Stochastic dynamics of stimulus encoding in schizophrenia: Theory, testing, and application

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A B S T R A C T

Cognitive processing among schizophrenia participants, entailing encoding of presenting stimulation into a format facilitating collateral processes (e.g., memory search), is examined in light of stochastic mathematical models of performance. Results implicate additional encoding operations (encoding subprocesses) as the source of schizophrenia encoding-process elongation. Convergent evidence for this inference, including that from auxiliary neuro-connectionist simulations, is brought forth. Developments from initial, fixed-parameter accounts include random-parameter mixtures, and their Bayesian extensions, formally mediating group-level results to assessment of individual performance. Outgrowths bear on model-selection methodology, according to coherence of group-level and individual-level model functioning (in part addressing the issue of “small-trial-sample model testing”); longitudinal monitoring of encoding-specific treatment response; evaluation of treatment-regimen efficacy with respect to encoding efficiency; and specification of times of measurement interest, in fMRI. The symptom significance of encoding elongation, strongly hinted at by model developments, along with a model-endowed window on exacerbating effects of stress, are drawn out.

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1. Introduction

Historical and contemporary descriptions of schizophrenia have featured thought disorder (delusions; disorganized incoherent speech) as a salient, characteristic symptom (International Statistical classification of Diseases, Injuries, and Causes of Death, Ninth Revision, World Health Organization (1977); Diagnostic and Statistical Manual of Mental Disorders (DSM), Third Edition, Revised, American Psychiatric Association (1987); DSM IV, Text Revised, American Psychiatric Association (2000); Kraepelin (1919); Maher (1966)). The centrality of this symptom prompted substantial research investment among clinical scientists of schizophrenia in information-processing paradigms. Certain paradigms were developed without reference to mainstream cognitive science, but many have borrowed directly from the field of experimental cognitive psychology (as reviewed in Broga and Neufeld (1981a); Neufeld (1991a); Schwartz (1978) and Wynne, Cromwell, and Matthyse (1978)). The study of cognitive functioning retains a prominent role in schizophrenia research. Included are extensions comprising cognitive neuroscience, and the evaluation of behavioral and pharmacological interventions (Carter & Barch, 2007; Nuechterlein et al., 2004; Keefe et al., 2007; Penn & Spaulding, 1997; Shuart, Spaulding & Poland, 2007). Moreover, as reported in this paper, and elsewhere in this issue, analytic modeling has gained impetus in the clinical cognitive science of schizophrenia, and impinges directly on the above investigative directions.

As stated above, salient in the symptom picture of schizophrenia is thought disorder. Thought-form disorder includes disorganized and incoherent expression of trains of thought that are loosely connected and subject to irrelevant intrusions. Thought-content disorder includes delusions with persecutory, jealous, grandiose, or religious content; delusions of reference entail convictions that common everyday occurrences, objects or people have special personal significance. Often-accompanying hallucinations, in turn, stand to be thematically coherent with delusional content. Thought-content disorder characterizes the paranoid schizophrenia subgroup.

Although the present developments are not unrelated to non-paranoid schizophrenia subgroups (Neufeld, 2007a), emphasis here is on that of paranoid schizophrenia, for the following reasons: (a) the relative empirical distinctiveness of their symptom picture (Neufeld, Carter, Nicholson, & Vollick, 2003; Nicholson & Neufeld, 1993); (b) relative incidence of this subtype, making

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upwards of 37% of schizophrenia diagnoses; (c) the comparative tendency toward more pronounced deviation among paranoid schizophrenia participants in cognitive functioning addressed in this paper; and (d) the tenability of linkage between (c), above, and thought-content disorder, as expanded upon quantitatively, below (see the Section 8.1). Of course, as Hoffman and McGlashan (2007) have noted, modeling the entire range of schizophrenia symptomatology is clearly an impossible task.

Among the thought-disorder, information-processing investigative paradigms implemented from experimental cognitive psychology, of note have been memory-search and closely-related tasks (see, e.g., Checkosky, cited in Sternberg (1966) and Wishner, Stein, and Paestrel (1978)). Prototypically, essential requirements of these tasks comprise ascertaining if a presented item (“probe item”) is a member of a previously memorized set of items (“memory set”), whose subspace size varies across trials. Component processes transacting the task are considered to be probe-item encoding, memory search, and response operations (for detailed elaboration, see Sternberg (1966, 1975), Townsend and Ashby (1983, Chap. 6); for a clinical-science slant, see Broga and Neufeld (1981a), Neufeld (1991a) and Neufeld, Vollick, and Highgate (1993)).

The focus of the present paper is on probe encoding, or the cognitive translation of the physical probe item into a task-facilitative format (e.g., extraction of physical features, in the case of template matching, where the probe and memory set comprise alphanumeric items).

This process appeared to be at the root of elevated latencies, among schizophrenia participants, on the intercept of the linear reaction time function of memory-set size, the data summary typically used to portray temporal aspects of performance (memory-scanning itself being spared, with schizophrenia according to analyses of reaction-time function slopes). Specifically, when experimental manipulations were introduced to eliminate the contribution of encoding to the latency intercept (Neufeld, 1977), group differences disappeared (error rates being non-contributory; Neufeld (1978)). Moreover, response operations, considered to represent extra-encoding effects on intercept values, were independently negated as a source of previous schizophrenia–control-group differences (reviewed in Neufeld (2007a) and Neufeld et al. (1993)).

A troubling dilemma surrounding inferred schizophrenia encoding deficit arose nevertheless with the following pattern of results: schizophrenia–control group differences did not increase under experimental conditions where the load on the encoding process was deemed to increase—specifically, where verbal versus pictorial probes were presented for memorial comparisons entailing non-verbal, spatial item properties (Highgate-Maynard & Neufeld (1986); based on Paivio (1975)). Similar perplexing results were obtained for other encoding-load manipulations (George & Neufeld, 1987, detailed below).

Conventional wisdom required that the two factors impinging on the encoding process, namely manipulated encoding load and “schizophrenia diagnostic status”, should produce a significant interaction on the dependent variable. Reconciling the initial evidence of schizophrenia encoding elongation, and the above additivity of factor effects, was found to require formal measurement models whose composition intersected with the substance of the encountered impasse (Townsend, 1984; Townsend & Ashby, 1983).

We now briefly describe this dissection of targeted data patterns with selected parametric process models. The empirical configurations are shown to be diagnostic of candidate-model architectures from amongst those considered, as well as of model parameters eligible to change with performance conditions and group status. We then present an empirically adherent model expansion, entailing the transition of model parameters from fixed to random status. The ensuing mixture-model structures potentially address individual differences in cognitive performance among participants, within groups. A natural outgrowth of mixture-model development comprises a Bayesian-conveyed mediation of group-level findings to individual participants. This extension in turn points to otherwise unavailable strategies for model testing, that include empirical evaluation of parameterized pathocognition at the individual level of performance. It points, as well to cognitive- and statistical-science principled methods of monitoring both individual treatment response, and the efficacy of treatment regimens (the latter at a premium notably in contemporary evaluation of pharmaceuticals).

Turning to cognitive neuroscience, the mixture-model estimated temporal trajectory of the encoding process is shown to inform demarcation of measurement epochs, in the service of calibrating space–time coordinates of fMRI and related technology. Bayesian individualization moreover holds promise for forming parametrically homogeneous subgroups when applying such technology, with desirable consequences for statistical power, and for increased precision of functional inferences. Parameterized characterization of the encoding deficit furthermore enables a quantitative foray into prominent clinical science observations and propositions, and thereby endows them with a novel and potentially parsimonious explanation.

2. Model properties accommodating data configurations

Mathematical developments brought to bear on the above constellation of findings comprised models of stochastic cognitive processes presented by Townsend (1984) and Townsend and Ashby (1983). These developments, among other important advances in the field, appeared to speak quite directly to the present paradigms, data and issues. Certain model architectures, from among the consulted set, were found potentially to accommodate the above configuration of encoding latencies. Within these architectures, in turn, selective parameter changes across groups and encoding conditions were indicated. Discernment of the candidate changes carried substantive significance, in that meaning ascribed to the parameters, within the process model structures, impinged on prevailing conjecture concerning the makeup of schizophrenia cognitive debility.

A pair of parameters, \( k \) and \( v \), attend each of the tendered models. Substantively, they are deemed to convey the number of component encoding events (subprocesses) of the encoding process, and the base rate of subprocess completion per unit time. Note that this base amount applies directly to the subprocess-completion rate where \( k' = 1.0 \), but potentially is modified according to the model structure otherwise. In the case of an ordinary gamma distribution, such as the parallel model with fixed capacity and reallocation (RPFC; detailed below), \( v \) is the distribution’s scale parameter, and \( k' \) is its shape parameter.

Where memory search comprises rudimentary template matching, for instance, the \( k' \) subprocesses may align with \( k' \) physical stimulus properties, such as curves, lines and intersections, of an alphanumeric item. In other instances, such as where encoding of visual–spatial properties (e.g., “real-life size” of a named or drawn item) is required, they may exist essentially as the covert marshalling of the requisite (e.g., size) properties (see, e.g., Paivio, 1975), the relative quantity of which, like the value of \( v \), is a matter of parameter estimation. For the moment, however, focus is on qualitative patterns of the quantitative predictions made by models in which these parameters appear.

The assigned meaning of the present parameters was not trivial with respect to the cognitive psychopathology literature. No small emphasis was given therein to the construct of “cognitive capacity”; nevertheless without a clear delimiting of its usage from amongst those available (enumerated in Townsend and
Ashby (1983, chap. 4)). Reduction in cognitive capacity with psychopathology generally was deemed a disproportionate decline in performance speed, and/or accuracy, accompanying an elevation in task load.

Capacity, in models recruited to the present dissection, on the other hand, could be identified specifically with the parameter \( v \). Judgment therefore could be passed on the issue of psychopathology-affected capacity, now framed expressly as a reduction in this parameter’s value. Note that the interpretation of \( v \) retains currency, as a parametric-distribution expression of prominent, mathematically-defined cognitive-capacity estimates that are distribution-general (see, e.g., Townsend and Ashby (1983, pp. 249, 250) and Townsend and Wenger (2004)).

Tracking down qualifying architectures, and selectively tweaking their parameters pending the target pattern of predictions, is considered analogous to adding a reagent in chemical titration to identify the volume of a constituent in question. Moreover, it is squarely in the spirit of “abductive reasoning in clinical science”, a bona fide method of theoretical explanation where plausible pre-existing explanatory developments are judiciously appropriated to empirical observations (Haig, 2008). Exploration of the tendered models through such rudimentary operations rigorously shows that the latency-additivity property of exogenous – experimental-condition – and endogenous – diagnostic-group membership – sources of elevation in encoding load on response latencies (error-rates not compromising latency interpretation; see, e.g., Luce (1986), Pachella (1974) and Townsend and Wenger (2004)), is diagnostic of an increase among schizophrenia participants of encoding subprocesses \( k' \).

Each examined architecture renders a distribution that either is a member of the general gamma family (McGill & Gibbon, 1965), or is an ordinary gamma distribution. Hence, each entails the sequential completion of the \( k' \) encoding subprocesses of the encoding process; and the times for each subprocess completion, termed inter-completion times (Townsend, 1984), or stage-transition latencies (McGill & Gibbon, 1965), are exponentially distributed. Combinations of one or other parameter change across the two 2-level encoding-load factors are four in number: two sources of encoding increase, by two candidate-model parameters.

Certain architectures are ruled out because every combination of parameter change yields nonadditivity. One such model is the independent parallel, unlimited capacity model (IPUC; also known as a “pure death system with a linear death rate” see also Townsend, Fitić, and Neufeld (2007)). Here, the expected latency, \( E(T)_{\text{IPUC},v,v} \), is

\[
1/v \sum_{i=1}^{k'} 1/i.
\]

The second-order difference with respect to \( k' \), \( \Delta^2 E(T)_{\text{IPUC},v,v} \), which would be 0 in the case of additivity, instead is found to be subadditive:

\[
1/v \left[ \sum_{i=1}^{k'+2} 1/i - \sum_{i=1}^{k'+1} 1/i - \sum_{i=1}^{k'} 1/i \right] = 1/v \left[ 1/(k' + 2) - (1/(k' + 1)) \right] < 0.
\]

Similar results hold for the other combinations. For example, letting the manipulated increase in encoding load elevate \( k' \), and schizophrenia status deflate \( v \), the corresponding partial derivative of the first-order difference is

\[
d/dv[1/v(1/(k' + 1))] = -1/v^2[1/(k' + 1)] < 0.
\]

By similar analyses, more complex variations, such as either or both of schizophrenia-status and manipulated encoding load being accompanied by change in both \( k' \) and \( v \), generate non-additivity.

The following architectures do enable additivity, but only if the increase in encoding load of both sources is conveyed by elevation in \( k' \): the fixed capacity and reallocation parallel (RPFC) model, whose inter-completion times are equivalent to those of the standard serial model; an independent parallel model with moderately limited capacity (IPMLC); and a parallel model with unlimited capacity during the first stage, followed by reduction, and then partial recovery, across stages thereafter (PFSCU).

Expected encoding latency for these models, expressed in terms of their successive encoding-subprocess completion times, are, respectively

\[
E(T)_{\text{RPFC},k',v} = \sum_{i=1}^{k'} 1/(k' - i + 1)v/(k' - i + 1)^{-1},
\]

\[
E(T)_{\text{IPMLC},k',v} = \sum_{i=1}^{k'} 1/(k' - i + 1) \left[ v/(k') \left( \sum_{i=1}^{k'} 1/i \right) \right]^{-1},
\]

and

\[
E(T)_{\text{PFSCU},k',v} = \sum_{i=1}^{k'} \frac{1}{(k' - i + 1)} \frac{v^k}{\Gamma(k' - i + 1)}.
\]

Each of the above equals \( k'/v \), whereby each second-order difference with respect to \( k' \) produces the required additivity:

\[
\Delta^2 E(T) = (1/v)[(k' + 2) - (k' + 1) - (k' + 1) + k'] = 0.
\]

Other combinations of parameter changes violate the required additivity. An increase in \( k' \) assigned to the manipulated higher task load combined with schizophrenia-related capacity reduction, for example, is expressed as the derivative of the first-order difference,

\[
d/dv[(k' + 1) - k']/v = -v^{-2} < 0.
\]

Convening the above architectures, their parameters, and empirical latency patterns of interest, places the analysis of schizophrenia encoding on a formal if basic theoretical platform. At minimum, it permits a quantitatively grounded assault on the fabric of processing deficit in schizophrenia, including a judgment on capacity decline. The verdict is negative with respect to the latency-distribution hazard rate, or the sheer potential to finalize a subprocess in the next instant, given that it is not yet completed (see, e.g., Wenger & Townsend, 2000). Rather, the picture rendered by this assessment is one of reduced economy of constituent operations marshaled to the encoding process, formally stipulated as an increase in \( k' \).

Accordingly, the disorder arguably is selective when it comes to constituents of the processing apparatus that suffer. Spared are the mechanisms of memory search (reviewed in Neufeld (2007a), and Neufeld, Vollick et al. (2007)), and encoding subprocess-wise capacity. Efficiency of capacity deployment \( k' \), on the other hand, is triaged as disorder affected.

Convergent evidence for the above account has emerged from several directions. Included are formal models implementing \( k' \) as a source of increased duration and compromised integrity of multi-dimensional similarity judgments (Carter & Neufeld, 1999), as well as neuro-connectionist-model extensions of increased \( k' \) (Carter & Neufeld, 2007). In addition, aligning schizophrenia performance with incrementation in this parameter has been advantageous to the interpretation of results arising from divergent paradigms used in other settings (see, e.g., Kieffaber et al., 2006).

Pinpointing the precise cognitive-behavioral underpinnings of increased \( k' \) remains ongoing. Multiple empirical findings nevertheless are coherent with this parametric expression of encoding elongation. Candidates have been enumerated in Neufeld (2007a)}
and Neufeld and Williamson (1996). One example entails the insertion of “priming”, or “activating” of the processing system, which may comprise orienting or other functions prior to encoding itself (Russell & Knight, 1977). Compatible with this possibility are fMRI findings of abnormal “resting-state, default-network neurocircuity” (Bluhm et al., 2007; Fox, Zhang, Snyder & Raichle, in press; Liang et al., 2006; Murphy, Birn, Handwerker, Jones & Bandettini, 2009). Anomalous maintenance/default-system connectivity may signal reduced positioning of the system for resource deployment to the service of encoding (Braver & Barch, 2006), upon the latter’s instigation.

Another cognitive-behavioral candidate takes the form of repetition of subprocesses that have failed to be tagged as already dispatched (Hemsley, 1993, 1994), or deficient storage and recognition of redundancy characteristics of constituent encoding operations (Stefy & Galbraith, 1980; Stefy & Waldman, 1993).

Morphological abnormalities in schizophrenia may also speak to aberrant encoding process functionality. Extensive evidence exists for structural brain abnormalities in chronic schizophrenia, including reductions in cerebral grey matter volume (Fannon et al., 2000; Lawrie & Abukmeil, 1998; McCarley et al., 1999; Shenton, Dickey, Frumin, & McCarley, 2001). A meta-analysis of MRI studies found whole brain white matter volume decreases of one per cent and whole-brain grey matter volume decreases of two per cent in individuals with schizophrenia, in relation to comparison groups (Wright et al., 2000). Subtle white matter problems, such as reduced density, problems with myelination, or disruption in uniformity of neural direction within tracts, are seen as potentially resulting in impairments with intra-cortical communication (see, e.g., Burns et al. (2003), Kubicki et al. (2002), Kubicki et al. (2003) and Skelly et al. (2008)).

3. Mixture-model extension of fixed-parameter formulation

In the above data-accommodating models, $k'$ and $v$ were fixed. These parameters, however, stand to vary randomly across task trials and/or individual participants. In so doing, the parameters themselves are randomly distributed, making for a mixture model of performance latencies. With this extension, certain distributions described under Section 2, are eligible mixture-model base distributions, whose parameters are randomly distributed as parameter-mixing distributions; the latter, in turn, are governed by hyper-parameters, which in this case may be substantively significant in their own right (Neufeld, 1991b). Assigning random status to performance-model parameters goes beyond intuitive appeal. Resulting predictions also accord with empirical configurations, at minimum with respect to both means, and inter-trial dispersion of latencies (Neufeld & Williamson, 1996).

The mixture-model extension raises the prospect of expressing individual differences in performance-model parameter values. Individual variation in performance around the group central tendency then can be rendered as part and parcel of the expanded model, rather than as “model-exogenous noise” (see, e.g., Shavelson & Webb, 1991). A desirable by-product of the extension involves the possibility of addressing at least some of the dispersion of observations that exceeds what might be expected from models with fixed parameter status (Batchelder & Riefer, 2007). With multiple response categories, for example, overdispersion may evince a significant subjects-by-category $\chi^2$ contingency test on cell frequencies (Batchelder & Riefer, 2007), or Tukey’s (1949) test for non-additivity on non-frequency data (Neufeld, 2007b). Selective application of the coefficient of variation may aid in monitoring overdispersion in cases where data is not arranged into multiple response categories within participants, albeit without accompanying significance tests.

The base distribution of latencies $t$ selected for the present purposes is that of the RPFC model, above. Its density function $f(t)$ is $(vt)^{k'-1}/(k'-1)!e^{-vt}$. Combined with candidate mixing distributions, predictions are congruent with qualitative properties of focal quantitative data configurations; in the “worked example”, below, both means and inter-trial variances, averaged across participants, are additive. The current developments and substantive inferences nevertheless need not be restricted to such data properties (see, e.g., Neufeld and Williamson (1996) and Neufeld, Carter, Boksa, Jetté, and Vollick (2002)). Moreover, other base distributions render predictions that comply with the present group-data constellation (e.g., compound-Poisson, below).

The mixing distribution for the scale parameter $\nu$ is gamma, with shape hyper-parameter $k$ and scale hyper-parameter $\nu$. Its density function $f(\nu)$ is $(\nu)^{k'/2}/(\Gamma(k')(\nu^{-1}))$. That for $k'$ is Poisson, with probability function $P(k')$ being $m^k/k'!(e^{-m})$. Note that other mixing distributions, such as Jeffreys’s uninformed prior (see, e.g., Berger, 1985) were considered, but yielded no comparative advantage in the present context over those appropriated.

The hyper-parameter $k$ has been aligned with process-wise performer competence, and $\nu$ with susceptibility to stress effects on processing capacity (Neufeld, 1994, 1999, 2007c). Similar to the fixed-parameter status of $\nu$, its distribution is viewed as constant across diagnostic groups, and encoding-load conditions.

Unlike the case of $\nu$, and similar to the fixed status of $k'$, the distribution of $k'$ is deemed to change with both exogenous and endogenous sources of increased encoding load. The base value of $m$ is $m' > 0$ (non-schizophrenia, low encoding-load condition). It is incremented by $h$ and/or $g$, $h, g > 0$, with higher encoding-load conditions, and/or schizophrenia diagnostic status, respectively (specifics of accommodating data configurations in the Section 2).

The parameter $m$ is considered a subprocess-(dis)inhibitory parameter; as it increases, the mean and variance of $k'$, both of which equal $m$, increase. This interpretation can be examined in light of the binomial-distribution’s counterpart of $m$, $np$, where $n$, very large, is the number of “trials”, while $p$, very small, conveys the “probability of success” on any given trial. In the present case, $n$ represents the pool of encoding subprocesses potentially recruited to the service of encoding. The parameter $p$ portrays the probability of a latent subprocess being activated to form part of the encoding operation. As implemented here, $n$ is $m' + h + g + x$, with $x$ being very large. In turn, the parameter $p \approx m/n$, where $m$ retains a base of $m'$, and increases by $h$ and/or $g$, with the occurrence of exogenous and/or endogenous sources of increase in number of encoding subprocesses. Note that with

$$\lim_{n \to \infty} np = m$$

the binomial and Poisson distributions converge. In this way, the stochastic roles played by $p$ are homologous when it comes to encoding-(sub)process activation, and encoding-process magnitude $P(k')$—thus fortifying the present alignment of $m$ with subprocess (dis)inhibition.

3.1. Experimental paradigm and model-addressed data

Exposition of the present developments exploits encoding-performance data from paranoid schizophrenia and control participants, obtained under different conditions of encoding demands. Data from this patient subgroup is selected because: (a) evidence of encoding deficit, although not absent among non-paranoid subgroups, tends to be consistently more pronounced among paranoid subgroups (Neufeld, 2007a,c); (b) these subgroups differ on clinically significant dimensions (Nicholson & Neufeld, 1993); and (c) the encoding deficit, as parameterized here, can be formally
integrated with thought-content disorder, a central symptom of paranoid schizophrenia (Neufeld, 2007a).

We begin with a description of the paradigm and data summaries submitted to mixture-model predictions, which are followed by stipulation of the predictions themselves, parameter estimation, and tests of empirical fit. Included is a principled competing model invoking a compound-Poisson base distribution.

3.1.1. Paradigm

The paradigm was designed initially to examine schizophrenia deviations in the following functional brain asymmetry (George & Neufeld, 1987): a left “hemifield” (right visual field) advantage for processing verbal stimuli, and a right hemifield (left visual field) advantage for processing pictorial stimuli (see, e.g., Moscovitch and Klein (1980) and Sergent (1982)). Principal experimental manipulations involved composition of presented stimuli (four-letter words or pictures of faces), and visual field of presentation. Presentation of stimuli in the contralateral field of hemispheric specialization thus composed the lower encoding load, and presentation in the ipsilateral field composed the higher load (Neufeld & Williamson, 1996). Participants registered “as quickly and accurately as possible” if the presented stimulus was a match to the immediately preceding picture or word. The main experimental paradigm then can be viewed as comprising a visual or memory search task – early- or late-target presentation (Townsend & Ashby, 1983) – with a memory-set size, or visual-array size of one, there being two conditions of probe- or target-item encoding (see Highgate-Maynard & Neufeld (1986), Neufeld et al. (1993) and Neufeld et al. (2002)).

Additional aspects of procedure were addressed to questions auxiliary to the current focus. Briefly, they differentially tapped hemispheric resources considered to be particularized to the stimulus type for the given trial (left hemisphere, in the case of words, and right hemisphere in the case of faces). Justification for their inclusion, and associated procedures, are detailed in George and Neufeld (1987) and Neufeld, Vollick et al. (2007). Note that the auxiliary procedures did not detract from the validity of data for the present modeling purposes, and their presence is built into the model predictions, below.

Model exposition appropriates the first two of the study’s five groups of 14 participants each: (a) paranoid schizophrenia participants; (b) student controls; (c) non-paranoid schizophrenia patients; (d) non-schizophrenia psychiatric patients (13 of 14 having affective disorder); and (e) general controls (demographically similar to the three patient groups). The involvement of group (b), university undergraduates, was undertaken to ascertain generalization to the present manipulations of behavioral lateralities in the very population providing most of the literature data on the topic. Provisions for diagnostic rigor, and for clinical and demographic “nuisance variables”, such as a balance of males and females, verification of similar performance across sex, as well as monitoring for handedness, are specified in the original report. Observe that essential data patterns for groups (a) and (b) resemble those of other paranoid schizophrenia–control pairings; in this way inferences drawn from the extracted two (diagnostic-group) by two (encoding-load condition) factorial layout are more general.

During each task trial, a target item, composed of either a four-letter word, or black-and-white photograph of a face, was presented in the central, “neutral” visual field for 1.5 s, and immediately followed by a probe item. (Note that visual-search terminology arbitrarily is used for the first item, and memory-search terminology, for the second item.) In fact, this probe-item display was made up of 3 items. When the target was a word, a pair of words was presented, one in each visual field. If the target was a face, a pair of faces replaced words in the respective visual fields. In each case, the central field was occupied by an item corresponding to the hemisphere-load manipulations pertaining to the auxiliary task, to be finalized after the “yes”–“no” target–probe match–mismatch response.

Probes were presented for 20 ms, or 200 ms, for words and faces, respectively (crafted according to substantial pilot work). A “yes” was to be registered if a probe item in either visual field matched the target, and a “no” otherwise. There were 128 trials for each stimulus type. The randomly-ordered trials were balanced with respect to the auxiliary-task load, and matching and mismatching probes. Half the matching probes were presented in the left hemifield, and half in the right hemifield. This arrangement resulted in 32 trials for each encoding-load condition, for each stimulus type; only probe-match trials were included in the present analysis.

3.1.2. Results and model-examined data

The focal measure was latency of correct responses, as measured from probe offset (Sergent, 1982). There were significant group differences in performance accuracy (see Table 1), but not so as to compromise latency-rendered inferences (see, e.g., Pachella (1974); cf. Townsend and Wenger (2004); also, in the present instance, ANOVA’s on all latencies differed trivially from those on correct responses only; see George and Neufeld (1987)).

The chief result from the ANOVA on the correct response time data, apropos of the present application, was a Stimulus-Type × Visual Field interaction, F(1, 60) = 10.41, p ≈ .002. This interaction was consistent across groups, F for three-way interaction ≈ 1.085, p ≈ .37, with the groups main effect itself being highly significant, p < .00005. Note that the F value for the 3-way interaction approximated its expected value under the zero-effect null distribution (cf. Cohen (1988, pp. 16, 17)).

This data, then, was poised for appropriation of a 2 (diagnostic group) × 2 (lower–higher encoding–load condition), design, as follows. The second factor comprised right versus left visual field of a target-matching probe specifically for the verbal stimuli. Word-item latencies were singled out because compared to pictures, their simple main effects for visual-field, and those for picture versus words in the left hemifield, exceeded slightly the corresponding simple main effects for pictures.

Data entering cells of the 2 × 2 design took the following form. Performance latency was summarized as the mean latency, averaged over the four auxiliary-task loads, and then across participants; and, second as the inter-trial variance in latency, averaged across the loads after being computed within each, with the mean of these values then taken across participants within cells. The use of latency moments can be advantageous in dealing with the “small N model-evaluation issue”—somewhat ubiquitous in the realm of clinical science. They provide a tool for coming to grips with attendant model-exogenous noise, when prolonged testing of an individual is proscribed. Moreover, the use of moments is not anathema to model discrimination. In the case of mixture models in particular, the criterion of coherence of group-level, and (Bayesian mediated) individual testing of empirical fit can be brought to bear on the issue of model selection (below).

Data aggregation across participants nevertheless risks folding systematic individual differences into an amalgam that then emerges as unrepresentative of any of its constituents (Carter, Neufeld, & Benn, 1998; Neufeld & Gardner, 1990). An associated risk is the application of model predictions to an “unreal computational creation”. In addition, the present mixture model prescribes a common pair of mixing distributions for each cell of the design. The mixture model therefore stipulates that data summaries within a cell should emanate from a single population. Their distribution, moreover, may be approximately normal, owing in part to operation of the central limit.
Table 1

<table>
<thead>
<tr>
<th>Diagnostic group</th>
<th>Paraphrenia schizophrenia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lower encoding load</td>
</tr>
<tr>
<td>Mean (Average variance)</td>
<td>Error rate</td>
</tr>
<tr>
<td>Observed</td>
<td>.83</td>
</tr>
<tr>
<td>Model prediction</td>
<td>.844</td>
</tr>
<tr>
<td>Competing model prediction</td>
<td>.833</td>
</tr>
</tbody>
</table>

Kolmogorov–Smirnov tests for distributional departure from a single normal population therefore were applied to the participants’ means, and separately, standard deviations, as averaged across the four auxiliary-task loads (standard deviations, i.e., “square-root transformation of the variances”, to avoid conflating skewness with multi-modality).

Probability values for the tests on means, progressing from controls (low encoding condition) to paranoid schizophrenia (high encoding condition) were .905, .959, .433 and .997. Parallel values for latency dispersion were .913, .990, .373 and .703. These values accorded with the homogeneity implied by the mixture model, and contraindicated multiple systematic components composing the aggregate, in each case.

As a final preliminary, a coefficient of variation was calculated as the grand mean of the correct–response latencies divided by the average within–cell standard deviation of participant mean latencies. The obtained values was .297. Although such an amount appears “respectable” with regard to tenability of a mixture model, there is no normative distribution for this type of study in which to situate the value. This, however, together with the above Kolmogorov–Smirnov test results, mitigate somewhat against a mixture model representing a patently mis–specified structure.

3.2. Model predictions of group data, parameter estimation, and tests of fit

The “proposed model” and its selected competitor are described in the following two subsections. It should be underscored that a “principled competing model” was selected, inasmuch as its predictions conformed to the qualitative configuration of empirical group–level performance (essentially additivity of means and expected variances; Table 1), but differed subtly from the proposed model in realizing this configuration (detailed below). The similar models thus were poised to illustrate the potential empirical discernment available from the combination of testing at both group– and individual–performance levels of analysis.

3.2.1. Proposed model

Appropriating the RBFC\(k,v\) distribution as the base distribution of encoding latencies \(t\), mixed in turn on \(k\) and \(v\), the expected encoding latency takes the following form:

\[ E(T|k', v)_{\text{encoding}} = \sum_{k=0}^{\infty} P(k') \int_0^{\infty} f(v)k'/v \, dv. \]

Substitution of the Poissonm and \(\gamma_m\) expressions for \(P(k')\) and \(f(v)\), yields

\[ E(T|k', v) = k \cdot r, \quad m\]_{\text{encoding}} = mr/(k − 1), \tag{1} \]

to which is added residual processes \(Y\) and a value of .160. (Details of parameter estimation, estimated values, and empirical fit, are presented in Appendix) The term \(Y\) implements the time for processes appurtenant to encoding, including comparison between the encoded probe and memory–held target, along with central–field auxiliary–task processing antedating registration of the probe–target match–mismatch response. The value of .160 (s) represents response–movement time, as estimated by Woodworth and Schlosberg (1954). This value has served satisfactorily in similar circumstances (see, e.g., Townsend (1984); cf. Rouder, Sun, Speckman, Lu, and Zhou (2003) and Townsend and Honey (2007)); replacing it with an estimated parameter has resulted in similarly small values, with no improvement in empirical fit (Carter and Neufeld (1999); cf. Townsend and Wenger (2004)), and direct measures have indicated comparable movement time, and its inter–trial variance (below), among schizophrenia and control participants (Boksman et al., in preparation; Carter & Neufeld, 2007).

In like fashion, the expected inter–trial variance in encoding duration becomes

\[ E[(\text{Var}T|k', v)_{\text{encoding}} = \sum_{k=0}^{\infty} \int_0^{\infty} P(k')f(v)k'/v^2 \, dv; \]

\[ E[(\text{Var}T|k', v)_{\text{encoding}} = mr^2/((k − 1)(k − 2)). \tag{2} \]

To the above value is added \(Z\) and .036\^2. Here, \(Z\) expresses the variance in latency identified with the same sources as associated with \(Y\), above, and .036\^2 (s\(^2\)) is the between–trial variance in response–movement time (Woodworth & Schlosberg, 1954).

3.2.2. Competing model

The competing model invokes a compound–Poisson base distribution for the individual’s set of trials (see, e.g., Ross, 1996). Rather than varying over individuals within cells, \(k\) by this model is deemed to vary randomly over trials within individuals. The Poisson–distribution parameter \(m\) now prevails for the population of trials for each individual within a cell of the encoding–load by diagnostic–group layout, but again changes across cells. It once more assumes a base value of \(m\), which is incremented by \(h\) for the higher encoding–load condition, \(g\) for Paranoid Schizophrenia diagnostic status, or both. The parameter \(v\) remains \(\gamma_{m,v}\)–distributed, the values of \(v\) being dispersed across individuals within cells, and the \(\gamma_{m,v}\) distribution itself once more being the same for each cell of the encoding level by diagnostic–group layout.

The structure of the expected latency turns out to be identical to that for the RPF(\(k,v\)) participant–wise base distribution:

\[ E[(T|v); m],_{\text{encoding}} = \int_0^{\infty} f(v)m/\text{vdv}; \]

\[ E[(T|v); m, k, r],_{\text{encoding}} = mr/\text{(k − 1)}. \tag{3} \]

To round out predictions, added to the above are \(Y\) and .160, which play roles corresponding to those added to (1), above.

Because the variance of a compound–Poisson distribution, in the case of i.i.d. process inter–completion times (subprocess completion times) is \(m\text{E}(R^2)\) (see, e.g., Ross, 1996), the expected–variance structure for this model departs by a factor of
2 from that for the model with the RPFC_{k,e} participant-wise base distribution:

\[ E[\text{Var}(T|v); m]_{\text{encoding}} = \int_0^\infty f(v)m(Z/v^2)dv; \]

\[ E[\text{Var}(T|v); m, k, r]_{\text{encoding}} = 2mr^2/[(k - 1)(k - 2)]. \tag{4} \]

Added terms are Z’ and .036^2, paralleling those added to (2), above.

Both the proposed and competing models produced no non-trivial departures from the empirical data, \( p \geq .255 \). The question arises therefore as to whether the models are discernable at the level of individual performance values, as evaluated against a sample of participants.

4. Individualization of group-level findings, coherence of group and individual tests of model fit, and candidate-model selection

Tenability of the present mixture-model structure implies that the model’s base distribution in principle should apply to performance of individual participants in general; only the base distribution parameters need be customized. Tendered models therefore arguably can be judged according to the coherence of results from group-level and individual-performer tests of fit.

Mediating the mixture model to the level of the individual participant bespeaks Bayesian prediction of individual performance, considering the natural role of mixing distributions as Bayesian priors. Accordingly, individual encoding-latency samples were tapped from a selection of participants, predictions customized, and tests carried out against larger (validation) samples of individual performance. The proposed and competing models were compared to how well they fared in empirical tests at the participant level of evaluation.

4.1. Individualization of model predictions

Bayesian estimation of an individual’s latency moments about the origin are available as follows. First, we note that

\[ E(T^n|\{t_1, t_2, \ldots, t_N\}) = \int_0^\infty f(t|\{t_1, t_2, \ldots, t_N\})t^n dt \]

where \( f(t|\{t_1, t_2, \ldots, t_N\}) \) is the posterior density function for encoding latency \( t \), given a set of \( N \) sampled encoding latencies for the current participant (subsequently denoted \( \{\ast\} \)).

For the proposed model,

\[ f(t|\{\ast\}) = \int_0^\infty 1/\Theta \sum_{k=0}^\infty f(v)P(k') \prod_{i=1}^N f(t_i|k' \cap v)f(t|k' \cap v)dv, \]

where the normalizing factor \( \Theta \) is

\[ \int_0^\infty \sum_{k'=0}^\infty f(v)P(k') \prod_{i=1}^N f(t_i|k' \cap v)dv, \tag{5} \]

and where \( f(v), P(k') \) and \( f(t_i|k' \cap v) \) are as defined above.

For the competing model,

\[ f(t|\{\ast\}) = \int_0^\infty 1/\Theta f(v) \prod_{i=1}^N \sum_{k'=0}^\infty P(k')f(t_i|k' \cap v)f(t|v)dv, \]

where

\[ \Theta' = \int_0^\infty f(v) \prod_{i=1}^N \sum_{k'=0}^\infty P(k')f(t_i|k' \cap v)dv. \tag{6} \]

Note that the competing model supplies a reasonably strong challenge to the proposed model; the only difference in the structure of Eqs. (5) and (6) is the order of the summation and product operators.

In each case, individual model predictions were available according to

\[ E(T^n|\{\ast\}) = \int_0^\infty f(t|\{\ast\})t^n dt. \tag{7} \]

4.2. Testing of individualized predictions

4.2.1. Performance-sample extraction and model-predicted dataset

Computations were applied to performance data from a representative of each of the four encoding-condition X diagnostic-group cells. For each person, a sample of four performance latencies (i.e., \( N = 4 \)) was extracted. Each of the values comprised an average of four correct-response latencies, one from each of the four concomitant-task loads. This format was in line withnulling of model-extraneous noise, and the collective concomitant-task structure of Y, Z, Y' and Z', above. As stated in Eq. (7), the respective performance samples \( \{\ast\} \) were integrated with the parameter-mixing distributions for the individual's group (i.e., a common distribution of \( v \), but group-specific distributions of \( k' \) to yield \( E(T^n|\{\ast\}) \) and \( E(T^n|\{\ast\})^2 - E(T^n|\{\ast\})^2 \). The resulting predicted means and inter-trial variances were set against those computed from the individual’s full set of correct-response data emanating from the current model-prescribed conditions, as described above. Significance tests in this case implemented two versions of \( \chi^2 \) following the format of Eq. (A.1).

The first, with \( df = q \) (the individual’s total number of correct responses), was

\[ q(M_{\text{latency, observed}} - E(T|\{\ast\})_{k,r,m})^2/Var(T|\{\ast\})_{k,r,m} \]

\[ + \sum_{i=1}^q (M_{\text{latency, observed}} - M_{\text{latency, observed}})^2_{k,r,m}/Var(T|\{\ast\})_{k,r,m}. \tag{8} \]

The second, with \( df = 2 \), was

\[ q(M_{\text{latency, observed}} - E(T|\{\ast\})_{k,r,m})^2/Var(T|\{\ast\})_{k,r,m} \]

\[ + (V_{\text{latency, observed}} - Var(T|\{\ast\})_{k,r,m})^2/[2(Var(T|\{\ast\})_{k,r,m})^2/q]. \tag{9} \]

The logic underlying equations (8) and (9) is similar to that presented in Appendix. Between-participant components are absent from the above denominators, however, in that both predictions and observations are person specific.

Performance-latency samples for the cross section of four individuals are presented in Table 2, as are predicted data, model predictions, and results from the application of Eqs. (8) and (9).

The degrees of freedom in each case correspond to the full complement of observations. Note that the current performance samples were constituents of the predicted observations, and the latter in turn had composed a subset of the corpus of data from the individual’s group that provided hyper-parameter estimates for base-parameter mixing distributions (although to be sure a minor subset). Although not adjusted for incomplete separation of constructed predictions and predicted data, the adopted \( df \) nevertheless served the present purposes in their being somewhat incidental to the central computational message, and expediting its exposition.

Results indicated that the proposed and competing models, differing only with respect to their adopted base distributions of encoding latency, were distinguishable at the individual-performance level of evaluation. For the proposed model, the test statistic supplied by Eq. (8) was consistently less than the expected \( \chi^2 \) value (i.e., \( .00 \)). Using Eq. (9), two of the four tests were not
significant. The competing model produced one significant result by Eq. (8), and two very highly significant results, by Eq. (9).

Because independent $\chi^2$s are additive, $\chi^2$ values were summed across diagnostic-group members, separately for the low- and high-encoding loads. By the statistic expressed in Eq. (8), the summed $X^2_{(df=61)}$ for the proposed model’s predictions of the lower-encoding-load observations is 37.122, $p = .999$; the $X^2_{(df=58)}$ of the higher encoding-load observations is 42.45, $p = .937$. Corresponding values, using Eq. (9), are $X^2_{(df=4)} = 27.84$, $p \to 0$, and $X^2_{(df=4)} = 7.031$, $p = .134$.

Whereas the proposed model now was significant in one of the four cases, the competing model was significant in three of the four cases: values for Eq. (8) were $X^2_{(df=61)} = 56.52$, $p = .639$, and $X^2_{(df=58)} = 96.616$, $p = .001$, for the lower and higher encoding-load conditions, respectively; for Eq. (9), they were $X^2_{(df=4)} = 20.21$, $p = .0004$, and $X^2_{(df=4)} = 39.311$, $p \to 0$.

By a “box-score count” of the collective results, selection of the proposed model is indicated. The ascendance of one model over the other can be summarized further by assembling the predictions of the proposed and competing models presented in Table 2, into a discrete finite mixture-model format (see, Hettmansperger and Thomas (2000) and Raaijmakers, Dolan, and Molenaar (2001)). The mixture-model predictions were applied to the observed values, again separately using Eqs. (8) and (9). The mixing parameter $\pi$ was assigned to the proposed model, and its complement to the competing model. Terms in Eqs. (8) and (9) were constructed to accommodate the present mixture-model expansion by generalizing to the present context the term-composition procedures presented in Appendix. For example, the between-model component for the denominator of Eq. (9)’s latency-variance constituent (i.e., the second expression of its sum) – analogous to the corresponding between-participant component of Appendix – is

$$\text{Var}(E(\text{Var}(T|\text{specific model}))) = E(\text{Var}(T|\text{specific model}))^2 - (E(\text{Var}(T|\text{specific model}))^2 + (1 - \pi)(\text{Var}(T|\text{competing model}))^2 - (\pi \text{ Var}(T|\text{proposed model})) + (1 - \pi)(\text{Var}(T|\text{competing model})))^2.$$
the proposed model is removed, and to 1.0, where the competing model is removed. Each of these “reduced models” is nested in the finite mixture with π estimated.

By this criterion, Table 3 contains five pairs of entries where, contrary to removal of the proposed model, removal of the competing model did not lead to a significant decline in empirical fit. There was one instance where the opposite combination of proposed and competing model effects on fit occurred. Finally, there were four instances where the addition of either model let to a significant increment in model fit.

On balance, the above head-to-head model competition favored selection of the proposed over the competing model. This model now is implemented in computations designed to assess individual treatment response specifically with respect to disorder-significant cognitive functioning.1

5. Implications for assessment of disorder-significant cognition

With a viable mixture model of cognitive abnormality, and its tenable implementation of individual functioning, Bayesian-based procedures present themselves for clinical assessment of cognitive status. Moreover, because prior distribution information is brought to bear, this methodology conveys the potential for delivering relatively precise estimation of individual cognition-performance properties, with only modest performance-specimen demands on assessed individuals (see, e.g., Batchelder (1998), Basawa and Rao (1980), and O’Hagan and Forster (2004); for the present instantiation, see Neufeld et al. (2002)). The latter asset can serve to good advantage with possibly already distressed clinical participants.

This constellation of assets is poised for clinically assessing disorder-significant cognitive status, as follows. Posterior probabilities of an individual’s alliance with diagnostic categories of varying symptom severity are available through Bayesian integration of prior parameter-mixing distributions with the encoding-latency specimen at hand. To facilitate this development, we allow the factorial combinations appearing in Table 1 through 3 to pose as surrogates for diagnostic groups with progressively greater symptom severity (Nicholson & Neufeld, 1993); symptomatology is deemed as increasing from the Control—Low-Encoding Load combination through to the Paranoid—Schizophrenia—High-Encoding—Load combination. The diagnostic-group posterior probabilities, given the encoding-latency specimen, provide a stochastic cognitive-model profile of symptom-related mentation status. They do so because sampled performance is dovetailed with Bayesian priors whose properties have been identified with “varying states of symptom severity”. As translated into the present context, the Bayesian priors are parameter-mixing distributions, and their symptom-severity distinguishing attributes are hyper-parameters, notably the parameter m of the encoding-size k’ distribution. Such a Bayesian infrastructure raises the prospect of assessing cognitive status over a course of treatment, thus ascertaining if the assessed individual is edging closer to less versus more symptom-severity status, specifically in terms mixing distributions produced by the differentially symptomatic groups.

In effect, what is available is the estimated probability of obtained specimen [•] being associated with membership in group g, as follows. The posterior probability of the occurrence of group g, g = 1, 2, . . . , G, where in the present application G = 4, given encoding-latency specimen [•], is

\[
P(g[•]) = P(g)ujd(•)[g]/ujd(•),
\]

where ujd(•)[g], the k′ ∩ v-wise unconditional joint density function of {t1, t2, . . . , tN}, given k′ and v, is

\[
\sum_{k=0}^{∞} P(k′) f(v)k′ cjd(•)[k′ ∩ v]dv,
\]

and where cjd(•)[k′ ∩ v], the conditional joint density of {t1, t2, . . . , tN}, given k′ and v, is

\[
\prod_{i=1}^{N} f(t[i]k′ ∩ v)_{KPC}.
\]

The normalizing factor ujd(•)[g] is

\[
\sum_{g=1}^{G} P(g)ujd(•)[g].
\]

Diagnostic group prevalence, or base rates, applying to the context of assessment, going from less to more symptomatology, arbitrarily are .25, .50, .20 and .05. In practice, the necessary base rates in principle are available actuarily. They can also be estimated, using procedures described in Section 6.

Values of ujd(•)[g] for the representative encoding-latency samples [•] of the G respective cells are presented in Table 4; those for P(g[•]) are presented in Table 5. The latter predictably evince the interplay of the Bayesian likelihood functions and priors—specifically, the ujd(•)[g]’s and the base rates of the G “variously symptomatic groups.” For example, entries in the last two rows of Table 4 are similar; however, those in the corresponding rows of Table 5 differ roughly by a factor of 4, expressing the difference in associated base rates (.20 vs..05).

The confluence of Bayesian priors and likelihood functions also determine the values of ujd(•)[g] themselves, presented in Table 4. As stated in Eq. (11), the pertinent likelihood function now comprises cjd(•)[k′ ∩ v], and the prior is the probability density of k′ ∩ v. As the distribution of v is common across the progressively symptomatic groups, the marginal density function of t, given k′, is inspected. The marginal density function f(t|k′) in the vicinity of each encoding latency specimen [•] comes into play with respect to the present likelihood function. In particular,

\[
f(t|k′) = \int_{0}^{∞} f(v)h(v)f(t|k′ ∩ v)dv
\]

\[
= \Gamma(k + k′)r^{k′−1}/((r + t)^{k′+k′})\Gamma(k)(k′ − 1)!. \tag{12}
\]

The height of f(t|k′) for various values of k′ and t is shown in Fig. 1. The interplay of the likelihood function and prior in determining ujd(•)[g] may be seen most directly by considering the top row and left-most column of the ujd(•)[g] entries in Table 4. Note that f(t|k′ = 1.0) becomes heightened with lower values of t, such values comprising the encoding-latency specimen for the representative from the Control—Low-Load condition, whose mean is .13605. With increasing values of t – which happens for specimens proceeding rightward in the top row (the remaining means being .22306, 29506, and 43706) – f(t|k′ = 1) decreases.

---

1 Model comparison also entailed implementation of the Bayes Factor, whose format comprised the ratio of Eq. (5) (≡ Eq. (11)) to Eq. (6), there being one such ratio for each of the current performance samples (adjusted, as exemplified in Table 4 for the proposed model). Inferences from results essentially were coherent with those based on Tables 2 and 3. Moreover, results emanating from computations assuming the proposed-model base distribution, again invoking a cross section of four performance samples spanning the present factorial combinations, but with a different paradigm and subject sample, were compatible with the present comparative-base-distribution inferences (Neufeld, 2007c; Neufeld et al., 2002). Note, however, that the present model-comparison procedures in principle could be applied to each and every study participant (Neufeld et al., 2002), and that taken together could further support or indeed contradict the present inferences. In any case, the present developments are sufficient to establish the case that discernment of even closely related models in effect is available at this level of analysis.
Table 4
Unconditional joint densities of individual encoding-latency specimens.

<table>
<thead>
<tr>
<th>Diagnostic group</th>
<th>Control</th>
<th>Paranoid schizophrenia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lower encoding load</td>
<td>Higher encoding load</td>
</tr>
<tr>
<td>Encoding-latency specimen</td>
<td>[.15706, .02006, .29006]</td>
<td>[.43006, .13006, .11206, .22006]</td>
</tr>
<tr>
<td>Bayesian-prior parameter-mixing distributions</td>
<td>1.217026</td>
<td>.43603</td>
</tr>
<tr>
<td>Control Lower encoding load</td>
<td>2.06626</td>
<td>.74367</td>
</tr>
<tr>
<td>Paranoid schizophrenia Lower encoding load</td>
<td>4.16402</td>
<td>1.603364</td>
</tr>
<tr>
<td>Higher encoding load</td>
<td>4.01487</td>
<td>1.57853</td>
</tr>
</tbody>
</table>

* Encoding latencies are estimated by subtractively adjusting performance samples appearing in Table 2 for estimated collateral-task processing Y and movement time (see text and Appendix).

Table 5
Posterior probabilities of group membership, given encoding-latency specimens.

<table>
<thead>
<tr>
<th>Bayesian-prior parameter-mixing distributions</th>
<th>Diagnostic group</th>
<th>Paranoid schizophrenia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Control</td>
<td>Paranoid schizophrenia</td>
</tr>
<tr>
<td></td>
<td>Lower encoding load</td>
<td>Higher encoding load</td>
</tr>
<tr>
<td>Control Lower encoding load</td>
<td>.128328</td>
<td>.1238107</td>
</tr>
<tr>
<td>Higher encoding load</td>
<td>.43575</td>
<td>.4223272</td>
</tr>
<tr>
<td>Paranoid schizophrenia Lower encoding load</td>
<td>.351256</td>
<td>.364218</td>
</tr>
<tr>
<td>Higher encoding load</td>
<td>.084668</td>
<td>.089644</td>
</tr>
<tr>
<td>Assigned “Cell base rates”</td>
<td>.25</td>
<td>.50</td>
</tr>
</tbody>
</table>

* Encoding-latency specimens are presented in Table 4.

Fig. 1. Marginal density function of $t$, given $k'$.

Values for the prior probability $P(k')$ are depicted in Fig. 2. Observe in passing that $k' = 0$ figures prominently where $m < 1.0$, as the $k'$-distribution mode occurs at the largest integer below $m$. However, $P(k' = 0|+]) = 0$, because $f(t > 0|k' = 0) = 0$. This leaves $P(k' = 1)$ as the maximum throughout the present range of $m$. Its relative dominance for $m = .0971$, as seen in the first edge of Fig. 2, however, elevates values of $ujd([|]+|g)$ according to $f(t|k' = 1)$; hence, $ujd([|]+|g)$ progressively decreases with increasing encoding-latency values, along the top row of Table 4. Similar observations apply to the second row, where $m = .17885$.

Turning to the left-most column of entries in Table 4, because $f(t|k' = 1)$ is elevated for lower values of $t$, increased $ujd([|]+|g)$ to $f(t|k' = 1)$; hence, $ujd([|]+|g)$ progressively decreases with increasing encoding-latency values, along the top row of Table 4. Similar observations apply to the second row, where $m = .17885$. 
should be realized with greater prevailing \(P(k' = 1)\), for this column's relatively low \(\{\ast\}\) values. Referring to Fig. 2, as \(m\) increases, so does \(P(k' = 1)\), leveling off as \(m\) moves from 1.7764 to 1.85815. Accordingly, moving down the left-most column, values of \(ujd(\ast\mid g)\) evoke the expected pattern.

In these ways, inspection of relations between its constituents again discloses the Bayesian mechanism underlying its emergent values, in this case those of \(ujd(\ast\mid g)\). The profiles of \(P(g\mid \ast)\) for assessed individuals, and also the make-up of their respective profile elements, are available.

The validity of \(P(g\mid \ast)\) estimates themselves of course is predicated on the fulfillment of selected assumptions. The assumptive platform is embodied in that applying to procedures for evaluating treatment-regimen efficacy, detailed in the following section.

6. Assessment of treatment-regimen efficacy

In the preceding development, base rates, or the relative population prevalence of symptom groups \(P(g)\), bearing on posterior probabilities of group membership, given \(\{\ast\}\), were known a priori. It is quite conceivable, however, that information comprising such prevalence instead is in question. Increasingly, treatment-efficacy research has been directed toward cognitive functioning of treated individuals (see Section 1). The present quantitative infrastructure invites estimation of these unknowns, by implementing Bayesian posterior estimation of group prevalence within a multinomial likelihood format. By capitalizing on this computational option, moreover, estimation is firmly set on a foundation of parameterized pathognomonic-like cognition.

The posed scenario of application is one in which a representative sample of individuals from the population encompassing the \(G\) prevalence values in question have been variously classified to the \(G\) symptom-level categories. Assignment moreover has occurred at a selected time of interest following the launch of the investigated treatment-regimen, assumptively using the same method of diagnosis used to form groups from which the available Bayesian priors were obtained. Proceedurally, the sampling of encoding latency \(\{\ast\}\) tenably is appended to the diagnostic interview resulting in assignment of the entertained individual to one of the \(G\) diagnostic groups at hand.

The likelihood function to be maximized, with respect to the \(G\)-1 values of \(P(g)\), is the following:

\[
\prod_{j=1}^{J} \left( \frac{n_{j1}}{\sum_{g=1}^{G} n_{gj}} \right) \prod_{g=1}^{G} P(g\mid\ast)_{j}^{n_{gj}},
\]

where \(P(g\mid\ast)_{j}\) is the posterior probability of group \(g\), given encoding specimen \(\{\ast\}_{j}\), \(n_{j}\) is the number of individuals producing \(\{\ast\}_{j}\), and \(n_{gj}\) is the number of such individuals diagnostically assigned to group \(g\).

In practice, it may well be that a specific \(\{\ast\}_{j}\) will not be duplicated, or for that matter be nearly approximated, by another individual. If so \(n_{j}\) defaults to 1.0, and Eq. (13) reduces to

\[
\prod_{j=1}^{J} P(g\mid\ast)_{j},
\]

where \(g\) is the group to which the lone individual with \(\{\ast\}_{j}\) has been allocated.

We instantiate these computations in the present application, as follows. Let encoding-latency samples for the present purposes be obtained from four participants, the sets of values themselves being those presented in Table 4. Moreover, let diagnostic assignments of the four respective individuals producing these samples be to the “symptom-level groups” adjacent to the encoding-latency specimens themselves. Thus, the individual rendering (.15706, .02006, .07706, and .29006) has been allotted to the lowest symptom-level group, and so on. In this application, then, \(J = 4\) and \(G = 4\). Eq. (14) therefore is to be solved for \(P(g = 1)\) through \(P(g = 3)\), with \(P(g = 4)\) being

\[
1 - \sum_{g=1}^{3} P(g).
\]

In principle, the \(P(g)\)'s of (14) of course can be estimated through analytic optimization. It was found necessary, however, to appropriate a numerical search algorithm (Waterloo Maple 11, Waterloo, Canada) when estimation entailed more than two values of \(P(g)\).

Resulting estimates, with categories of increasing symptom-severity, were \(P(g = 1) = .466\), \(P(g = 2) = .325\), \(P(g = 3) = .099\), and \(P(g = 4) = .11\). Based on these estimated base rates, symptom severity status of the majority of the implicated individuals is aligned with that of the two lower-severity classifications. “Implicated individuals” are those of whom the current sample is representative. The population of individuals implicated by the current sampling, for instance, may comprise only those whose symptom severity has warranted treatment intervention, such as participants initially falling into either of the two more symptom-severe categories. Or, it may comprise some broader constituency, such as one equally distributed across the \(G\) groups at the outset, this population subsequently supplying the representative sampling of \(J\) individuals from whom the \(J\) encoding-latency specimens \(\{\ast\}_{j}\) are obtained. Either way, the news at the current point of assessment essentially is “good”, with respect to reduction in the relative presence of pathognomonic-like elevated hyper-parameter \(m\)-related – cognition.

Moreover, the accuracy of these estimates stands to benefit from prior information being imported by each of the \(J\) terms of Eq. (14). Eq. (10) and (11) state that the \(G\) prior parameter mixing distributions successively enter into the respective numerators of the \(J\) terms, according to group assignment. In addition, they are invoked by each of the \(J\) normalizing factors. The \(G\)-1 estimates of \(P(g)\) likewise participate in the \(J\) terms of Eq. (14). Hence, each estimated parameter \(P(g)\) is anchored in the \(G\) prior parameter-mixing distributions lodged in these Bayesian formulae. Overall, the \(P(g)\) estimates can be viewed essentially as maximally reconciling the distribution of diagnostic-group categorization of the sample of \(J\) individuals, with the associated array of \(ujd(\ast)\).

A non-trivial asset of the present application prototype is that its estimates of \(P(g)\) can access the Bayesian variance-reducing influence on parameter estimates, relative to classical frequentist-based estimation (i.e., Bayesian-based “shrinkage”; O’Hagan and Forster (2004)). In this case, shrinkage acts through the stabilizing influence of the constant set of parameter-distribution priors to which the \(J\) specimens \(\{\ast\}_{j}\) are referred.

As with individual assessment [Section 5], the present estimation procedures can be applied at selected intervals of interest. Doing so, however, demands the tenability of several assumptions.

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2 Obtained values nevertheless were proportional to analytic optimization applied to a subset of these values of \(P(g)\) requiring two estimates, the third being their complement. Resulting estimates also approximated amounts obtained by minimizing the Pearson \(\chi^2\) implementation of “event probabilities” \(P(g\mid\ast)\) – and observed “frequencies” – diagnostic assignment(s) to groups \(g\) – appearing in Eqs. (13) and (14) (as might be expected, even though multinomial-\(L_{G}^2\) and Pearson \(\chi^2\) converge only with large \(n_{j}\)). Here again, moving beyond two estimates of \(P(g)\) became intractable analytically, even with the aid of computer-algebra software (Maple 11), as had been found when working with Eq. (14) itself, or with its logarithmic transformation.
One is that the classificatory schema hosting the establishment of the prior parameter-mixing distributions remains in place. Second, the mixture-model structure, including the architectures of the parameter-mixing and base distributions, must remain viable. Third, the priors themselves, including their hyper-parameter values, need to retain their association with the G classificatory divisions. This association risks being eroded, for example, by practice effects on encoding-latency specimens, possibly invoking the need for substitution of task items (analogous to parallel-form procedures used with psychometric tests).

In addition, a provision for possible differential “drop-out” rates across the G candidate classifications is called for. Factors aligned with participant access over the course of treatment may be entwined with treatment efficacy itself, thus compromising the representativeness of the G categories with respect to the J samples of \( \{s\}_j \) (see, e.g., Luke & Homan, 1998). Specifically, an improvement in targeted aspects of cognitive functioning may contribute to participant geographical mobility. To the degree that diminished symptom severity is allied with such improved functioning, more participants with an improved symptom picture may inadvertently have vacated a study’s catchment area. Inspection of Eqs. (10), (13) and (14) indicates that if comparatively fewer treated individuals currently characterized by less severe symptomatology remain present to render their healthier encoding-latency samples \( \{s\}_j \), the estimated \( P(g = 1, 2) \) would tend to move downward. Accordingly, the estimated- \( P(g) \)-based appraisal of treatment-regimen efficacy (of course duly adjudged against corresponding results from treatment-control participants), would tend to be negatively biased.

Conversely, the opposite combination of participant availability and attendant self-selection bias in \( P(g) \) estimation may occur. Individuals with higher symptom severity, for example, may avoid contact with study personnel, thereby reducing the relative presence of their corresponding categories and \( \{s\}_j \)’s, amongst the overall J-sized sampling. On balance, such possibilities highlight the importance of implementing contemporary outcome-trial logistics for tracking participant whereabouts, in the service of preempting representativeness issues such as these.

Albeit salient, such assumptive stipulations arguably are no more troublesome than are those attending the use of conventional “cognitive-assessment batteries”. The latter tend to include traditional psychometric and cognitive-neuropsychology measures. We note, in passing, that they also may well include certain behavioral cognitive tasks, which if not contrived in isolation of cognitive science proper, nevertheless often fail to exploit interpretative advantages endowed by mathematical performance models. In any event, because of the formal environment in which analytic modeling operates, frequently couched in its flagging of inferential boundaries are candidate directions of redress (e.g., certain provisions for practice effects, and for risks of sampling bias).

In all events, the enumerated provisos should not detract from the thrust of the present offerings. Unlike “off-the-shelf measurement batteries”, possibly combined with generic statistical methods (e.g., selected structural equation modeling), these developments embed the dynamical monitoring of individual treatment response, and treatment-regimen efficacy, directly in the stochastic dynamical model of pathognomonic cognition itself (see, e.g., McFall & Townsend, 1998). Moreover, the methodology, instantiated here for schizophrenia cognition, nevertheless is general, available for adaptation to specific cases in point of cognitive deviation.

7. Implications for epochs of vascular- and electro-neurophysiological measurement

An additional endowment of stochastic analytic modeling of pathocognition pertains to estimated neurocircuitry of disorder-affected processes. A rigorously developed model can serve as a map for navigating key times of measurement during cognitive-paradigm trials. Doing so allows for maintenance of the target process (e.g., item encoding) within the context of task-transacting collateral processes (e.g., memory search and response operations). Importantly, the approach arguably preserves the condition of the triaged process, over and against possible distortion that comes with dismantling a multi-process assembly, and examining its constituents in isolation. Ensuring the presence of a process – if flawed with psychopathology – devoted expressly to conversion of presenting stimulation into a format facilitating memory search requires that memory search itself remain part and parcel of successful trial completion.

Moreover, addressing neurocircuitry corresponding to measurement epochs, as prescribed by a tenable stochastic performance model, lends a potentially important methodological angle on so-called event-related recording. Typically, the “events” onto which brain region, and region–network responses, are superposed, comprise physical-stimulus transitions (e.g., visual target movement). Actual events of interest, however, are not the physical transitions themselves, but rather the cognitive/perceptual processes that are set in motion (e.g., deployment of visual attention). A tenable stochastic model that allows for estimation of the dynamic trajectory of an activated process of interest can provide for informed process-wise temporal navigation in fMRI (as well as related measurement technology, such as magnetoencephalography, and EEG). In this way, mathematical cognitive psychology is uniquely poised to advance the “functional side of cognitive neuroscience” in general, and clinical cognitive neuroscience in particular. Along the way, to the degree that model-parameterized pathocognition is symptom significant, clinical mathematical cognitive psychology stands to broker relations between deviations in patterns of regional neuro-(co)activation, on the one hand, and symptomatology on the other (Carter et al., 1998).

If allowing the target process to remain in situ, vis-à-vis collateral processes, demands the chronometric template conveyed by a viable stochastic process model, its latency-distribution property of choice is the estimated survivor function—in the present case, specifically of the encoding process (Neufeld et al., 2002). The upshot is that encoding elongation supervenes on the neurocircuitry probabilistically enveloped by selected values of the encoding-process’ survivor-function estimate.

Summarily presented here is a specific instantiation of the above measurement strategy, as applied to schizophrenia stimulus encoding (Boksmann, 2006; Boksmann et al., in preparation). The cognitive paradigm comprised a Sternberg memory-search task (i.e., “early-target presentation”; Townsend and Ashby (1983)), the memory set being composed of consonants, with set size varying from 1 through 4. The memory set composition remained constant, within set sizes (i.e., a fixed-set procedure). Participants indicated “as quickly and accurately as possible”, via a peripheral finger keyboard press, if the presented probe was a member of the memory set for that trial.

For the low-encoding-load condition, the probe consonant was always presented using an Arial typeface, in line with memory-set presentation during preliminary learning trials. For the high-encoding-load condition, the probe was presented in randomly varying non-Arial fonts (Binner, Gothic, Westminster, etc.). Randomly interspersed among memory-search trials were “baseline trials”; for these trials, the probe consonant was replaced with an arrow, whose direction simply indicated the key to be pressed.
are conditions, respectively. Hyper-parametervaluesforthemixingdistributionof
where (see, e.g., encoding means and variances, using the implemented hyper-parametervalues
allowing for encoding-residual constituents) were encompassed by modeled
representative estimated encoding-process survivor functions (i.e., $1 - F(t)$, where $F(t)$ is the cumulative distribution function) used to establish the encoding-process fMRI measurement epoch. Plots are for schizophrenia participants and approximately matched controls, performing under lower, and higher encoding conditions, respectively. Hyper-parameter values for the mixing distribution of $\nu$ are $k = 2.5044$, $t = 0.9735$, for both groups; and $m = 70.0001$ and 19.73901, for the schizophrenia and control participants, respectively.

Focusing on the experimental trials, performance accuracy was high, varying between 1% and 6% across experimental conditions, with no significant main effects or interactions involving groups (analyses nevertheless were restricted to correct trials throughout). The latency results essentially aligned with experimental manipulations, including significant main effects only for groups and encoding load with respect to mean latencies. Note that this paradigm had been extensively piloted, incorporating a 0-Tesla, mock MRI environment (Boksman, 2006). The survivor function of the present mixture model was deemed applicable to estimation of encoding duration:

$$\sum_{k} \Gamma(j + k)(t + t)^{i+k}.$$ (15)

Its hyper-parameters were varied, however, accommodating the current MRI-friendly paradigm (see Fig. 3).

The selected post-probe “epoch of interest”, addressing the encoding process, was 500 ms. This interval was considered optimal with respect to encompassing this schizophrenia-affected process, along with necessary duration for fidelity of associated high-field MRI (4.0 T) monitoring of blood-oxygen-level-dependent (BOLD) responding.$^3$

Reference to estimated encoding-process survivor functions $S(t)$ indicated that the probability of encoding-process presence during the demarcated interval, in the case of schizophrenia participants, ranged from 1.0 at $t = 0$ to .54 at $t = .5$ s; that for controls ranged from 1.0 to .0136. Accordingly, the adopted measurement interval allowed for the intended emphasis on encoding in schizophrenia, if necessarily less so for controls because of endogenous differences in its duration. The comparative values of $S(t)$ thus potentially make for an informed, albeit stochastic judgment, as to the degree and nature of group cognitive-process differences during a designated measurement period. Abetting interpretation in the present instance is experimental evidence that encoding precedes collateral processes (Boksman et al., in preparation; Highgate-Maynard & Neufeld, 1986; cf. Neufeld, 2007c).

Encoding-related neurocircuitry was estimated according to time-series covariance, between selected task-versus-baseline-activated “seed voxels”, and other monitored voxels (“whole brain analysis”; Boksman et al. (2005), Fox and Raichle (2007) and Friston et al. (1997)). Covariance values computed from those BOLD-response signals monitored during the designated encoding epoch, significantly exceeding corresponding covariance values as computed from signals monitored during other measurement epochs, were deemed to selectively express encoding–process, seed–voxel connectivity. The other epochs included those of the memory-search paradigm preceding probe presentation (cuing of the memory-set for the trial, to probe appearance), the interval from the post-probe 500 ms to response, plus the baseline post-“probe” (i.e., arrow)-to-response interval.

Results from schizophrenia participants for one of the seed voxels, representing the left anterior cingulate cortex (LACC), are presented in Fig. 4. Apparent is somewhat pervasive LACC connectivity characterizing the monitored encoding period. To be sure, considering the complexities of BOLD-response dynamics, a portion of the observed results are conveyed by pre-probe cognition; and certain time-series covariance concomitants of encoding-affiliated neurocircuitry inevitably escape detection, their being evinced only after the delimited encoding measurement period.

Any dissociation of patterns between the high- and low-encoding conditions, as exemplified in Fig. 4, nonetheless are ascribable to associated differences in the preceding probe stimulation, because the trials were identical in every other respect. An added level of dissociation, supportive of drawn encoding-specific inferences, takes the form of a pattern of seed-searched-voxel co-activation opposite to the present one—obtained for the left precuneus seed voxel (not shown). In this case, substantially more diffuse connectivity attended the low encoding-load condition (Boksman, 2006; Boksman et al., in preparation).

On balance, the picture of neurocircuitry emanating from this analysis was one of comparatively intense connectivity with midline structures, tapering off to more diffuse connectivity elsewhere. In this way, encoding elongation supervenes on less channeled, seemingly more entropic neuro-connectivity.

The chief point to be made, apropos of the present issue, is that navigating measurement intervals based on stochastic process-modeling of behavorial latency data can yield significant and meaningful results with respect to neurocircuitry of clinically significant functions. Convergent support for the present observations, for example, are those of less centered connectivity attending lexical encoding among first-episode, never-treated schizophrenia participants (Boksman et al., 2005). As well, magnetic-resonance spectroscopy studies of never-treated schizophrenia participants have reported elevated levels of LACC glutamine (Théberge et al., 2002), and a positive correlation between duration of untreated psychosis and LACC choline (Théberge et al., 2004).
Turning to $k'$, $E(k'|[t_1, t_2, \ldots, t_N])$ is found to be

$$\sum_{k'=0}^{\infty} \frac{\Gamma(Nk'+k+1)}{[r+\sum t_i]^{Nk'+k+1}} \frac{m^k}{k^k} \prod_{i=1}^{N} \frac{k'-1}{(k'-t_i+1)^{k'-1}} ^{k'-1} = D,$$

from which is available the posterior mean and variance of $k'$, from $n = 1; 1, 2$. Again, excluding all terms referring to the performance sample reduces (19) to $k'$'s mixing-distribution mean $m$ for $n = 1$, and to $m + m^2$ for $n = 2$. The involvement of the maximum likelihood solution for $k'$ is less transparent, in this case, as even that for a fixed value of $v$ entails the simultaneous solution of two equations, one being (18), above, and the other incorporating the PSI (or digamma) function (see, e.g., Evans, Hastings, and Peacock (2000)).

Overall, the above developments show how symptom significance can be brokered to MRI-monitored neuro-(co)activation by overlaying onto MRI-signal analysis the estimated stochastic dynamical template of a symptom-significant cognitive function (Carter et al., 1998); potential symptom significance of the present function is taken up further in the Section 8.1.

8. Discussion

Process modeling of schizophrenia cognitive performance tenably conveys assets involving theoretical analysis, empirical testing, clinical assessment, and measurement at complementing levels of study. Such assets have presented themselves over the course of the present model developments, from the latter’s beginnings in formally accounting for summary statistics of empirical studies, through to viable mixture-model and Bayesian extensions. The translation and readying of presenting stimulation into a format facilitating collateral processes has been triaged as a disorder-significant process. Parametric dissection, in turn, has identified its elongation specifically with the involvement of additional subprocesses. This theoretical formulation has stood up to both fixed and random status of the subprocess-number parameter.

Convergent lines of support for this expression of deficit have included connectionist modeling of schizophrenia processing of facial affect, where the subprocess-transaction rate has been computationally distinguished from the subprocess amount (Carter & Neufeld, 2007); stochastic modeling of the speed and content of multidimensional similarity judgments, where group inequality in subprocess number has been integrated with Chechile’s (1987) theory of memory-trace susceptibility (Carter & Neufeld, 1999); and various behavioral observations, including those bearing on eye-movement dysfunction, a much studied abnormality in schizophrenia (Mather, Neufeld, Russell, and Merskey (1989), Mather, Neufeld, Merskey, and Russell (1992), Neufeld, Mather, Merskey, and Russell (1995) and Neufeld and Williamson (1996); cf. Shelhamer (2008)).

A by-product of these developments is an additional test of model selection, potentially complementing the existing armory of Bayesian-based testing and selection methods (Karabatsos, 2006; Rubin, 1984; Wagenmakers, Ratcliff, Gomez, & Iverson, 2004). It comprises coherence of model performance at group and individual levels of prediction. This option also takes account of clinical constraints on the magnitude of cognitive-performance specimens, and thus speaks to the challenge posed by model-exogenous noise when it comes to small-$N$ model selection (cf. Neufeld and Gardner (1990)).

The homologous link between Poisson-distribution expressed process magnitude, and its binomial-distribution counterpart, theoretically connects increased process magnitude with subprocesses.
dis-inhibition. The estimated encoding-process survivor function conveyed by the present multiple-mixture model, in turn, supplies a “temporal phalanx” for fMRI estimation of associated neurocircuitry. The observed over-connectivity of sites tenably subserving stimulus encoding (Fig. 4), on the one hand, is not out of keeping with the diminished encoding efficiency as parameterized here, on the other. This combination, in turn, raises the prospect of monitoring over a course of treatment (pharmacological or otherwise) patterns of neuronal co-activation, specifically via model-informed MRI measurement, in lockstep with individualized assessment of encoding, embedded in the same model infrastructure (see Wykes et al., 2002).

An additional potentially non-trivial spinoff-in-waiting comprises estimation of cognitive aspects of treatment-regimen efficacy. This application is entrenched in quantitatively modeled pathocognition as coupled with statistically disciplined estimation of the target parameter (in this case, symptom-level prevalence).

8.1. Extensions to thought-content disorder (delusions and thematic hallucinations), and effects of stress

At several junctures of the above presentation, encoding protraction has been tendered as being symptom significant. Previous treatments of this proposal essentially have held that aborted encoding, given time constraints, or undermining of just-encoded material by prolonged encoding of remaining material, compromise the complement of intake from which inferences are drawn (Neufeld, 1991a, 2007a; Neufeld et al., 1993). This vulnerability to fractional information is deemed exacerbated by a heightened propensity, among paranoid schizophrenia patients, toward over-inferring the presence of stimuli or stimulus properties (Broga & Neufeld, 1981b). The present extension lends a formal aspect to the symptom-significance issue.

Exposition of this aspect is expedited through reference to Fig. 5. The curved response surface depicts the probability of encoding-process completion at or before a given time interval \( t \) (\( t = 3 \) for illustrative purposes), in terms of the cumulative probability distribution function \( F(t) \) for the present mixture model (i.e., \( F(t) \), where \( 1 - F(t) \) is stated in Eq. (15)).

The decline in completion probability with increased \( m \) is enhanced with increased values of \( r \); as \( r \) increases, the subprocess-completion rate stochastically decreases, the distribution of encoding-process latencies for any given value of \( m \) moves to the right, and the probability of process completion for a selected time window \( t \) goes down.

Values of \( r \) parsimoniously are considered common to the schizophrenia and control groups. Elevation in \( r \) is identified with a reduction in stimulus salience. Reduced salience may be effected, for example, by lowered stimulus intensity or clarity (see, e.g., Townsend, 1984), placement in the peripheral visual field (see, e.g., Neufeld, Townsend, & Jetté, 2007), oraccompaniment by undermining stimulation (see, e.g., Kieffaber, O’Donnell, Shekhar, & Hetrick, 2007).

Salience may suffer, as well, if stimulation holds a peripheral versus central status within the “attentional domain”, as follows. Features of a stimulus complex may be degraded according to the individual’s deployment of attentional resources. Such functional variation in salience of selected aspects of a stimulus array can be expressed quantitatively, as modeled rates of processing segments of a visual display that differentially bear on task success (Neufeld, 1996; Neufeld, Townsend et al., 2007). Analogously, outside the experimental setting, certain portions of the attentional field may be degraded relative to others. Less engaging portions nevertheless may be vital to endowing the stimulus constellation at large with its objective significance. In the employment setting, for instance, two fellow workers quietly laying out plans for a recreational hunting trip may have beside them a set of hunting regulations and a topographical map. Allowing these objects to be less central in the observer’s attentional sphere, they may not be registered within a brief encoding interval, more so if the observer’s encoding process itself is elongated. That which is successfully taken in, such as the presence of a quiet conversation, rather than being anchored in its objective significance, becomes exposed to mis–interpretation. A quiet conversation, for example, may be taken as personally significant, even conspiratorial (cf. Maher, 1988). In general, owing to differences in ranges of \( r \), successfully encoded material stands in peril of being unsorted by cues conferring its objective significance. Allowing .80 to represent a hypothetical benchmark for integrity of interpretation, increased \( m \) is seen to hasten the latter’s failure. In these ways, “context deficit” in schizophrenia (Dobson & Neufeld, 1982; George & Neufeld, 1985), an idea evidently retaining heuristic currency (see, e.g., Cohen, Barch, Carter, and Servan-Schreiber (1999) and Kerns and Berenbaum (2003)), is lent a somewhat analytic infrastructure.

Reasons posited for a negative bent to inferences drawn from truncated information (e.g., persecutory or jealous delusional content) appeal to the role of possible protective functions. Quite simply, false imputation of malevolent intent stands to be less hazardous to well being than false imputation of positive intent (see, e.g., Neufeld, 1991a).

An additional candidate source of stimulus degradation is psychological stress (definitions, and formal treatments of stress effects on cognition are presented in Paterson and Neufeld (1989), Neufeld (1990), Neufeld (1996), Neufeld and McCarty (1994), and Neufeld, Townsend et al. (2007)). Psychological stress thus may further stochastically erode processing rate (increasing hyper-parameter \( r \)). Inspection of Fig. 5 discloses how pre-existing impairment to the economy of subprocess recruitment (increased \( m \)) is poised to exacerbate adverse effects on encoding-process completion, of stress encroachment on process-wise capacity. Context-grounding cues therefore are placed in yet greater jeopardy, adding more to the risk of compromised inference veridicality.

In this way, these formal developments bear on diathesis-stress postulates of symptomatology, prominent in clinical science.
(see, e.g., Zubin & Spring, 1977). Norman and Malla (1994) rigorously have documented the positive association between symptomatology and antecedent stress; secondary analysis yields a highly significant relation between their total “hassles” scores (Kanner, Coyne, Schaefer, & Lazarus, 1981), and subsequent-month “reality distortion” (delusions and hallucinations), p → 0. Pursuant to this relation, the current proposal goes so far as to funnel stress effects into its parameter r, and the “diathesis” component of the diathesis–stress duo, into the parameter m.

Furthermore, to stress–symptom relations, stress itself stands to impair its own resolution, more so with increased m. Certain forms of coping are cognition intensive; “decisional control”, a prominent form of coping (Averill, 1973), consists of engaging the least-threatening option of a multifaceted stressing situation (Lees & Neufeld, 1999). Identifying the option of minimal social or physical threat from a presenting assortment arguably requires the marshaling of visual- and memory-search operations, including their encoding constituents (Kukue & Neufeld, 1994; Morrison, Neufeld, & Lefebvre, 1988). From this standpoint, if negotiation of decisional control is compromised by impaired encoding, so is stress resolution, thereby compounding the reduction in r. In this way, an elevation in m indirectly increases r by impairing processes involved in cognition-intensive coping (Neufeld, 1999).

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Appendix. Mixture model fit and test

In the present application, the parameter k is set to 30, a defensible value considering the present rudimentary encoding requirements combined with analytical construct validity of k as a task-wise performer–competence parameter (Neufeld, 2007c). Doing so left six free parameters per model: m, h, g, r, Y, and Z.

Parameter estimation recruited a moment-fitting cost function (Townsend (1984), Townsend and Ashby (1983); see also Neufeld and McCarty (1994) and Carter and Neufeld (2007)), minimized with STEPIT 7.4 ((Chandler, 1975), with application cross-checked against a search algorithm of Waterloo MAPLE 9; Waterloo, Ontario, Canada). For the proposed model, with a RBFC$_{c,v}$ base distribution, values for the remaining parameters were .0971 for m, .08175 for h, 1.6793 for g, 10.734 for r, .64794 for Y, and .031842 for Z.

The testing of model predictions against observed latencies and mean inter-trial variance, presented in Table 1, used the following ANOVA-based $\chi^2$ format (Carter & Neufeld, 1999; Snodgrass & Townsend, 1980):

$$\sum_{w=1}^{W} (x_{\text{observed},w} - \mu_{\text{model–predicted},w})^2/\sigma^2_{\text{model–predicted},w},$$

(A.1)

In this case, the term $x_{\text{observed},w}$ is the empirical mean latency, or inter-trial variance, hypothetically sampled from the model-defined population; $\mu_{\text{model–predicted},w}$ is the model-prediction of the sample value, or equivalently the model-based population value of the sampled statistic; and $\sigma^2_{\text{model–predicted},w}$ is the model-stipulated variance of the statistic. It is assumed that sample values are normally distributed, which is reasonable considering the sample statistics’ aggregate form conjoint with the central limit theorem. In essence, the test interrogates if the proposed model prescribes a population whose summary statistics are hospitable to empirical sample values. With W of Eq. (A.1) being 8, and six parameter estimates, degrees of freedom were 2.

The format of the numerator for the mean latency statistic is $n(M_{\text{observed}} - \mu_{\text{model}})^2$, n being 14, the number of participants serving in each cell of the design. The corresponding denominator $\sigma_{\text{model–predicted}}$ is made up both of within- and between-participant components (see, e.g., Kirk (1995) and Parzen (1962, p. 52)). For the proposed model, the within-participants component, or $E(\text{Var}(T|k' \cap v))$, is

$$1/q \sum_{k'=0}^{\infty} P(k') \int_{0}^{\infty} f(v)(k'/v^2 + Z + .036^2)dv$$

$$= [mr^2/((k - 1)(k - 2)) + Z + .036^2]/q,$$

(A.2)

where q is the average number of correct trials for individuals within the cell (unweighted approximation), and $f(v)$ and $P(k')$ are as stated in the text.

Observing that $\text{Var}(x) = E(x^2) - (E(x))^2$, the between-participant component, or $E(\text{Var}(T|k' \cap v))$, in turn, is

$$\sum_{k'=0}^{\infty} P(k') \int_{0}^{\infty} f(v)(k'/v^2)^2dv - \left(\sum_{k'=0}^{\infty} P(k') \int_{0}^{\infty} f(v)k'/vdv\right)^2$$

$$= mr^2((k - 1) + m)/[(k - 1)^2(k - 2)].$$

(A.3)

Turning to mean inter-trial variance, the numerator takes on the same form as that for the mean latency, above. Because the variance in sample variance, given $k' \cap v$, with a sample size of q is 2(population variance)/q, the within-participant component of the denominator, or $E(\text{Var}(T|k' \cap v))$, is

$$\sum_{k'=0}^{\infty} P(k') \int_{0}^{\infty} f(v)(2k'/v^2 + Z + .036^2)^2qdv$$

$$= 2((m + m^2)r^4/((k - 1)(k - 2)(k - 3)(k - 4))$$

$$+ Z(Z + .00259) + 2mr^2/((k - 1)(k - 2))$$

$$\times (Z + .001296) + .168(10^{-3})]/q.$$ (A.4)

Division above is by q rather than q - 1, because maximum-likelihood computations were used for empirical variances (Evans et al., 2000). The between-participant term, in turn, or $E(\text{Var}(T|k' \cap v))$, is

$$\sum_{k'=0}^{\infty} P(k') \int_{0}^{\infty} f(v)(k'/v^2)^2dv - \left(\sum_{k'=0}^{\infty} P(k') \int_{0}^{\infty} f(v)k'/vdv\right)^2$$

$$= mr^4(4mk + k^3 - 3k + 2)/[(k - 1)^2(k - 2)^2(k - 3)(k - 4)].$$

(A.5)

Incorporating these amounts into (A.1), a $\chi^2$ of 1.3356 was obtained, with p = .513.

For the competing model, with a compound-Poisson base distribution, $k = 30$, $m' = .534$, h = .202, $g = 3.037$, $r = 5.88125$, Y' = .565, and Z' = .00019. The within-participant component for the denominator of the mean-latency constituents of (A.1) now becomes

$$1/q \int_{0}^{\infty} f(v)(m(2v^2) + Z' + .001296)dv$$

$$= [2mr^2/((k - 1)(k - 2)] + Z' + .001296]/q.$$ (A.6)
The between-participant component is
\[ \int_0^\infty f(v) (mv)^2 dv - \int_0^\infty f(v) m^2 dv = m^2 r^2 / ((k - 1)^2 (k - 2)). \]  
(A.7)

The within-participant component for the denominator of Eq. (A.1)’s latency-variance constitutes is
\[ \int_0^\infty f(v) 2m(2v^2 + Z^2 + 0.36v^2) qdv = 2/q \left( 4m^2 r^2 ((k - 1)(k - 2)(k - 3)(k - 4)) + Z^2 + 0.168(10^{-5}) + 4m^2 r^2 ((k - 1)(k - 2))(Z^2 + 0.001296) + 2Z^2 + 0.001296 \right). \]  
(A.8)

The between-participant component becomes
\[ \int_0^\infty f(v) 2mv^2 dv - \int_0^\infty f(v) 2m^2 dv = 8m^2 r^4 (2k - 5) \left[ k^2 (k - 2)(k - 3)(k - 4) \right]. \]  
(A.9)

The \( \chi^2 \) test of fit gave a value of 2.739, \( p = .255 \). Here too, there was no non-trivial departure of empirical observations from model predictions.

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