Online Supplementary Material

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1) Narrative for \( E(t) \) for first-order scenario NC, including the operation of the Hypergeometric Distribution (Patil & Joshi, 1968).

For Scenario NC,

\[
E(t) = \frac{1}{p} t_1 + \frac{p-1}{p} \cdot \frac{q}{pq-1} \sum_{i=2}^{pq} H(q-1; pq-2, pq-i', q-1) \cdot t_i
\]

Expected threat \( E(t) \) for NC comprises the sum of two expressions. The first again is simply the product of \( Pr(t_1) \) and \( (t_1) \); that is, the bin containing \( t_1 \) is assigned with probability \( 1/p \), and if assigned, \( t_1 \) is chosen. The first term of the second expression, \( (p-1)/p \), represents the complement of \( Pr(t_1) \), the probability that a \( t_i \) other than \( t_1 \) will be selected. The second fraction \( q/(pq-1) \) represents the probability that for any particular \( t_i \), the latter is located in a bin other than that of \( t_1 \), that is \( (p-1)q/(pq-1) \), multiplied against the probability that the bin containing \( t_i \) is assigned given that the one containing \( t_1 \) has not been assigned, or \( 1/(p-1) \). Each of the summed terms in the second expression involves the hypergeometric distribution (e.g., Patil & Joshi, 1968), used to obtain the probability that \( t_i \) is the lowest threat value in its bin, given that the bin with \( t_1 \) is not assigned and that \( t_i \) is not in \( t_1 \)'s bin. The Hypergeometric distribution is used to assess the probability of a given \( t_i \) thus being selected, which is tantamount to the probability
that all other elements in its current bin have higher threat values. Paralleling bin-model
terminology, $H(q-1; pq-2, pq-i', q-1)$ is the probability that out of a random sampling of
$q-1$ (mutually independent) balls, without replacement (cf., Milenkovic, 2004) -- that is,
$q-1$ (mutually independent) threat values $t_i$ -- from a bin containing a total of $pq-2$ balls
(both $t_1$ and the $t_i$ under consideration themselves are ineligible), $pq-i'$ of which are white,
-- that is, $pq-i'$ for which $t_i$ exceeds the particular $t_i$ under consideration, -- all $q-1$
sampled balls are white -- that is, all sampled $t_i$ values exceed the currently entertained $t_i$.
An analogous format of the hypergeometric distribution is used when the latter is called
upon in obtaining the remaining mathematical expectations of threat.

Note that $H(q-1; pq-2, pq-i', q-1)$ is equal to $1 - \sum_{j=0}^{q-2} H(j; pq-2, pq-i', q-1)$.

Moreover, obtaining $q-1$ white balls in a random sampling of size $q-1$ implies obtaining 0
black balls (i.e., obtaining no $t_i$ values lower than the considered $t_i$), with the equivalent
probability now expressed as $H(0; pq-2, i'-2, q-1) = 1 - \sum_{j=1}^{q-1} H(j; pq-2, i'-2, q-1)$.

Along with the $pq$ element encounters being mutually exclusive and exhaustive, whereby
constituent probabilities involved in $E(t)$ sum to 1.0, these relations are available as
computational checks regarding $E(t)$ for structure NC, and by extensions prescribed by
their host scenario structures, all other $E(t)$ formulae.
2) Attribute values and narrative formulation of $E(t)$ for second-order scenario UCN

Scenario UCN.

$Pr(t_1) = 1/Pq$

$RSS = p$

$$E(t) = \frac{1}{P} \left( \frac{1}{q} t_1 + \frac{q - 1}{q} \cdot \frac{p}{Ppq - 1} \sum_{i=2}^{Ppq} H(p - 1; Ppq - 2, Ppq - i, p - 1) \cdot t_i \right)$$

$$+ \frac{P - 1}{P} \cdot \frac{(P - 1)p}{Ppq - 1} \cdot \frac{1}{P - 1} \sum_{i=2}^{Ppq} H(p - 1; Ppq - 2, Ppq - i', p - 1) \cdot t_i$$

$OSS = Pp.$

The value of $Pr(t_1)$ corresponds to the inverse of the product of the set sizes ($P$ and $q$) of the hierarchy levels at which free choice is not exercised (U and N). The $RSS$ corresponds to the set size ($p$) of the tier with free choice. The $OSS$ is the product of the two set sizes of the tiers at which there are multiple possibilities ($P$ and $p$ under U and C, respectively).

There are two larger expressions in the UCN $E(t)$ formula. The first fraction applied to the entire first expression is $1/P$, the probability that $t_1$'s bin set is selected. Within the large bracket, the first term $1/q$, as combined with $1/P$, accounts for $Pr(t_1)$, which is scaled by $t_1$. The first fraction in the second term within the bracket represents the complement of $1/q$, the probability that $t_1$ is not assigned ($(q-1)/q$). The second fraction $p/(Ppq - 1)$ represents the probability that $t_i$ is one of those assigned into $t_1$'s bin set, given that $t_1$ itself is not assigned. The appearance of the hypergeometric distribution within this first larger expression conveys the probability that $t_i$ is the least of the $t_i$'s.
assigned in $t_1$’s bin set, given that $t_1$ itself is not assigned. The hypergeometric-distribution probabilities multiplying respective $t_i$ values are summed over $i'$ values.

The second larger expression starts with the probability that $t_1$’s bin set is not assigned($\frac{(P-1)}{P}$). The next fraction ($\frac{(P-1)p}{(Ppq-1)}$) represents the probability that $t_i$ is one of the assigned $t_i$ s located in bin sets other than $t_1$’s bin set. The fraction $\frac{1}{(P-1)}$ then accounts for the probability that the bin set containing $t_i$ is assigned, given that $t_1$’s bin set has not been assigned. The second use of the hypergeometric distribution yields the probability that $t_i$ is the least of the $p$ $t_i$ s assigned in $t_i$ ’s bin set given that $t_1$’s bin set is not assigned. This second larger expression then entails the summation of these probabilities of $t_i$ encounters combined with the respective $t_i$ values.
3) Threat Expectation $E(t)$ and Unpredictability of Threatened Events

Implementation of decisional control conveys specific values of threat expectation $E(t)$ according to a scenario structure’s prevailing choice conditions. Decisional-control determined $E(t)$ also enters into adverse-event predictability, as follows. Level of threat identified with scenario-element $i$, defined as the probability of adverse-event occurrence $t_i$, does not directly stipulate the impact, or magnitude, of the stochastic event itself (cf., Neufeld, 1990; Paterson & Neufeld, 1987). However, implicit in the current expression of structure-wise threat level $E(t)$ are two event magnitudes ($m$), specifically 1 “unit of severity”, corresponding to event occurrence, and 0 units, corresponding to event non-occurrence:

$$
\sum_{i=1}^{(P, pq)} Pr(t_i)[t_i (1.0) + (1-t_i)(0)] = \sum_{i=1}^{(P, pq)} Pr(t_i) t_i = E(t). \quad (5)
$$

This dichotomous format of event magnitude nevertheless is coherent with associational-memory (“categorical memory”) accounts of probability learning (Estes, 1975; 1976; 1977), which repeatedly have been shown to extend to predictive judgments of stressor-event occurrence (Morrison, et al, 1988; Mothersill & Neufeld, 1985; Neufeld & Herzog, 1983; Lees & Neufeld, 1999). Greater variation in event magnitude $m$, however, can be accommodated in the present $E(t)$ computations; where the adverse event corresponding to scenario-element $i$ is of an unique magnitude $m_i$, for example, its cross-product with the event’s probability of occurrence $t_{m_i}$, that is $t_{m_i} m_i$, can be stipulated to equal the element’s value of $t_i$ in the present layout. $\text{Var}(m)$, in turn becomes

$$
\sum_{i=1}^{(P, pq)} Pr(t_{m_i}) t_{m_i} m_i^2 - \left[ \sum_{i=1}^{(P, pq)} Pr(t_{m_i}) t_{m_i} m_i \right]^2.
$$
Exercise of available control thus affects level of stressor-event threat, as expressed in $E(t)$; however, it also impinges on stressor-event predictability in quantifiable ways directly incorporating $E(t)$. Note that stress activation is deemed to be driven upward as predictability of stressor characteristics, including event magnitude, declines (see, e.g., Denuit & Genest, 2001; Osuna, 1985; Paterson & Neufeld, 1987; Smith, 1989; Suck & Holling, 1997). Accordingly, variance in event magnitude $\text{Var}(m)$ can be shown to equal $E(t)[1-E(t)]$. Formally,

$$m \in \{0,1\}; \quad t_i \in [0,1];$$

$$\text{Var}(m) = E[E(m^2 | i)] - [E(E(m | i))]^2$$

$$= \sum_{i=1}^{(P) pq} Pr(t_i) [t_i (1^2) + (1-t_i) (0^2)] - \{ \sum_{i=1}^{(P) pq} Pr(t_i) [t_i (1.0) + (1-t_i) (0)] \}^2$$

$$= \sum_{i=1}^{(P) pq} Pr(t_i) (t_i) - \{ \sum_{i=1}^{(P) pq} Pr(t_i) t_i \}^2 = E(t) [1 - E(t)]. \quad (6)$$

Consequently, $\text{Var}(m)$ is maximized where $E(t) = 0.5$. This value turns out to be approximated by the larger values of $E(t)$ obtained in the simulation results presented in Tables 3 and 4 (see main document; further considerations surrounding $\text{Var}(m)$ are available as a .pdf document from the first author.) In this way, those scenario structures with elevated threat $E(t)$ also are accompanied by elevated unpredictability $E(t)[1-E(t)]$.

Other indexes of (un)predictability, framed within the present quantitative structure, convey additional situation properties that stand to be psychological-stress significant. One such index, which ignores event magnitude, is $\text{Var}(t_i)$,

$$\sum_{i=1}^{(P) pq} Pr(t_i) t_i^2 - \{ \sum_{i=1}^{(P) pq} Pr(t_i) t_i \}^2.$$
Another, which circumvents event magnitude, is Shannon-Weaver information entropy:

\[
- \sum_{i=1}^{(P)_{pq}} Pr(t_i) \log_2 [Pr(t_i)] = \\
- \sum_{i=1}^{(P)_{pq}} Pr(t_{m_i} m_i) \log_2 [Pr(t_{m_i} m_i)]; \\
Pr(t_i) = Pr(t_{m_i} m_i).
\]

References not listed in main document


