful events and conditions" (p. 150). Support for the importance of flux as elemental to stress transactions has been amply supplied by laboratory and field studies (e.g., Folkman & Lazarus, 1985; Lazarus, Averill, & Opton, 1974; Neufeld, 1976).

The present article is addressed to a theoretical modus operandi commensurate with the preceding conceptualization of stress and coping. It begins with enumeration of attributes implied by the view of the subject matter as progressive and nonstatic. Basic, single-variable models illustrate certain features of formalizing dynamic properties, not unlike those needed for multivariable models of the present subject matter. These methods usher in the latter more complex formulations. Such formulations bring into play a network of stress, coping, and related variables interacting over time. At this point, the representation takes on somewhat of the dynamic, systemic flavor suspected as being quintessential to stress-coping phenomena.

Axiomatic Attributes of Stress and Coping

Verbal conjectures regarding the fluid nature of stress-coping networks have emerged from anecdotal observations, field studies, and laboratory experiments (e.g., Folkman & Lazarus, 1985; Lees & Neufeld, in press; Neufeld, 1976). Attributes embodied in this perspective are listed in Table 1 as verbal (vis à vis formal mathematical) axioms. Also listed are selected sources of the axiomatic attributes, as well as means by which the attributes are realized in formal dynamic systems models.

The descriptors in the first row of Table 1 are used more or less interchangeably in the cited sources and elsewhere. The descriptors seldom, if ever, are operationally separated. In general, they convey intervariable relations involving modification of one variable's behavior—inhibition, enhancement, direction-reversal—according to prevailing values of another variable (resembling analysis-of-variance interactions). They convey as well "reciprocal determinism" (Bandura, 1978; Lazarus, Delongis, Folkman, & Gruen, 1985), whereby the direction of such influence between system-dimensions is two-way (cf. Staddon, 1984).

Recursion is separated from reciprocal determinism as follows. As a qualitative dimension-behavior, recursion refers to a perpetual repetition of pattern, or repetition pending some specific condition. Pursuant to this property, nonlinear dynamic systems can display limit cycles made up of repeated oscillations; even linear systems can evince periodicity. In addition, nonlinear systems can display damped oscillations or other trajectories eventuating in point attractors or equilibria (see, e.g., Roughgarden, 1979). Recursion, in a mathematical sense, requires that a function be applied to the values it generates (Borowski & Borwein, 1989). This characterization is exemplified where the value of a dynamic-system dimension at time \( t \) enters into the function specifying rate of change at \( t \).

The term has also been used to indicate that "outcomes can influence antecedent variables depending on where in the flow of
### Table 1
**Verbal Axioms Regarding Attributes of Stress and Coping, Representative Sources, and Attribute Implementation in Formal Models of Dynamical Systems**

<table>
<thead>
<tr>
<th>Axiomatic attributes</th>
<th>Representative sources</th>
<th>Attribute implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interactive; interdependent; interconnected; transactional; self-regulatory; reciprocally determining; systemic</td>
<td>Delongis, Lazarus, &amp; Folkman, 1988; Folkman, Lazarus, Gruen, &amp; Delongis, 1986; Gruen, Folkman, &amp; Lazarus, 1988; Lazarus, Deese, &amp; Osler, 1952; Pearlman, Menaghan, Lieberman, &amp; Mullan, 1981</td>
<td>Momentary change of each dimension of the network of dimensions composing a system is a specific function of its own status and that of the other system dimensions. System dimensions can be highly coupled.</td>
</tr>
<tr>
<td>Dynamic</td>
<td>Caplan, 1983; Lazarus, 1990; McGrath &amp; Beehr, 1990</td>
<td>Behaviors of system dimensions are defined in terms of their instantaneous rates of change at each point in real (continuous) time. Changes in interdimensional influences potentially occur with progression of time. The status of each system dimension at each instant evolves from prior continuous changes, and remains in a state of flux, or potential flux, at each point in time.</td>
</tr>
<tr>
<td>Processlike</td>
<td>Cohen &amp; Edwards, 1989; Edwards, 1992; Endler, 1989; Lazarus, 1966; Lehman, 1972; Spielberger, 1972</td>
<td>System dimensions are responsive to each other; system dimensions can accommodate the behaviors of other system dimensions.</td>
</tr>
<tr>
<td>Adaptational</td>
<td>Lazarus, 1990; Lazarus &amp; Folkman, 1984</td>
<td>Differential equations composing a dynamic system involve &quot;functions applied to their own values.&quot; Possible behavior patterns include limit cycles and point equilibria.</td>
</tr>
<tr>
<td>Recursive</td>
<td>Lazarus, DeLongis, Folkman, &amp; Gruen, 1985; Lazarus &amp; Folkman, 1987</td>
<td>Differential equations composing a dynamic system involve &quot;functions applied to their own values.&quot; Possible behavior patterns include limit cycles and point equilibria.</td>
</tr>
</tbody>
</table>

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Psychological events one chooses to begin and end the analysis" (Lazarus et al., 1985, p. 777). Recursion now presumably is used in the sense of ergodic Markov-chains, whereby the same state of an entity is cast alternatively as antecedent and outcome with the progression of state transitions. The preceding slant on recursion may have a further nuance: anticipation of a given dimension's future state can influence the dimension's state during the current period (cf. Caplan, 1983).

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**Some Instructive Unidimensional Linear Dynamics**

Attention now is turned to certain methodological unpinnings of dynamic-systems modeling. Consider a simple, yet methodologically informative one-dimensional system. The dynamics of the system are straightforward: The rate of increase in a variable constantly decreases as it approaches its equilibrium value. Let the variable \( y \) at time \( t \) be \( y(t) \), its equilibrium value be \( y_e \), and its momentary rate of change be \( y'(t) \). Then, translating the above verbal statement,

\[
y'(t) = m[y_e - y(t)],
\]

where \( m \) is a constant. For convenience, \( y_e \) is set equal to 1.0. It is desired to trace the path of \( y(t) \) over an interval of \( t \), following a hypothetical dislodgment of \( y(t) \) from its equilibrium. To do so, it is necessary to know the values of \( y(t) \), say, from the point of dislodgment to the end of the interval. Notice that the above equation provides the momentary rate of change in \( y(t) \) for each value of \( t \), but not \( y(t) \) itself. Technically, \( y'(t) \) is a differential equation because it is defined in terms of the unknown and sought-for function, \( y(t) \). Also, it is linear, in that there is no cross-product between \( y(t) \) and \( y'(t) \), nor is \( y(t) \) raised to a power other than 1.0.

The exact solution for this differential equation must be an explicit function of \( t \). With some additional information, \( y(t) \) can be solved through established methods. The additional information is the value of \( y(t) \) at the beginning of the interval, denoted \( y(0) \), which is 0. In this case, the exact solution turns out to be \( 1 - \exp(-mt) \). This solution is perfectly consistent with Equation 1: Differentiation of \( 1 - \exp(-mt) \) with respect to \( t \) yields \( m e x p(-mt) = m[y_e - y(t)] = m[1 - \{1 - \exp(-mt)\}] \).

The initial value of \( y, y(0) \), of course need not be 0. If not, the trajectory, \( y(t) \), will change, but it inevitably will converge onto 1.0 as time progresses. The more general exact equation for \( y(t) \), with constraints lifted from \( y(0) \), is \( 1 + [y(0) - 1] \exp(-mt) \). The second term obviously becomes negligible as \( t \) becomes large. Figure 1 displays the trajectories of \( y(t) \) for initial conditions of 0, 0.1, and 0.5, with \( m \) equal to 0.1. Note that this insensitivity to initial values does not necessarily extend to nonlinear representa-

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\( ^1 \) This aspect of recursion can present potential problems for expression in nonlinear dynamic models, the very type implied by attributes in Table 1. Such problems illustrate vital constraints on verbal propositions imposed by formal reasoning. Let the state of dimension \( x \) at time \( n + 1, x_{n+1} \), be an outcome that is anticipated at antecedent time, \( n \), and whose anticipation hypothetically determines the state of \( x \) at \( n \). Consider the logistic equation, a well-known elementary difference equation that nevertheless can exhibit important earmarks of nonlinearity (see Townsend, 1992a): \( x_{n+1} = f(x_n) = ax_n(1 - x_n) \), where \( a \) is a constant. Accordingly, \( x_n \) is found to be either \( 1/(2a) \) \( (a - [a^2 - 4ax_n])^{1/2} \) or \( 1/(2a) (a + [a^2 - 4ax_n])^{1/2} \), an impossibility if only one state of \( x \) is allowable at time \( n \). Technically, \( f \) is not 1-to-1, and does not have an inverse. Further branching of possible values takes place with additional steps backward (e.g., \( x_{n-1} \) necessarily has four possible values). Note also that \( x_n \) takes on imaginary values when the square-bracketed term is negative. Representation of (cognitively mediated) retroactive influences clearly requires additional qualifications surrounding admissible values of \( x_n \).
Figure 1. Trajectories of $y(t)$ against time, where $y(t) = 1 + [y(0) - 1] \exp(-mt)$, for $m = 0.1$, and $y(0) = 0, 0.1$, and 0.5.

sections, which are discussed later. Despite its simplicity, this model can convey certain theoretically poignant features of propensity to engage in available coping options (Neufeld, 1989).

Essentially identical curves to those of Figure 1 are available by working directly with the differential equation, Equation 1, along with $y(0)$, and proceeding "numerically" (see, e.g., Braun, 1983; Vetterling, Teukolsky, Press, & Flannery, 1985). A benefit of explicit versus numerical solutions for $y(t)$, however, is the insight into behaviors of variables conveyed by the structure of the equations. Inspection of the exact closed-form solutions, $y(t)$, to Equation 1, immediately reveals equilibrium properties that are robust to initial conditions. Moreover, numerical solutions sometimes can be inaccurate, as judged against their analytical counterparts (see, e.g., Koçak, 1989; cf. Parker, Vollick, & Redmount, 1997). Nevertheless, if used with caution, numerical methods can provide important insight into behavioral dynamics according to computed trajectories of variables unfolding over time. Moreover, they bypass the need for often intractable explicit solutions, allowing us to attack the essentially substantive aspects of the focal topic domain (see, e.g., Boyce & DiPrima, 1977; Braun, 1983; Vetterling et al., 1985). Such methodology now is immersed in the target domain of stress and coping.

Stress, Coping, and Related Variables Composing Multidimensional Dynamic Systems

The observation that nonlinear dynamics are indispensable to expressing interactions endemic to personality and social psychology has been present for well over a decade. Staddon (1984) showed how even a simple three-dimensional nonlinear system could be used to explore important and complex dynamics involving social stressors. The primary stressor, in this instance, was the irritating behavior of a periodically unruly child and the "stressee" was an attending guardian. The coping response of the stressee was to apportion punishment roughly parallel to the magnitude of offence. Staddon's pithy treatise, which drew from much earlier work on the use of dynamic systems in mathematical ecology, was cogent, rigorous, precise, largely unheeded, and surprisingly unacknowledged (e.g., Vallacher & Nowak, 1994). In Staddon's portrayal, the interacting parties display a damped oscillation following the displacement from equilibrium levels of behavior (e.g., a rise in the child's aggression in response to an external stimulus). The parties reciprocally increase and decrease their respective behaviors, but progressively to lesser degrees, until each settles back to his or her pre-roused value.

Effects of stress and coping on one another do not necessarily lead to stable equilibria, as Staddon noted in the commentary on his dynamic prototype. In some instances, resting conditions are better represented by constant motion. Included in this representation are "limit cycles" and "chaotic states" (cf. Skarda & Freeman, 1987). The former is a stable oscillation, with a given period and amplitude, and the latter is nonrepetitive, lacking the predictability either of an equilibrium point or limit cycle. In the six-dimensional model, developed later, perturbation of the system by external agency results in the progressive reattainment of an equi-
librium state. Periodicity as well as equilibrium properties previously have been implemented in the stress-coping domain by appropriating two-dimensional Volterra-Lotka predator-prey systems (Nicholson & Neufeld, 1992), as well as "first and second-order linear systems" (Neufeld, 1985, 1997).

A Six-Dimensional Prototype Accommodating Complex Nonlinear Dynamics of Stress and Coping

I now consider some of the more intricate reciprocal influences and interdependencies among stress, coping, and certain variables implicated by these constructs. The form of coping to which the following layout applies is designated "decisional control." Averill (1973) described this form as varying with the number of alternatives available to an individual for negotiating stressful situations (cf. Thompson, 1981).

Appropriating decisional control, then, entails predictive judgments surrounding the amount of threat associated with alternatives embodied by the presenting stressor environment. Such judgments are formulated so as to engage the alternative least threatening. Cognitive activity, therefore, is at the root of negotiating through a stressful situation to the most advantageous resolution. In a situation of social evaluation, or "ego threat," for example, an individual may selectively engage others based on judged minimum likelihood of a gauche outcome. In a physically threatening setting (e.g., hazardous occupational setting), an individual may engineer contact with the estimated least dangerous or uncomfortable alternative (specific occupational task undertaken). A mathematical elaboration of these points, along with empirical support from psychophysiological and behavioral stress measures, have been presented elsewhere (Morrison, Neufeld, & Lefevbre, 1988; Kukde & Neufeld, 1994; Paterson & Neufeld, 1995; Neufeld, 1990b).

The first dimension of the system $Y_1(t)$ represents the level of collective external-stressor properties, accessible to decisional control. Other types of stressors are negligible in the environment to which the system applies (an additional, explicit simplifying assumption). Such environments may include, for instance, academic settings, and those of executive decision making.

The second dimension, $Y_2(t)$, depicts stress arousal levels. Various indicants of this construct have been discussed in other sources (e.g., Goldberger & Breznitz, 1982; Neufeld, 1989; Wong, 1990). For manageability of the present endeavor, $Y_2(t)$ is taken to indicate some summary value of stress arousal at time $t$. Note that $Y_2(t)$ is considered to increase with exogenous stressor properties, $Y_1(t)$, but also with the "cognitive work" involved in information-processing-based coping activity, $Y_3(t)$, discussed later (e.g., Kukde & Neufeld, 1994; Solomon, Holmes, & McCall, 1980; Townsend & Ashby, 1978).

A third dimension, $Y_3(t)$, implicated by the present stressors and coping activity, is that of cognitive efficiency (e.g., M. W. Eysenck, 1989; M. W. Eysenck & Calvo, 1992). As stress arousal increases, the efficiency of cognitive functions impinging on decisional control potentially decreases (Fisher, 1986; Hockey, 1979; Neufeld & McCarty, 1994).

The fourth dimension, $Y_4(t)$, involves coping activity in the form of decisional control (e.g., formulating and implementing threat-minimizing predictive judgments). Dimension 5 portrays the degree to which the stress-arousal mechanism is sensitive to its sources of activation, $Y_1(t)$ and $Y_2(t)$. Finally, Dimension 6 conveys the responsivity of coping-activity level, $Y_3(t)$, to changes in the level of exogenous stressor properties, $Y_1(t)$.

The respective differential equations appear as follows. First,

$$ Y'_1(t) = a - bY_1(t)Y_4(t) - cY_1(t), $$

where $Y'_1(t)$ is the rate of change in stressor level at time $t$, and $a$, $b$, and $c$ are parameters. The parameter $a$ reflects the degree to which the individual's environs tend to propagate stressor properties in the absence of countering influences; $b$ indicates the extent to which coping activity $Y_4(t)$, scaled by efficiency of this activity $Y_3(t)$, diminishes such properties; and $c$ marks the extent to which the level of stressor properties is inclined to increase more when it is relatively lower, and less when it is already higher. The term $cY_1(t)$ roughly reflects regulatory factors intrinsic to the environment. Stressor events may proliferate more when they are low, but slow their increase or reverse it, as a saturation point is approached (e.g., Neufeld & Nicholson, 1991).

Next,

$$ Y'_2(t) = Y_3(t)\left(\left[ a - bY_3(t)Y_4(t) - cY_1(t) \right][1.0 + Y_6(t)] - eY_3(t) \right) - \left[ fY_3(t)Y_4(t)Y_1(t) - g \right], $$

where $Y'_2(t)$ is the rate of change in stress arousal at time $t$, and $e$, $f$, and $g$ are additional parameters. This equation is best understood by considering it term by term. As indicated earlier, $Y_3(t)$ weights the impact on stress arousal of changes in the latter's sources. The value of $Y_3(t)$ itself is affected by the current values of stress arousal sources, $Y_1(t)$ and $Y_2(t)$, as well as by its own extant value. The exact specification of changes in $Y_3(t)$ is taken up later in this article. Momentary shift in $Y_3(t)$, as depicted in Equation 2, appears as the first square-bracketed expression within braces. Turning to the curl-bracketed expression within the braces, the term $Y_3(t)$ implements the second source of stress arousal, that of coping activity compelled by exogenous stressor level. Responsiveness of coping activity to changes in stressor level is represented by a dynamic-weighting coefficient assigned to the latter, namely $Y_6(t)$. As in the case of $Y_1(t)$, this weight varies with other variables in the network (see later in this section). Coping activity, in the form of decisional control $Y_4(t)$ is considered in the present system to be affected by factors other than exogenous stressor level, as detailed momentarily. However, $Y_4(t)$ is deemed to have the main influence on stress-arousal properties of $Y_2(t)$ (Neufeld, 1990a). The product $eY_3(t)$ is a self-regulatory expression affecting coping outlay, analogous to that of Equation 2, $cY_1(t)$, pertaining to exogenous stressor level.

Note that change in stress arousal parallels change in stressor level, rather than stressor level in and of itself. This linkage is designed to express documented variation in psychophysiological and other measures of stress activation with variation in situational stressor properties (e.g., probability, as well as magnitude of physical or social-evaluative threat). It is possible that stress arousal increases while stressor level remains constant, but a more accurate representation appears to be one where stress arousal tends to track stressor properties, at least under conditions of systematic measurement (for reviews, see Neufeld, 1982; Paterson & Neufeld, 1987).

The curve-bracketed expression, $fY_3(t)Y_4(t)Y_1(t) - g$, introduces the concept of cognitive appraisal (Arnold, 1960; Lazarus,
Effectiveness of coping, in terms of efficiency of cognitive functioning impinging on decisional control, $Y_s(t)$, is thrown into relief according to the degree to which decisional-control activity is being exercised, $Y_c(t)$. Perceived rate of yield on coping investment, in turn, involves the apparent intensity with which exogenous stressor level currently is reducing. The magnitude by which exogenous stressor level is depleted depends on the magnitude in place, $Y_c(t)$. Put another way, the salience of decisional-control activity, as moderated by efficiency of its cognitive apparatus, varies with $Y_c(t)$. (The relation is not unlike one in mathematical ecology, where the rate of predators' consumption depends on the density of prey to which the predatory behaviors are applied.) The perceived rate of return on coping investment is compared to a subjective standard, represented by the parameter, $g$. Values below $g$ increase stress activation, and those above $g$ reduce it.

Before proceeding to Equation 4, a word is in order regarding parallels within Equation 3 to the concepts of primary and secondary stress-related appraisal (Lazarus, 1966). Primary appraisal of exogenous threat as a source of stress activation is represented by the placement of $Y_c(t)$, scaled by 1.0 in this equation. Secondary appraisal, along with monitoring of personal coping efficacy (reappraisal), in turn is represented by the final curve-bracketed term. The presented interplay of variables affecting primary and secondary appraisal in dynamic form may be more accurate than earlier quasi-formal portrayals involving simple ratios of "appraised exogenous-stressor potency to appraised personal coping efficacy" (Neufeld, 1976). The latter efforts appeared to fall short because their predictions remained static across time.

The fourth equation is relatively uncomplicated, as follows:

$$ Y_c(t) = h - iY_c(t), \tag{4} $$

where $h$ and $i$ once again are parameters. Cognitive efficiency pertaining to decisional control varies inversely with stress arousal, $Y_s(t)$. An individual’s vulnerability to the impingement of stress on processing resources theoretically is evinced as lower values of the parameter $h$. The parameter therefore accommodates individual differences in subjective aversiveness, specifically with respect to associated toll on attentional resources (see M. W. Eysenck, 1989, 1992). Note that, theoretically, under certain conditions stress activation may improve cognitive performance (Anderson, 1990; cf., Neiss, 1990). The present system is restricted to the domain of performance decline (see, e.g., Fisher, 1986; Neufeld & McCarty, 1994).

Equation 5 describes movement in levels of decisional-control coping activity:

$$ Y'_c(t) = Y_c(t)[a - bY_c(t)Y_s(t) - cY_c(t)] - jY_c(t) \tag{5} $$
$$ + \left\{ [k(fY_c(t)Y_s(t)Y_c(t) - g)] + d \right\}, $$

where $j$ and $k$ are additional weighting parameters. The remaining added parameter, $d$, depicts a base tendency for decisional-control related activity to drift upward (or downward), in the absence of other influences on the dimension (cf. Staddon, 1984). Approaching this more complex equation, once again term by term, note first that the square-bracketed expression puts in place the change in stressor level to which coping activity is responsive. The degree of responsibility is depicted by the dynamic weighting factor $Y_s(t)$. A product $fY_c(t)$ once more represents a self-regulating mechanism, slowing or reversing the increase in $Y_c(t)$ when it is at higher levels, and doing the opposite when $Y_c(t)$ assumes lower levels. Cognitive appraisal, as weighted by the parameter $k$, is brought into play by the expression in braces. The expression implements incentive properties of appraisal: Values above the subjective standard of coping efficacy $g$ tend to increase coping activity, and values below tend to decrease it.

Equation 6 defines sensitivity of stress-arousal mechanisms to increases in exogenous stressor level and to elevation in decisional-control coping activity (cognitive work) impelled by such increases (see Equation 3):

$$ Y_s(t) = 1.0 - Y_s(t)[Y_c(t) + Y_s(t)] - Y_s(t). \tag{6} $$

The value of 1.0 is a constant around which sensitivity $Y_s(t)$ can increase or decrease, as determined by the other terms in the equation. The second term modulates changes in sensitivity according to the latter’s extant value, as set against cumulative sources of potential stress activation, $Y_s(t)$ and $Y_c(t)$. Recall that in Equation 3, $Y_s(t)$ weighted only that portion of $Y_c(t)$ affected by fluctuation in exogenous stressor level. In the present equation on the other hand, $Y_s(t)$ is diminished with higher levels of coping activity, regardless of origin. This configuration appropriates the assumption that a change in demands on expenditure of cognitive effort arising from alteration in level of exogenous stressors depends on ongoing cognitive investment, regardless of the sources of such investment. The final term in the equation, $Y_s(t)$, represents a self-regulatory component similar to those of Equations 2, 3, and 5. In this instance, the extant value of the dimension $Y_s(t)$ simply is weighted by the constant of 1.0. This weight is sufficient to produce apparently credible behavior of the system (see later sections, System Behavior Over Time and Additional Exemplary Properties of the System); the weight can be altered, however, to accommodate theoretical or empirical contingencies.

Observe that the constant of 1.0, appearing at the beginning of the equation, can be modified according to individual differences in sensitivity to sources of stress activation. If preferred, the constant readily could be replaced by an adjustable parameter. A constant was used in the present case so as to avoid further proliferation of parameters. The point to be made is that nonlinear dynamic systems, such as the one described here, have little difficulty in providing for stress-relevant individual differences. The present constant could vary with differential responsiveness to sources of stress activation, for example, associated with individual differences stemming from sensitivity of “neurological punishment/reward centers” (Gray, 1970, 1971), or functioning of the limbic and ascending reticular activating systems (H. J. Eysenck, 1967, 1970; H. J. Eysenck & M. W. Eysenck, 1985). The constant under consideration conveys between-subject differences in stress responsivity, whereas the parameter $h$ of Equation 4 conveys variation among individuals in the degree to which stress activation encroaches on the efficiency of deployed cognitive resources.

Equation 7 indicates the tendency to marshall changes in decisional-control coping activity in response to those in exogenous stressor level. Specifically,

$$ Y'_s(t) = \left[ 1.0 - Y_s(t)Y_c(t) \right] - Y_s(t). \tag{7} $$

The square-bracketed expression states that elevation in coping activity in response to elevation in exogenous stressor levels $Y_c(t)$
is modulated as the combination of existing responsivity, \( Y_s(t) \), and \( Y_e(t) \) presently is high. The term \( Y_e(t) \) again is an autonomous self-regulatory factor implicitly weighted by the coefficient 1.0.

**System Behavior Over Time**

To fathom the dynamic properties of the assembled system, it is necessary to trace the numerically computed path of its dimensions. Doing so requires first the specification of the system parameters and constants. In fields of application such as heat conduction and fluid dynamics, required values potentially are obtained from past observations or direct measurement. Typical of behavioral sciences is initial selection based on reasonable values. First “guesstimates” later can be refined according to compliance of the resulting dimensional behaviors with known facts and correspondence between model-predicted and observed dimension trajectories.

Stress-and-coping systems ordinarily resolve themselves without suddenly attaining unrealistically high values. Accordingly, a system that is “bounded” within the designated time interval is indicated (see, e.g., Boyce & DiPrima, 1977; Braun, 1983). Parameter values meeting this minimal condition include 1.0 for parameters, \( d, e, f, h, i, \) and \( j \); 2.0 for parameter \( a \); 0.1 for parameters \( b \) and \( c \); 0.15 for parameter \( g \); and 0.5 for parameter \( k \). Another set of values requiring specification are those apportioned to the respective dimensions at the beginning of the epoch under consideration (i.e., where \( t = 0 \)). In the present case, the initial value for each dimension simply is 1.0.

Given the preceding settings, the system can be set in motion for inspection of its dimension trajectories over time. To simplify the task, two dimensions, \( Y_s(t) \) and \( Y_e(t) \), or stressor level and coping activity, are taken up first. Figure 2 presents these dimensions’ integral curves and phase portrait (a plot of one dimension against

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**Figure 2.** Phase portrait and integral curves for dimensions of stressor level, \( Y_s(t) \), and coping activity, \( Y_e(t) \), of a six-dimensional system. Specifications: Runge-Kutta algorithm, step-size of .025, and \( t = 0 \) to \( t = 20 \).
the other, with time progressing in the direction of the arrows). At the outset, both $Y_1(t)$ and $Y_4(t)$ increase, with $Y_4(t)$ accelerating, and $Y_1(t)$ decelerating. With a continuing increase in $Y_4(t)$, $Y_1(t)$ begins to decrease, whereupon $Y_2(t)$ declines below its initial value, allowing $Y_1(t)$ increases once more. The latter movement, in turn, “prompts” elevation in $Y_3(t)$, which results in another reduction of $Y_1(t)$. After a further oscillation—essentially a damped version of its predecessors—both dimensions appear to decline to an equilibrium value. Some insight into this mutual descent to equilibrium values is available from the full set of equations.

Returning to the equational system at large then, it is evident that increasing $Y_3(t)$, cognitive efficiency pertaining to decisional control, indirectly can attenuate $Y_1(t)$ as well as $Y_4(t)$. The attenuation occurs in part by elevating the stressor-reducing dividends accruing to any rise in $Y_4(t)$. Note, however, that the dimension $Y_3(t)$ tends to be influenced upward with a reduction in stress arousal $Y_2(t)$. Examination of the integral curves presented in Figure 3, which includes the first four dimensions of the system, unveils precisely this combination of transactions.

Increases and decreases in $Y_5(t)$ follow decreases and increases, respectively, in $Y_6(t)$. Extension of $t$ to 200 (with a numerical-algorithm step-size of 0.001) indicates that $Y_6(t)$ eventually stabilizes at a value of 40 (arbitrary units). A question arises as to why stress arousal should dip so severely at the outset, when sources of stress arousal are on the rise (i.e., when $Y_1(t)$ and $Y_4(t)$ are positive). That is to say, the first square-bracketed term of Equation 3 fully includes Equation 2, and would appear to figure prominently in Equation 5. The combination, considered in isolation, would suggest that $Y_5(t)$ should be positive when $Y_1(t)$ and $Y_4(t)$ are positive. Apropos to this inference, separate inspection of the dynamic weights involved in defining $Y_5(t)$, specifically, $Y_3(t)$ and $Y_6(t)$, indicates they themselves remain positive throughout the interval.

Examination of Equation 3 reveals that the reduction of stress arousal can be understood in terms of stress-mitigating effects of favorable cognitive appraisal concerning coping efficacy. Recall

![Figure 3](image_url)

**Figure 3.** Integral curves for stressor level, $Y_1(t)$, stress arousal, $Y_2(t)$, cognitive efficiency, $Y_3(t)$, and coping activity, $Y_6(t)$, of the six-dimensional system. Numerical methods are identical to those in Figure 2.
that cognitive appraisal of coping efficacy involves the cross-product of $Y_i(t)$, $Y_j(t)$, and $Y_k(t)$. With the equation in mind, the role played by formally defined appraisal can be appreciated by examining the paths of these three dimensions in relation to that of $Y_2(t)$. Such a pattern of results cogently illustrates the fact that quantitative models readily accommodate covert processes, including the subjective monitoring of coping efficacy, provided the workings of such processes are made explicit within the formal deductive system (e.g., Braithwaite, 1968; Staddon, 1984).

Additional Exemplary Properties of the System

Before leaving this six-dimensional model, some additional characteristics are noted. These pertain to properties of interdependence and self-regulation.

Consider, for example the two-dimensional phase portrait of $Y_2(t)$, and $Y_3(t)$ as shown in Figure 4. The variables initially are located at their equilibrium values of 1.0 (arbitrary units), for $Y_2(t)$, and 40.0 for $Y_3(t)$. (Equilibrium values for $Y_2(t)$, $Y_4(t)$ and $Y_6(t)$ are 0.5, 0.7, and 1.0, respectively.) According to the phase portrait, the variables are uprooted from their equilibrium points, leading to an excursion away from the stable values, and an eventual resetting at the attractor locations. The movement away from the equilibrium appears enigmatic pending consideration of other aspects of the system. The broader perspective reveals that $Y_1(t)$ has been displaced by an external event or influence from its own equilibrium value of 0, to a value of 2.0. Because of its coupling with the other dimensions, the entire system is disturbed by $Y_1(t)$. The episode is revealed with respect to $Y_2(t)$, $Y_3(t)$, and $Y_1(t)$ by tilting the phase portrait so as to expose the time course of all three dimensions simultaneously (Figure 5). The effects of the $Z$ axis, on which $Y_1(t)$ is plotted, becomes apparent by rotating the $X$ axis 40 degrees, and the $Y$ axis 25 degrees (Figure 5).

![Figure 4](image).

**Figure 4.** Phase portrait for stress arousal, $Y_2(t)$, and cognitive efficiency, $Y_3(t)$, dislodged from equilibrium through "latent events" comprising incrementation of exogenous stressor level $Y_1(t)$ by a system-external agent(s). Specifications: Runge-Kutta algorithm, step-size of 0.001, and $t = 0$ to $t = 200$. 
Figure 5. Displacement of $Y_1(t)$ from 0 to a value of 2.0, exposed by rotating the horizontal axis of Figure 4 $40^\circ$ and the vertical axis $25^\circ$. This displacement of $Y_1(t)$ is the instrument of unsettling all dimensions of the system, eventuating in a realignment at equilibrium status. Numerical methods are identical to those in Figure 4.

A lesson conveyed by the foregoing incident goes beyond properties of interdependence and self-regulation in certain respects. It provides some insight regarding system incompleteness. The perturbation of $Y_1(t)$ from equilibrium in this case was imposed by an effect emanating from outside the system as formulated. A more comprehensive system would incorporate the factors responsible for the dislodgement of $Y_1(t)$. Nevertheless, by commencing with a seed-core of variables, others impinging on the content domain cumulatively can be added, making for progress toward the goal of system self-containment.

It can be shown that the property of interdependence, illustrated earlier, may be varied in such a way as to shape favorably or unfavorably the behavior of the system in certain significant ways. It has been conjectured in several sources that networks of variables pertaining to stress, coping, and appraisal can be tightly coupled (e.g., Folkman, Lazarus, Gruen, & DeLongis, 1986). An increase in coupling of variables in the present system leads to substantially greater robustness, in terms of resistance to destabilization with deflection by external forces from equilibrium values. A simple modification resulting in heightened interdependency increases the potency of the equilibrium attractor to recapture perturbed dimension values and to retain their bounded properties. It seems axiomatic that a system is more plausible if its behavior becomes more veridical subsequent to structural modifications indicated by natural observation.

The current instance of increased coupling encompasses adding the element of cognitive appraisal as expressed in Equation 5—$-k[Y_2(t)Y_3(t) - g]$—to the right-hand side of Equation 7. As a result, responsivity of coping activity $Y_6(t)$ to changes in stressor level, $a - bY_4(t)Y_6(t) - cY_1(t)$, or $Y_7(t)$, is modulated according to appraised coping efficacy. Sensitivity increases if appraisal is positive (in both a mathematical and psychological sense) and the
opposite. It appears reasonable, at least in a face-valid way, that as confidence in coping increases, so does the acuteness of matching coping activity to exogenous stressor levels.

To elaborate on changes consequential to the elevated coupling, numerical analyses indicate that relocating dimensions away from their equilibrium values by certain amounts can metamorphose trajectory behavior. A breach can occur, whereby rather than returning to a state of equilibrium, dimensions become unbounded, taking on infinite values. For purposes of demonstration, let the values at \( t = 0 \) for each dimension be 0.5, except \( Y_1(0) \), which will be allowed to vary. Whether cognitive appraisal is used to define \( Y_0(t) \) or not, a gradual increase in \( Y_1(0) \) eventually generates unbounded values for \( Y_1(t) \) and the other dimensions of the system. However, in the case of tighter coupling of the form described earlier, the value of \( Y_1(0) \) that results in the system becoming unbounded is 2.15; its counterpart in the less coupled system is only 0.75. By extension, the former system is better able to withstand higher (imposed) levels of environmental stressors. To the degree that more rather than less resilience characterizes the focal content domain, the more resilient hypothetical system stands a better chance of being correct. Other forms of stress-coping system modification affecting stabilization have been drawn out with reference to the Volterra-Lotka equations described by Stadler (1984; Neufeld & Nicholson, 1991; Nicholson & Neufeld, 1992).

**Augmenting Formal Approaches**

Differential equations representing nonlinear systems are flanked by other formal approaches to the field of psychological stress and coping. These approaches may contain a dynamic element in their own right (e.g., stochastic process models); they can inform the construction and modification of systems formulations (see Busmeyer & Townsend, 1993; Townsend, 1992b). The specification of dimensions of the six-dimensional dynamic model, discussed earlier, and the nature of interdependence among its dimensions have been determined largely by quantitative axiomatic arguments concerning the structure of decisional control (Morrison et al., 1988; Neufeld, 1990a). Structural features bearing on the systems setup have encompassed relations among the following quantified dimensions: (a) availability of decisional control in the encountered stressor situation, (b) associated amount of potential threat reduction, and (c) cognitive transactions mediating the available threat reduction. Placed within the framework of axiomatic arguments, discussed earlier, the six-dimensional dynamic model assumes that the context of stress negotiation is one rich in the availability of decisional control for potential threat reduction.

Other formal impinging on dynamic systems formulations include measure-theoretic analysis of the subjective significance of environmental stressors (Birnbaum & Sotoodeh, 1991). Such analyses interface with the operationalization of dimension \( Y_2(t) \) discussed earlier. They also include formal analyses of stress effects on cognitive efficiency and individual differences in susceptibility to these effects (Neufeld, 1994; Neufeld & McCarty, 1994). The latter impinge on the composition of \( Y_1(t) \), discussed earlier.

On balance, nonlinear dynamic systems, as instantiated by the proposed six-dimensional system, depict the topography of interactions over time of the specified variables. Other formal accounts, such as those enumerated earlier, and possibly integrative connectionist models (Mischel & Shoda, 1995), are required to address specific mechanisms whereby quantified changes take place across time (see also Booth, 1985; Deffenbacher, 1994).

**Levels of Empirical Support**

As with nonlinear dynamical models generally, empirical outworking of the presented model is multi-tiered. Model viability is first addressed at the simulational level, where general but important properties of system behavior are exposed. Such properties speak to capacity of the model to quantitatively implement axiomatic properties of the content domain (see Table 1), vis-à-vis existing knowledge. Examples consist of boundedness, plausibility of intervariable relations computed over time, and response of the system to innovations in structure, such as enhanced interdimensional coupling. General but important dynamical system qualities are revealed at this level. Included is the character of adaptation, self-regulation and interdependence, as well as nonintuitive behavior trajectories whose complexity vastly exceeds that of the defining equations.

Simulations also provide for refinement of models subjected to empirical data evaluation. In this way, results from computer simulations form predictions for the next level of empirical support. It is contended here that informational returns on investment in prospective studies (see e.g., Folkman et al., 1986) will be closely linked to preliminary formal efforts. Efficiency in this regard can be enhanced by putting tendered models through their simulational paces, with an eye to differential plausibility of computed behaviors. In principle, each dimension of the present system is measurable. Diary checklists are applicable to Dimensions 1, 2, and 4 (Sadle, Sutherland, & Harris, 1982; Bolger, 1990; Bolger & Zuckerman, 1995; Guttalschink, Bauer, & Whybrow, 1995). Measures of cognitive efficiency, or psychometric surrogates (Carter, Neufeld, & Benn, 1998) can be used to tap Dimension 3. Indexes of stress sensitivity have been briefly discussed earlier, with respect to Dimension 5 (Birnbaum & Sotoodeh, 1991); other candidates have been enumerated elsewhere (Neufeld, 1989). Finally, Dimension 6 calls for measures of coping propensity (discussed in Lees & Neufeld, in press; Neufeld, 1982).

It must be recognized that the present system dynamics operate in the company of measurement error and noise variance. A relevant designation of the empirical process, therefore, is that of "percolation" (Frisch & Hammersley, 1963), which is a "process of deterministic flow through a stochastic medium" (Winsor, 1995, p. 181). The deterministic predictions thus are perturbed by a

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2 Parameter values for this demonstration are as follows: \( a = 0.5 \); \( b = c = d = e = g = h = i = j = 1.0 \); \( f = 0.1 \); \( t = 0.5 \).

3 For both the higher and lower coupled versions of the system, the gradual return to equilibrium versus eventual excursion into unbounded values becomes apparent fairly early: at approximately \( t = 40 \), in the case of less coupling, and approximately \( t = 100 \) in the case of more coupling. If a downward trajectory is not apparent in these vicinities, unbounded values eventually ensue. In the former case, unboundedness occurs at approximately \( t = 130 \), and in the case of tighter coupling, at approximately \( t = 900 \). In the absence of a downward-trajectory lead-indicator at the stated times, stressor level depicted by \( Y_1(t) \) remains in an "elevated zone of danger" pending its final escape from finite values.
stochastic element (not unlike error components in analysis-of-variance linear models).

Certain statistical diagnostics (essentially lag-correlation procedures) nevertheless may be employed to probe for the presence of nonlinear system properties (Gottschalk et al., 1995; Redington & Reidbord, 1992; Wolf, 1994). Application of more stringent numerical diagnostics (e.g., the Lyapunov exponent and the critical dimension-value of a Cantor set), however, is problematic in the case of fallible measurement. A sophisticated description of these and other mathematics of nonlinear dynamics and chaos has been presented in a tutorial by Townsend (1992a).

A second approach to empirical assessment involves more traditional aspects of model testing, as follows. The conformity of observed trajectories of dimension measures are directly compared to those predicted by the model (e.g., Staddon, 1984). The degree of correspondence (“goodness of fit”) potentially is subject to testing for statistical significance of the degree to which observations are congruent with predictions. For example, values of model parameters may be estimated from observations using an appropriate search algorithm (e.g., Hall, 1991) that optimizes the values according to some specified criterion (e.g., the method of least squares, or maximization of the cross-correlation between observed and predicted values). The correlation between predicted and observed values may in turn be tested for statistical significance. Degrees of freedom can be adjusted according to the number of model-parameter estimates and the operation of autocorrelation effects within predicted and observed sets of values (Holtzman, 1963).

A recurring difficulty concerning empirical data employed to test formal models entails the occurrence of consistent individual differences and data instability (Luce, 1997; Suck & Holling, 1997). The former source potentially is made up of two components. One component is variation across relatively homogeneous groups of individuals in model structure (e.g., perturbations of the right-hand sides of Equations 2 through 7). The second component entails individual differences in values of model parameters, given a common structure. Instability, on the other hand, refers primarily to effects of random measurement error, or measure variation uncorrelated with the principal variables under study (see, e.g., Embretson, 1983; Neufeld & Gardner, 1990). Aggregation of observations across relatively homogeneous clusters of individuals often is the only available means of increasing data stability to the point of allowing viable tests of proposed models (associated issues and methods are presented in a recent series of articles; see Neufeld, 1998).

Sources of variance extraneous to the formally posed dimensional network may encompass systematic effects, rather than random noise. In other words, dimensions not expressly incorporated into the modeled structure may impinge on the latter’s dimensions. This eventuality is illustrated in Figures 4 and 5, where displacement of the first dimension (incrementation of stressor level) is imposed by a source(s) external to the six-dimensional system. Also apropos of Dimension 4, in addition to decisional control, instrumental forms of coping (Averill, 1973; Pearlin, Menaghan, Lieberman, & Mullan, 1981) may be undertaken (e.g., change in occupational or marital status, engagement of social or therapeutic support). Again, such behaviors are outside the purview of the presented model. Staddon (1984), for one, has referred to such system incompleteness as “path dependence.” To illustrate within the system as presented, a dimensional subset of this network (e.g., Dimensions 1, 2, and 4) is path dependent with respect to a hypothesized path-independent six-dimensional set. Path dependence risks distortion of estimated parameters and poor empirical fit of otherwise viable models.

It is imperative, therefore, to specify inferences according to the dimensional specifics implicated by the model architecture (as exemplified in this article’s obsessional qualifications of Dimensions 1 and 4). A major asset of formal models is the salient constraints on deductions they impose. Meanwhile, empirical pursuits evidently should include the monitoring of events bearing on the tested system (questionnaires, interviews, etc.), affording a more informed judgment about the encountered context of operation and implicated upgrading of system composition (e.g., Ciompi, 1989).4

All things considered, correspondence of the quantitative trajectories traced by model predictions and measurement observations appears to be the most exacting of the possible routes to dynamic model testing. The present category of evaluation clearly meets the criteria for a “strong test” (i.e., Popperian “bold conjecture”; see, e.g., Meehl, 1978).

Conclusion

A considerable arsenal of tools for expressing interactions among stress and coping variables across time is available from the field of quantitative dynamics. Representative avenues of integrating substantive conjecture with mathematical systems formulations, complementing streams of formal and quasi-formal modeling (e.g., formal stochastic process models), as well as tactics of empirical testing, have been examined.

It is evident that appreciation of the dynamics of informal postulates about the temporal unfolding of behaviors among variables in this field demands application of appropriate quantitative translations of verbal arguments. Despite the mathematical tractability of linear models, for example, explicit solutions for Y(t), nonlinear models appear indispensable to such quantitative translations, given the accommodation of behavioral complexities involved (e.g., Levine & Fitzgerald, 1992). What appears to be unmistakable is that knowledge about temporal interactions will be accelerated by approaching, rather than avoiding, the considerable presenting arsenal of relevant quantitative dynamics.

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4 Note that a tentatively “reduced system” still affords predictable responses to abrupt dislodgment of its dimensions by external events, as is evident in numerical results presented here.

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