

Tonal Functions and Active Synthesis: Hugo Riemann, German Psychology, and Kantian Epistemology

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One could in fact set the whole first part of the Transcendental Doctrine of Elements of the *Critique of Pure Reason* to music [...].

Carl Stumpf, *Tone Psychology* (1883, viii)

Introduction

Music theorists often analyze a piece of music without reflecting on the philosophical basis of their theoretical approach. This lack of reflexivity creates epistemic blind spots, which can conceal important methodological assumptions. The most common way to promote epistemological reflection is to analyze existing theories, attempting to uncover their implicit philosophical commitments. This strategy is exemplified by Henry Klumpenhouwer's recent examination of the debate between David Lewin and Nicholas Cook over active versus passive listening.¹ But despite such examinations, many theorists share presuppositions that go unrecognized. One way around this problem of shared—and thus invisible—epistemological assumptions is to take a historical approach, investigating the philosophical and intellectual context of influential figures in the history of music theory. In this essay, I apply such a historical approach to the thought of the German music theorist Hugo Riemann (1849-1919). An analysis of Riemann's work is crucial because, despite his critics, many theorists still share his harmonic functionalism, as well as his basic belief in the primacy of the active, synthetic, logical mind of the listener in musical experience. I suggest that when the ideas of musical logic, tonal function, and active synthesis are placed in their intellectual context, it becomes clear that the problems with which Riemann struggled are relevant

¹ Klumpenhouwer 2006, 286-288.

to more recent discussions of the relation between music theory and music cognition.²

I will argue that Riemann's epistemology—his theory of *Musikbören*—sits squarely within the Kantian tradition of nineteenth-century German psychology. Kevin Korsyn, in his pioneering study of Heinrich Schenker, has shown that we must be aware of the Kantian ideas behind Schenker's influential theories if we want to understand how graphic analysis depends on our ability as listeners “to transcend discrete instants and hear temporal unities.”³ Previous authors have briefly alluded to Riemann's connection to the Kantian tradition, referring to his work as “genuine Kantianism” or as “quasi-Kantian,” but have not explored it at any length.⁴ I will demonstrate that Riemann, like Schenker, was familiar with Kantian ideas, and that these ideas shaped his understanding of musical experience. For Riemann, influenced by the Kant-inspired logical and psychological views of his contemporaries, analyzing music and hearing music were a single, unified task of the active mind.

Although the historical arguments presented in this essay have implications for modern functionalism, I will not discuss the theoretical perspective of neo-Riemannianism.⁵ My reason is simple: the goals and analytical strategies of neo-Riemannian theorists are quite distant from Riemann's own, and thus Riemann's epistemological views are not necessarily relevant to the work of these theorists. Although the idea of chords having functions is today shared by many music theorists, and not just by neo-Riemannians, the term ‘function’ conceals a variety of assumptions.⁶ Current theorists sometimes invoke the concept of mathematical function when pressed, but Riemann—often hailed as the father of modern functionalism—had a quite different understanding of the term. At the end of the paper, I will offer a reinterpretation of Riemann's influential theory of tonal functions in light of his intellectual context.

² Agmon 1995; Rings 2006, chap. 2; Klumpenhouwer 2006.

³ Korsyn 1988, 56.

⁴ Handschin 1948, 126; Mooney 1996, 131.

⁵ For example: Hyer 1995; Cohn 1998; Rings 2006.

⁶ See Agmon 1995 and Kopp 1995.

Over the past fifteen years, a growing scholarly literature on Riemann's work and its implications has emerged.⁷ Several recent studies have focused on the general context of Riemann's theories, in particular Chien-Chang Yang's dissertation *Music as Knowledge: Hugo Riemann's Theory of Musical Listening* and Alexander Rehding's book *Hugo Riemann and the Birth of Modern Musical Thought*.⁸ Although Yang and Rehding have greatly aided our understanding of the intellectual climate of Riemann's time, our picture of the more specific epistemological context of his writings remains incomplete. To explore this context more closely, I draw primarily on those philosophical and psychological works that Riemann explicitly cited in his own writings. I will also occasionally discuss the works of Riemann's teachers, making it clear in each case whether Riemann explicitly cited the text in question. It is important to state at the outset that there is no direct evidence that Riemann was familiar with the works of Kant; however, all of his predecessors were steeped in Kant's ideas, and many even played central roles in the rise of neo-Kantianism.⁹ Thus, my goal is to

⁷ Recent historical studies include Waldbauer 1989, Burnham 1992, Wason & Marvin 1992, Harrison 1994, Arntz 1999, Mooney 2000, Kopp 2002, and Rehding 2003. For theoretical investigations, see Lewin 1982 and 1987, Waldbauer 1989, Hyer 1995, Cohn 1996 and 1997, and the special issue on "Neo-Riemannian Theory" in *Journal of Music Theory* 42/2 (1998). Dissertations include Mooney 1996, Hunnicutt 2000, Yang 2002, Kim 2003, and Baker 2003. Two recent German volumes also contain essays on Riemann: Böhme-Mehner & Mehner 2001 and Motte-Haber & Schwab-Felisch 2005, §4. Some of Riemann's works have only recently been translated into English, for instance Riemann (1914/15) in 1992, Riemann (1872) in 2000, and Riemann (1903) in 2000. This has even extended to the translation of secondary works on Riemann, e.g. Münnich (1909) in 1995.

⁸ Yang 2002 and Rehding 2003.

⁹ Köhnke 1986. The term 'neo-Kantian' is used in different ways. In the narrow sense, it refers to a set of turn-of-the-century German philosophers, in particular the Marburg School (Hermann Cohen, Ernst Cassirer, etc.), the Southwest School (Wilhelm Windelband, Heinrich Rickert, etc.), and others like Hans Vaihinger (Adair-Totef 2003, 33-42). In the broader sense, however, it refers to a variety of German philosophers from the mid-to-late nineteenth century who, in reaction to naive scientific materialism, embraced aspects of Kant's transcendental philosophy. It is in the latter sense, the sense in which I use the term, that it includes figures like Johann Friedrich Herbart, Hermann Lotze, and Hermann von Helmholtz (Köhnke 1986; Sullivan 1990).

uncover Riemann's debt to Kantian epistemology by exploring the work of his immediate intellectual predecessors: Hermann von Helmholtz above all, but also Riemann's teachers Adolf Trendelenburg, Christoph Sigwart, Moritz Drobisch (a student of J.F. Herbart), and Hermann Lotze. Although previous historical studies have explored Kant's influence on the psychological study of sensation in this period, they have only rarely discussed auditory perception. My own investigation of Riemann's theory of musical hearing in the context of nineteenth-century German psychology thus supplements existing work in the history of psychology that has focused almost exclusively on the visual.¹⁰ In sum, building on the substantial amount of recent work on Riemann, my study provides a thorough overview of his immediate epistemological context.¹¹ Without this context, we miss a central insight of one of the influential figures of modern music theory—the connection between the logical functions of the mind and the tonal functions of chords.

Moving beyond Helmholtz's emphasis on sensation, Riemann made the synthetic activity of the representing and judging mind the foundation of his musical epistemology. This epistemology remained stable from the late 1870s to his death in 1919, and lies unnoticed in the background of his famous theory of tonal functions. In the first part of the paper, I will introduce the importance of Kant's ideas to nineteenth-century scientists, discussing Helmholtz's work and Riemann's reaction to it. In the central part of the paper, I will explore the characterization of logical activity in the writings of Riemann's teachers and show how Riemann's musical logic depends on the Kantian idea of active synthesis. Finally, in the third part of the paper, I will offer a new reading of Riemann's theory of tonal functions that emphasizes its roots in the psychical functions of Carl Stumpf and the logical

¹⁰ For example, Hatfield 1990. For a very brief introduction to psychology in the latter part of the nineteenth century, see Hatfield 2003.

¹¹ Several German authors have mentioned the importance of Kant, Sigwart, and Drobisch for understanding Riemann's conceptions of musical logic and tonal function (Novak 2001, 45-48; Motte-Haber 2005, 218-221). Another role of this paper is to expand upon these brief suggestions, painting a more complete picture of his intellectual environment.

functions of Kant. For Riemann, the functional significance of a chord could not be separated from the logical activity of the mind. In understanding Riemann's epistemology, we gain insight into our own epistemological assumptions about how we hear and comprehend music.

Helmholtz, Riemann, and *Musikwissenschaft*

The work of Immanuel Kant was influential throughout the nineteenth century, not just in philosophy but also in the natural sciences. His texts and ideas, however, were notoriously flexible, lending themselves to projects as seemingly unrelated as G.W.F. Hegel's philosophy and Hermann von Helmholtz's science.¹² The latter was, according to Helmholtz himself, a kind of Kantian physiology. As we will see, Riemann used a selective reading of Helmholtz as a foil for his own theory of musical hearing. In Riemann's post-Helmholtzian project, *Harmonielehre* became a part of natural science—not Kantian physiology, but Kantian psychology.

In nineteenth-century Germany, music and sound were the territory not only of musicians and theorists, but also of psychologists, physiologists, and physicists. This scientific colonization of the musical world can be traced, at least in part, to the Kantian argument that reason can provide the foundations for all natural science worthy of the name. Kant claims, in his *Metaphysical Foundations of Natural Science*, that proper science is based on laws known *a priori*, as opposed to empirical laws: "A rational doctrine of nature thus deserves the name of a natural science, only in case the fundamental natural laws therein are cognized *a priori*, and are not mere laws of experience."¹³ One year

¹² For an argument that Hegel is best seen as a reader of Kant, see Pippin 1989. Köhnke 1986, 151-158 suggests that Helmholtz was part of the earliest neo-Kantian program.

¹³ Kant 1786, 4:468. When citing the works of Kant, I will provide the volume and page numbers of the *Akademie* edition. These numbers are given in the margins of the Cambridge Edition translations of Kant's works. The one exception is the *Critique of Pure Reason* (1781/87), which I will cite with the 'A' and 'B' numbers for the first and second editions, respectively.

later, in the introduction to the second edition of the *Critique of Pure Reason*, he emphasizes that in showing how synthetic *a priori* judgments are possible, the way is opened to employing “the pure use of reason in the grounding and execution of all sciences that contain a theoretical *a priori* cognition of objects.”¹⁴ For Kant, only mathematics and “*physica pura*” fit this description, and he explicitly rules out empirically grounded sciences like chemistry and psychology.¹⁵ Later thinkers, however, threw off these limits, attempting to discover a rational, scientific basis for a variety of human endeavors—what one might call a will to science. Most relevant for our purposes is the rise of German *Musikwissenschaft*, the science of music, a term that was fully established by the early twentieth century when Riemann published his general survey *Grundriß der Musikwissenschaft*.¹⁶ Physicists like Hermann von Helmholtz and Arthur von Oettingen, together with psychologist-philosophers like Johann Friedrich Herbart and Moritz Drobisch, were at the forefront of this new scientific approach to music. The prominence of such figures is confirmed by Hermann Lotze, himself a philosopher, who opens the chapter on “Music” in his *History of Aesthetics in Germany* with a discussion of the works of Herbart, Helmholtz, and Drobisch.¹⁷

Although these researchers embraced Kant’s doctrine that knowledge requires both the receptive faculty of sensibility and the active faculty of understanding, they gave his work an empirical twist. They believed that the spontaneous and universal rules by which we actively order experience were susceptible to physiological and psychological explanations.¹⁸ Helmholtz was a central practitioner of this sort of physiologized Kantianism, as he

¹⁴ Kant 1781/87, B20.

¹⁵ *Ibid.*, B21, 4:468; cf. 4:279.

¹⁶ Riemann, 1908. For a more extended discussion of the rise of *Musikwissenschaft*, see the fourth chapter of Yang 2002.

¹⁷ Lotze 1868, 461-478.

¹⁸ Kant 1781/87, A51/B75. For Kant, in contrast, arguments for the universality of the ordering activity of the understanding could not appeal to empirical results – for him the fundamental rules of ordering were *a priori*, prior to all experience. In the 1870s, many neo-Kantians argued against the ‘psychologistic’ reading of Kant (Hatfield 1990, 311n7).

demonstrates in a speech that Klaus Köhnke presents as a key neo-Kantian text:

[...] what the physiology of the senses has proven with respect to the course of experience in modern times, Kant already before sought to prove generally for the representations of the human mind, in that he explained the part played by the particular, innate laws of the mind, the organization of the mind, in our representations.¹⁹

Helmholtz distinguished these representations of the mind (*Vorstellungen des Geistes*) from mere sensations (*Empfindungen*): the latter are “excitations of nerves” whereas the former are the resulting connections between sensations and inferred external stimuli.²⁰ The fact that external objects are only inferred is what makes this view Kantian: we experience only the appearances of things, not the things themselves. It is only the organization of the mind, shared among all rational beings, that guarantees objectivity.²¹

Helmholtz is perhaps most famous for his psychological explanations of human vision—these are what most often earn him the label ‘neo-Kantian.’²² However, as mentioned above, Riemann’s problem with Helmholtz’s theory of musical hearing was that it was not psychological enough. Helmholtz spells out some of the reasons why psychology is less important for a theory

¹⁹ Helmholtz 1855, 99. As Koenigsberger 1902/03 documents, Helmholtz wrote to his father about the main theme of this lecture: “Last Tuesday [...] I gave another lecture upon ‘Human Vision’, in which I tried to put forward the correspondence between the empirical facts of the physiology of the sense-organs and the philosophical attitude of Kant” (1:242/138; cited in Köhnke 1986, 152).

²⁰ Helmholtz 1855, 115 and 1870, 6/4. For translated primary sources, I will cite the pagination of the German edition followed by that of the English translation. That objects are only inferred and not directly experienced is part of Helmholtz’s theory of signs, as discussed in Friedman 1997 and McDonald 2002. For a recent overview of Helmholtz’s work, see Patton 2008.

²¹ Helmholtz 1855. For more on Helmholtz, music, and objectivity, see Rieger 2006.

²² Cf. Hatfield 1990, 171: “Helmholtz devoted more pages of his published work to the psychology of visual spatial perception than to any other single topic. This is somewhat ironic, for he considered himself to be a physicist and physiologist but not a psychologist.”

of hearing than for a theory of vision at the beginning of his *Theory of Tone Sensations*.²³ Presenting a threefold division of labour, he declares that physics studies the stimuli themselves, physiology studies the interaction between the stimuli and our nervous apparatus, and psychology studies how this interaction prompts representations of external objects.²⁴ However, in the case of music Helmholtz elects to concentrate on physiology, since

[...] music stands in a much closer relation to pure sensations [*Sinnesempfindungen*] than all the other arts, which have much more to do with sense-perceptions [*Sinneswahrnehmungen*]; that is, with representations of external objects, which we first extract from sensations by means of psychical processes.²⁵

He continues: “in music, it is actually the tone sensations that constitute the material of the art; we do not build from these sensations, at least insofar as they are shown to advantage in the music, any representations.”²⁶ Thus he argues that music has little to do with perception (*Wahrnehmung*) or representation (*Vorstellung*), both of which lie in the domain of psychology and play a prominent role in his theories of vision. It is not surprising, then, when after pointing out that both physiological and psychological studies of acoustics are lacking, he chooses to concentrate on the former: “it is especially the physiological part, the study of the sensations of hearing [*Gebörempfundungen*], from whose results the theory of music as a natural science must learn.”²⁷ Thus

²³ Helmholtz 1870. Riemann first read Helmholtz in 1869/70 while attending the University of Tübingen (Gurlitt 1950, 1868). I am using the 3rd edition (1870) of Helmholtz’s *Theory of Tone Sensations* (1st ed., 1863), which Riemann cites in his dissertation (Riemann 1874, 5). I have consistently modified Alexander Ellis’ translation. In August of 1876, Riemann wrote to Helmholtz asking for a recommendation for a professorship in Bonn (Hörz 1997, 410-412).

²⁴ Helmholtz 1870, 6/4. For more on this tripartite division of the sensory process, see Vogel 1993, 282-287.

²⁵ Helmholtz 1870, 3/2. Alexander Ellis’ 1875 translation of Helmholtz uses the now old-fashioned word ‘psychical’ to translate the German word ‘*psychisch*’. I will follow this usage in my translations and discussions, primarily to avoid the modern connection between the word ‘psychic’ and clairvoyance.

²⁶ Helmholtz 1870, 4/3.

²⁷ *Ibid.*, 6/4.

Helmholtz, in the introduction to his book, minimizes the role of psychology in the scientific investigation of music.²⁸

Because Riemann's reading of Helmholtz's work passed over its psychological aspects, he was able to cast his own psychology as a natural sequel to Helmholtz's physiology.²⁹ From the mid-1870s onwards, Riemann used this physiological reading of Helmholtz as a foil for his own psychological studies of musical hearing. In his dissertation, published under the title *Musical Logic*, Riemann argues that a "logical law" must underlie our perception of musical relations, and points to the importance of the "activity of our mind" in comparing tone representations.³⁰ He employs the term 'tone representations' (*Tonvorstellungen*) to describe the "reaction of our psyche [*Seele*] to [...] a tone stimulus," recalling and opposing the 'tone sensations' (*Tonempfindungen*) of Helmholtz's title. This contrast is made even more explicit in Riemann's next book, *Musical Syntax*, where his mature epistemology is presented for the first time:

Musical hearing is thus a *selection from the sound-material brought to the ear* [...]; hence it is no longer a physical suffering, but rather a *logical activity*. It is even a *representing*, a unifying, separating, comparing, and relating of *representations* [...] – *tone representations*.³¹

For Helmholtz, on Riemann's reading, musical hearing was a passive process involving "the natural laws of the activity of our

²⁸ Gary Hatfield 1981, 302 states that this emphasis on physiology stems from the fact that "elements must be regarded as more basic than what they come to constitute," but this ignores Helmholtz's comments about the unique character of music.

²⁹ For some of the psychological aspects of Helmholtz's theory, see Steege 2007. Despite Helmholtz's early distinction between "the corporeal ear of the body and the mental ear of the representational faculty" (Helmholtz 1857, 143/63-64; translation modified), his account of the specifics of harmony in the third part of *Tone Sensations* is primarily aesthetic-historical rather than psychological. At the end of the book, Helmholtz writes that he has only carried his work "as far as the physiological features of auditory sensation exercise a direct influence on the construction of the musical system, that is, as far as the work must fall principally to a natural scientist [*Naturforscher*]" (1870, 579/371).

³⁰ Riemann 1874, 41; emphasis in original.

³¹ Riemann 1877, viii; emphasis in original.

ear;” for Riemann, in contrast, musical hearing involved not just the activity of the ear but also the logical “activity of the mind.”³² Riemann’s emphasis on the psychological as opposed to the merely physiological was at the center of his epistemology by the mid-1870s, and remained so until his death. In a late essay, he echoes the 1877 passage and calls it the “guiding principle” of his music-theoretic work: “musical hearing is not only a passive suffering of sound effects in the ear but rather a highly developed activity [...] of the human mind.”³³

Riemann thus sees himself as replacing a passive physiology of hearing involving tone sensations with an active psychology of hearing involving tone representations. This distinction between physiology and psychology becomes even more striking in Riemann’s essay “The Nature of Harmony;” he echoes Helmholtz’s triadic division of labor—physics studies “sounding bodies,” physiology studies “tone sensations,” and psychology studies “tone representations.”³⁴ However, Riemann, in contrast to Helmholtz, argues that psychological rather than aesthetic principles determine our harmonic system; thus psychology must play a central role in any theory of harmony. Whereas Helmholtz claimed that “the construction of scales and of harmonic texture is [...] in no way immediately given by the natural structure or the natural activity of our ear,” and thus provided not natural-scientific but aesthetic explanations of these phenomena, Riemann declared that natural science, and in particular psychology, could explain harmonic relations.³⁵ For Riemann, “the theory of harmony [...] is a part of the science of music [*Musikwissenschaft*], in particular of the natural science of music [*musikalische Naturforschung*].”³⁶

³² Helmholtz 1870, 568/365 and Riemann 1874, 41.

³³ Riemann 1914/15, 1/81; translation modified.

³⁴ Riemann 1882, 160/3-4.

³⁵ Helmholtz 1870, 568/365.

³⁶ Riemann 1882, 159/3. I have consistently modified J.C. Fillmore’s translation of this essay. Cf. Dahlhaus 1990, 61: “While Riemann accentuates the ‘nature of things’ as the true foundation of the theory of harmony, Helmholtz emphasizes a ‘stylistic principle’ as the goal realized through the various means of tonal harmony.”

Later in the same essay, Riemann offers, as an example of the importance of psychology, the concept of *Klangvertretung*, in which a single tone is perceived as the representative (*Vertreter*) of a chord or harmony (*Klang*): “the principle of *Klangvertretung* belongs not in physics, nor in physiology, but rather in psychology.”³⁷ According to Riemann, this “latest advance of scientific knowledge” transforms harmonic theory “from a theory about the mathematical relations of musical intervals to a theory about tone representations and their connection” (185/28). Physics and physiology will still have some importance, but only as *Hilfswissenschaften*, ancillary sciences. Thus, although Riemann opposes his active theory of musical hearing to the passive theory of Helmholtz, he also sees himself as extending Helmholtz’s scientific project into the psychological realm. Helmholtz had done this for visual, but not for auditory perception. For Riemann, then, the science of music could explore all of its aspects, including the “harmonic texture” that Helmholtz saw as primarily aesthetic and cultural. Helmholtz was practicing Kantian physiology, but this did not produce a scientific theory of harmony. As we will see in the next section, Riemann turned instead to Kantian psychology.

Sigwart, Drobisch, and Logic as Active Synthesis

Riemann used the term ‘musical logic’ throughout his career, and in a variety of senses. Alfred Nowak suggests three: the logic of the cadence,³⁸ the inner logic of the composition,³⁹ and the logical connection of tone representations.^{40,41} I will concentrate on the third of these conceptions, tracing its provenance to the logical

³⁷ Riemann 1882, 184-184/27. When Riemann uses the term ‘*Klang*’ in the context of *Klangvertretung*, he is referring to a chord or harmony (thus the single note ‘C’ can represent a C major chord or harmony). This usage is obviously very different from that recommended by Helmholtz’s technical definitions, in which a *Klang* is a complex tone and a *Ton* is a simple tone (Helmholtz 1870, 39/23).

³⁸ Ries 1872.

³⁹ Riemann 1889.

⁴⁰ Riemann 1914/15.

⁴¹ Nowak 2001.

and psychological work of Riemann's predecessors.⁴² Contrary to Nowak's suggestion, however, Riemann's idea of logic as the active synthesis of representations was not limited to the period following his "Theory of Tone Representations,"⁴³ but stretched from the publication of his dissertation in 1874 to his death in 1919.⁴⁴ In this section, I will demonstrate that the epistemological underpinnings of Riemann's musical logic are derived from the work of Kant—either more directly, via Riemann's reading of Lotze's summary of Kant's views, or less directly, via the Kant-inspired writings of Sigwart, Herbart, Drobisch, and Fechner.

Riemann's introduction to philosophical thinking came with private study in the philosophy of law (*Rechtsphilosophie*) with Adolf Trendelenburg in 1868/69 in Berlin, where Riemann was undertaking his military training.⁴⁵ Trendelenburg's work is never discussed by Riemann, and it is thus impossible to ascertain the extent of his influence. However, if Riemann did examine Trendelenburg's *Logical Investigations* at the time, this would have been his first taste of modern views of logic.⁴⁶ Moreover, Trendelenburg was at the heart of contemporary neo-Kantian debates: Riemann studied with him immediately prior to the publication of the older philosopher's infamous *Kuno Fischer and his Kant*,⁴⁷ which attacked Fischer's interpretation of Kant's doctrine of space.⁴⁸ Thus, although Riemann did not study logic with Trendelenburg, this was likely his first exposure to the importance of Kant's ideas for contemporary philosophical debates.

Riemann's first known contact with Kant's views came late in 1873, when he was preparing to defend a revised version of his

⁴² The earliest conception, that of the logic of the cadence, is derived from Hegel and J.G. Fichte by way of Hauptmann (Seidel 1966, 48-49; Dahlhaus 1990, 51-53; Mooney 2000, 83). The Hegelian and psychological conceptions blur somewhat at the time of Riemann's dissertation, given the prominent role of synthesis in, e.g., Hegel's dialectic and Drobisch's logic.

⁴³ Riemann 1914/15.

⁴⁴ Nowak 2001, 45-48.

⁴⁵ Gurlitt 1950, 1866; Arntz 1999, 54.

⁴⁶ Trendelenburg 1840.

⁴⁷ Trendelenburg 1869.

⁴⁸ Adair-Totef 2003, 31-32.

dissertation before Hermann Lotze and Eduard Krüger at Göttingen.⁴⁹ Michael Arntz, Riemann's biographer, describes how Riemann was still frantically reading Lotze's *History of Aesthetics in Germany*⁵⁰ a few hours before his dissertation defense.⁵¹ This book contains a full chapter on Kant's views, including a concise summary of the active role of the understanding in Kant's *Critique of Pure Reason*:

Our conception of the world is not mere intuition; behind the coexistence and succession of the appearances, we presuppose an inner connection [*Zusammenhang*] among them from which their spatial-temporal orderings and alterations first flow. Moreover, the picking out of this connection, which is the task of the understanding, only succeeds with the help of principles that we borrow not from the testimony of experience, but rather possess prior to all experience as innate rules by which our cognition prescribes necessary, inherent forms of reciprocal relation to the manifold of perception.⁵²

Thus for Kant, the understanding is an active faculty of the mind that prescribes rules of relation to our manifold perceptions, making possible the coherent experience of the world.⁵³ As we will see, this active role of the mind in synthesizing representations is one of the cornerstones of Riemann's thought.

⁴⁹ Gurlitt 1950, 1872, and Mooney 2000, 83.

⁵⁰ Lotze, 1868.

⁵¹ Arntz 1999, 68; cf. Gurlitt 1950, 1872. Riemann also mentions Lotze's "Geschichte der Aesthetik i D" in an 1876 letter to Helmholtz, although an incorrect title is given by the editor (Hörz 1997, 411-412). Seidel 1966, 43 traces Riemann's focus on mental activity in musical hearing to Lotze's influence. Yang 2002 and Rehding 2003 also point to the importance of Lotze. For an overview of Lotze's work, see Sullivan 2006.

⁵² Lotze 1868, 37.

⁵³ It may seem somewhat strange that I refer only to the *Critique of Pure Reason* (1781/87) and not to the *Critique of the Power of Judgment* (1790). This is because German logic and psychology, the decisive influences on Riemann's epistemology, were influenced much more by the former. However, Riemann was probably familiar with Lotze's discussion of the latter work (Lotze 1868, chap. 2). Kant's opinion of music, incidentally, was not entirely favorable: "it is, to be sure, more enjoyment than culture (the play of thought that is aroused by it in passing is merely the effect of an as it were mechanical association); and it has, judged by reason, less value than any other of the beautiful arts" (1790, 5:328). For the intellectual background to Riemann's aesthetics, see Yang 2002, 80-98.

Riemann was also indirectly familiar with Kant's epistemology via the logical writings of his various teachers. Christoph Sigwart taught Riemann his first logic course at the University of Tübingen in 1869/70.⁵⁴ Although the first volume of Sigwart's *Logic* was published three years after his contact with Riemann, who does not cite this work, it is likely that Sigwart's logic course, which Riemann attended, dealt with similar topics and ideas.⁵⁵ In the opening passage of the book, Sigwart defines logic as a "*theory of the art of thinking*," and claims that to "determine what *thinking* is in general, how it distinguishes itself from other mental activities, and in what relations it stands to them, is primarily a matter for psychology."⁵⁶ Thus logic and psychology are inextricably entwined for Sigwart. He then explains that thinking is "*an activity of representation*" that has no connection to objects, unlike perception or intuition: "thinking describes a *pure inner vitality* of representing, which thus even appears as a spontaneous action arising from the force of the subject alone."⁵⁷ If logic is the art of thinking, and thinking is a spontaneous activity of representation, then the change in Riemann's dissertation title—from *On Musical Hearing* (1873) to *Musical Logic* (1874)—becomes less mysterious. Musical hearing, for Riemann, requires logical activity, which is a spontaneous, representative function of the human intellect. Thus the title *Musical Logic* refers to a logic not of the music itself, but of our psychical activity.

Sigwart's idea of the spontaneity of human mental activity is derived from Kant, who writes that "the faculty for bringing forth representations itself, or the **spontaneity** of cognition, is the **understanding**."⁵⁸ For Kant, logic is "the science of the rules of

⁵⁴ Gurlitt 1950, 1868; Arntz 1999, 55. Ernst Schröder, perhaps the most famous logician in late nineteenth century Germany (his work was read by the likes of C.S. Peirce and Gottlob Frege), proclaimed himself "a follower of Sigwart in regard to the fundamentals of logic" (cited in Peirce 1903, 165). Peirce criticized Sigwart's conception of logic for being too psychologistic (1903, 166).

⁵⁵ Sigwart 1873. Nowak 2001 mentions the influence of Sigwart, but links his ideas to the period of Hauptmann's influence rather than to Riemann's epistemology in general (40-41).

⁵⁶ Sigwart 1873, 1, emphasis in original.

⁵⁷ *Ibid.*, 1-2, emphasis in original.

⁵⁸ Kant 1781/87, A51/B75, emphasis in original.

the understanding in general,” and the understanding is a “**faculty for judging**,” i.e. a “faculty for thinking.”⁵⁹ Thinking, in turn, is defined by Kant as knowledge through concepts, which provide unity and order to the synthesis of the manifold in experience. Through Sigwart’s logical ideas, Riemann was indirectly exposed to the Kantian claim that concepts spontaneously give order to the “manifold content of representation.”⁶⁰ Sigwart even discusses musical perception in these terms:

Even if our representational world were limited to the twelve simple tones of an octave, as we noticed each individual tone and its certain difference from the others, which prevents any confusion, we would achieve all that is necessary to raise our representations to conceptual determination; and with the representations of the individual tones and the consciousness of their differences, we would have been given the complete material of our concepts in definite order.⁶¹

That Sigwart chooses to elucidate this idea of the ordering of representations by concepts—i.e., the organization of experience—with a musical example indicates that he too, like Riemann, viewed musical hearing as an active process. Sigwart’s psychologistic logic, inspired by Kant’s epistemology, provided Riemann with a model of logic as mental activity, and specifically representative activity involving the conceptual ordering of the manifold.

Other psychologists and logicians with whose work Riemann was familiar also held broadly Kantian views. For example, at the beginning of his dissertation Riemann cites Johann Friedrich Herbart’s “Psychological Remarks on the Theory of Tone,” which both foreshadows Riemann’s critique of Helmholtz and offers a Kantian approach to music theory.⁶² He even uses Riemann’s favourite term, *‘Tonvorstellungen’*:

⁵⁹ *Ibid.*, A52/B76, A69/B74.

⁶⁰ Sigwart 1873, 280.

⁶¹ *Ibid.*, 280.

⁶² Herbart 1811. Riemann also cites Herbart’s “Psychological Investigations” (1839). Although she helpfully points to the Riemann-Herbart connection, Youn Kim 2003, 246 claims that “it is very doubtful whether Riemann ever directly studied Herbart’s treatises.” In fact, Riemann cites Herbart’s writings on music in

[...] physics is not psychology; vibrating bodies are not representations of tones; indeed, the existence of vibrating bodies was denied by idealism, whereas the psychological fact that we have tone representations [*Ton-Vorstellungen*] and receive such and such impressions from their associations cannot be denied.⁶³

Herbart also warns against “the useless attempt to push between physics and psychology a physiological hypothesis about vibration relations acquired unscathed through the nerves of the psyche [*Seele*].”⁶⁴ This aggressive questioning of physical and physiological approaches to musical hearing could have been one of the things that recommended a more psychological approach to Riemann. A few pages later, Herbart speculates about the possibility of applying Kant’s “Transcendental Aesthetic” to music:

[...] one could decide to adopt, for the harmonic relations of certain intervals [...], a series of a priori laws, and to explain music via the pure intuition of the tone-line and hence the pure forms of synthesis associated with it, just as geometry and the pure theory of nature are explained via the pure intuition of space.⁶⁵

It is obvious that Herbart endorsed Kant’s claim that “the fundamental natural laws” of a natural science must be “cognized *a priori*”; this, combined with the stress on psychology and tone representations, probably shaped Riemann’s ideas as he prepared his dissertation.

The main influence on Riemann’s burgeoning logical ideas, however, was the philosopher-psychologist Moritz Drobisch, who, along with Oscar Paul, rejected Riemann’s “post-Hauptmannian” dissertation at Leipzig.⁶⁶ Despite this rejection, Riemann was still engaging with Drobisch’s ideas in the late 1870s. For example, he

his dissertation (1874, 3), and it seems unlikely that he did not at least peruse Herbart 1811, only nineteen pages long.

⁶³ Herbart 1811, 100; cf. 1839, 54.

⁶⁴ *Ibid.*, 100.

⁶⁵ *Ibid.*, 102; cf. 1839, 50. Stumpf cites a similar passage from Herbart in the preface to the first volume of *Tonpsychologie* (1883, vii-viii). He may have had this passage in mind, but the wording as quoted is different (see the epigraph to this paper).

⁶⁶ Mooney 2000, 82.

footnotes the third edition of Drobisch's *New Presentation of Logic*⁶⁷ at the end of the opening passage of *Musical Syntax*:

[...] it is a strange thing that psychology could leave it unremarked for so long that the understandable in pieces of music, and even in individual chord progressions, requires an activity of logical functions and cannot be a mere tickling of the senses [*Sinnenkitz[e]*] or psychical passivity.⁶⁸

In the footnote, Riemann quotes Drobisch, who sees thinking as “an *integrating* of the *many* and *manifold* into a *unity*. What gets integrated,” he continues, “are not actual objects, but rather *representations*.”⁶⁹ In the same footnote, Riemann mentions the following “noteworthy” thought: “That synthesis which presents an *inner* connection of associated concepts yields *synthetic concept-forms*, which are called in the narrower and more proper sense *relations*.”⁷⁰ The implication is clear: to find out what Riemann means by the “activity of logical functions,” we need only look to his footnote, which cites a book on logic. For Riemann, then, following Sigwart and Drobisch, logic fundamentally involves the active synthesis of representations by the human mind.

Both Riemann's contrast of passivity and activity as well as the focus on active synthesis in the logical works of Drobisch can be traced to the work of Kant, who begins the subsection of the *Critique of Pure Reason* entitled “On logic in general” with a discussion of “two fundamental sources in the mind,” the receptive faculty of sensibility and the active faculty of understanding.⁷¹ It is the active, spontaneous faculty that is engaged in the crucial work of synthesis: “By **synthesis** in the most general sense, however, I understand the action of putting different representations together with each other and comprehending their manifoldness in one cognition.”⁷² For experience to be possible, Kant argues, we

⁶⁷ Drobisch 1863.

⁶⁸ Riemann 1877, 1.

⁶⁹ Drobisch 1863, 5; Drobisch's emphasis; cited in Riemann 1877, 1n.

⁷⁰ *Ibid.*, 34. Riemann leaves some words out of this quotation, but emphasizes the term ‘synthetic concept-forms’. Drobisch emphasizes the words ‘inner’ and ‘relations’.

⁷¹ Kant 1781/87, A50/B74.

⁷² Kant 1781/87, A77/B103; emphasis in original.

cannot just receive the plurality of sensations; we must also synthesize and bring unity to that plurality via rules and concepts. Kant's basic idea of a synthesis of the manifold of representation plays, as we have seen, a vital role in Drobisch's view: "Each thought is, generally speaking, an *integrating* of the *many* and *manifold* into a unity."⁷³ Drobisch's logical edifice, so influential on Riemann, is built on a Kantian foundation.

Thus the idea of logic as involving the active, spontaneous synthesis of the manifold of representations comes to Riemann from Kant, albeit filtered through the logical and psychological writings of Lotze, Sigwart, Herbart, and Drobisch. Words based on the adverb '*zusammen*' (together) are central to this discourse of synthesis: in Lotze's description of Kant's epistemology, experience presupposed an inner connection (*Zusammenhang*) of appearances, and in Drobisch's work, thought is the integrating (*Zusammenfassen*) of the manifold into a unity. Not only was Riemann taught by a logician, Sigwart, who emphasized this spontaneous activity; his dissertation was also supervised by another logician, Drobisch, who defined thought itself as the synthesis of the manifold of representations. Moreover, when Riemann mentions the "activity of logical functions" that psychology has neglected, he appends a long footnote focusing on the idea of active synthesis. This includes quotations from Drobisch, as we have seen, but also from the psychologist Gustav Fechner: "*The most fundamental law* [relating to the pleasure/agreeability of music] *seems to me still to lie in darkness* [...]. I think that *the principle of the unified connection of the manifold* plays a key role."⁷⁴ For Riemann, therefore, as Helga de la Motte-Haber has recently written, musical logic involves the "activity of the mind," i.e., "putting what follows in relation to what precedes, as well as the integration of both into a higher unity."⁷⁵ In the next section we will investigate the connection between this logical activity and Riemann's theory of tonal functions.

⁷³ Drobisch 1863, 5.

⁷⁴ Fechner 1876, 1:164; Riemann's emphasis.

⁷⁵ Motte-Haber 2005, 218.

Kant, Stumpf, and Tonal Functions

The theory of tonal functions of chords, or ‘harmonic functionalism’ as it is now often called, is Riemann’s best-known contribution to music theory. However, there is still a great deal of disagreement over the provenance of the term ‘function,’ and thus over the epistemological presuppositions of the theory. In this section, I will argue that two hitherto unexamined ideas lie hidden in the background of Riemann’s theory of tonal functions: the logical functions of Kant and the psychical functions of Stumpf. For Riemann, the tonal functions of chords are directly related to the activity of the logical functions of the human mind. Hence, in his view, the act of analysing music cannot be abstracted away from the act of listening to music.

Most commentators have assumed that Riemann’s idea of a tonal function is a metaphor “borrowed from mathematics.”⁷⁶ It is initially suggestive for this view that Riemann’s brief stint at the University of Göttingen (to defend his revised dissertation) overlapped with that of the mathematician and logician Gottlob Frege, who defended his own dissertation a few months before Riemann’s arrival.⁷⁷ Frege, later in his career, even wrote an essay called “What is a Function?”⁷⁸ Like Riemann, he was deeply influenced by Lotze and the neo-Kantian tradition, as Gottfried Gabriel and David Sullivan have shown.⁷⁹ However, Riemann was only in Göttingen for a few months, and even if the two did meet, Frege was focused on mathematics and geometry at the time—he had yet to publish any work in logic, and did not discuss the idea of

⁷⁶ Hyer and Rehding 2007, §2; cf. Dahlhaus 1966 and Hyer 1989, 99-107.

⁷⁷ Riemann’s doctorate was granted on November 30, 1873, and Frege’s was granted on December 12 of the same year. However, Frege orally defended his thesis on August 8, after it was submitted to and assessed by the mathematician Ernst Schering. Riemann did not arrive in Göttingen until late September or early October (Kreiser 2001, 92-93; Mooney 2000, 83, 96n5).

⁷⁸ Frege 1904.

⁷⁹ Gabriel 1986 and 2002, and Sullivan 1990 and 2002. At HOPOS 2004 in San Francisco, I presented an earlier version of this paper that focused on these connections. For more on the importance of Lotze’s work, see Sullivan 2006. For recent historical work on Frege, see Kreiser 2001 and Milkov 2001.

function until the early 1890s.⁸⁰ These facts, combined with Riemann's silence on mathematical topics, make it unlikely that the idea of mathematical function was the source for Riemann's concept of tonal function. That said, however, many recent interpretations of Riemann's functionalism have arrived at a mathematical conception of function that accurately captures his epistemological insights.⁸¹ Thus, although 'tonal function' may not have been a mathematical metaphor for Riemann, the mathematical interpretation represents a fruitful elaboration of Riemann's general approach.

Looking at Riemann's own work, an important predecessor to his 1893 account of the tonal functions of chords is his earlier claim that musical hearing requires "an activity of logical functions."⁸² Although I have not been able to locate the phrase 'logical functions' (*logische Funktionen*) in the works of Sigwart, Drobisch, or Lotze, it does appear prominently in the work of Kant, and with a definition that resonates strongly with Riemann's views on active synthesis. Kant's famous table of categories—pure concepts that give unity and structure to any experience—is derived from his table of logical functions of judgment.⁸³ For Kant, a function is "the unity of the action of ordering different representations under a common one," and "judgments are functions of unity among our representations."⁸⁴ Each of Kant's categories (negation, necessity, causality, etc.) corresponds to a particular logical function of judgment: "there arise exactly as many pure concepts of the understanding [...] as there were logical functions of all possible judgments [...]: for the understanding is

⁸⁰ Frege's dissertation was entitled "On a Geometrical Representation of Imaginary Figures in the Plane." He did take Lotze's course on the philosophy of religion, but his thesis was submitted to the mathematician Ernst Schering. Lotze had no influence on Frege's oral defense, having asked to be excused from the proceedings (Kreiser 2001, 87, 92-93).

⁸¹ Hyer 1989, 99-107; Rings 2006, chap. 2.

⁸² Riemann 1877, 1.

⁸³ Kant 1781/87, A70/B75, A80/B106.

⁸⁴ *Ibid.*, A68/B73. I have recently discovered that Nowak 2001 also cites this definition, connecting it as I do to the "logical functions" passage of *Musical Syntax* (47). Cf. Motte-Haber 2005, 218.

completely exhausted and its capacity entirely measured by these functions.”⁸⁵ Kant’s idea of a logical function as a unifier of representations is strikingly similar to the synthetic activity of the mind discussed in the work of Sigwart and Drobisch, and is thus a possible source of Riemann’s 1877 terminology.

Motte-Haber has recently remarked that Kant’s definition of function as the act of ordering and unifying representations is so illuminating in light of Riemann’s overall epistemology that “it leaves the occasionally astute debates over a possible parallel with the mathematical concept of function appearing not especially necessary.”⁸⁶ However, it is not completely clear that Kant’s term ‘logical function’ is the direct predecessor of Riemann’s ‘tonal function,’ despite their strong resonance. Another likely candidate, and one to which we know Riemann was exposed, is Carl Stumpf’s concept of a psychological function, which he employs in his book *Tone Psychology*.⁸⁷ This work was published a full ten years before Riemann’s *Harmony Simplified, or the Theory of the Tonal Functions of Chords*.⁸⁸ In the preface, Stumpf says that his book “is called *Tone Psychology* because it describes the psychological functions that are excited by tones.”⁸⁹ Such functions are not inherent in the sensations themselves, but are mental activities: “Thus no association and no relation of two sensations or representations is in and for itself already a judgment of them. This judgment, the affirming or negating relation, arrives as a new and heterogeneous function.”⁹⁰ In a footnote, Stumpf refers to his teacher Franz Brentano’s distinction between a judgment and a representation: in the latter, a simple or compound object is merely represented to

⁸⁵ Kant 1781/87, A79-80/B105.

⁸⁶ Motte-Haber 2005, 218n70.

⁸⁷ Stumpf 1883.

⁸⁸ Riemann 1893. Riemann studied *Tone Psychology* in the years preceding the presentation of his own function theory. For one citation, see Riemann 1891, 92. Münnich 1909 analyses the later debates between Riemann and Stumpf.

⁸⁹ Stumpf 1883, v.

⁹⁰ *Ibid.*, 4. The idea of a psychological function, at least as Stumpf 1906, 4-5 defines it, is quite general: “We designate as *psychical functions* [...] the noticing of appearances and their relations, the integration of appearances into complexes, the formation of concepts, grasping and judging, the emotions, desiring and willing.”

consciousness, whereas in the former, “it is simultaneously represented and affirmed or denied.”⁹¹ Brentano argues that “in every act of consciousness, however simple it may be, e.g. the act in which I represent a tone, not merely a representation, but also at the same time a judgment, a cognition, is resolved.”⁹² Judgment, as a psychical function, is present whenever anything is represented—for Stumpf, when we experience tone representations or tone sensations, there is always an accompanying affirmation or denial of their associations and relations to one another. This active notion of function obviously meshes easily with Riemann’s psychological picture of musical experience, described above.

Stumpf’s psychical functions relate to Riemann’s tonal functions inasmuch as both involve how we hear particular tones or chords. For Stumpf, a sense-judgment determines whether “we designate a sensation as the tone A or as the third of F,” and psychical functions of this sort are constantly acting when we listen to music.⁹³ This active picture of musical hearing is similar to Riemann’s. On the face of it, however, Riemann’s tonal functions seem quite passive—mere functional roles taken on by particular chords. In *Harmony Simplified*, he equates tonal function with a chord’s meaning or significance (*Bedeutung*) within a key: “There are only three kinds of tonal functions of harmony (meanings within a key), namely that of the tonic, dominant, and subdominant.”⁹⁴ Earlier in the introduction, Riemann glosses the meaning of a tone as its “aurally and exactly known relation to other tones of the same melody.”⁹⁵ But where does this relation come from? From the activity of the mind, as Riemann describes in *Musical Syntax*—this relation is one of those “*synthetic concept-forms*” that Riemann highlighted in his citation of Drobisch, “which are called in the narrower and more proper sense *relations*.”⁹⁶ Thus a tonal function in Riemann’s sense appears to be a special kind of Stumpfian psychical function or Kantian logical function, in which a chord’s

⁹¹ Brentano 1874, 289/221; translation modified.

⁹² *Ibid.*, 296/225.

⁹³ Stumpf 1883, 3.

⁹⁴ Riemann 1893, 9.

⁹⁵ *Ibid.*, 2.

⁹⁶ Drobisch 1863, 34; cited in Riemann 1877, 2n.

meaning within a key is logically determined by the listener's mental activity.

Some German scholars have emphasized this connection between Riemann's functionalism and his epistemology. Carl Dahlhaus puts the point this way: "The fundamental idea of Hugo Riemann's theory of functions is 'that the act of listening to music [...] is [...] a highly developed application of the logical functions of the human mind.'"⁹⁷ Motte-Haber makes a similar claim:

Riemann made the "activity of the mind" [*Geistestätigkeit*] a central factor in both his theory of meter and his theory of harmony. This means putting what follows in relation to what precedes, as well as the integration of both into a higher unity [...]. To understand what follows in relation to what precedes means to undertake a comparison ('relation', 'comparison', and 'higher unity' are words used often by Riemann), to recognize a functional connection. Riemann used the concept of function in his theory of harmony to embrace the comparative and synthesizing achievements of consciousness, but it shaped all of his thinking.⁹⁸

This is why Riemann speaks of the tonal functions of tonic, dominant, and subdominant as "the three main pillars of the *harmonic-logical* construction."⁹⁹ The function of chords is defined as their meaning "for the logic of the composition depending on their position relative to the tonic."¹⁰⁰ But according to Riemann, *Harmonielehre* itself is "[t]he theory of the meaning of harmonies (chords), i.e., *the explanation of the thought-processes in musical hearing*."¹⁰¹ Thus Riemann's theory of tonal functions cannot be divorced from the logical and psychical functions of his theory of *Musikbören*.

The idea that Riemann's tonal functions are not passive roles played by chords but rather products of the active synthesis of tone representations can be illustrated by a musical example. At the very end of his career, Riemann published three volumes containing analyses of all of Ludwig van Beethoven's piano sonatas. His analysis of the opening of the second movement of Piano Sonata No. 21 in C major, Op. 53, nicknamed the

⁹⁷ Dahlhaus 1990, 47; citing Riemann 1914/15, 1.

⁹⁸ Motte-Haber 2005, 218.

⁹⁹ Riemann 1912, 214; quoted in Dahlhaus 1966, 93; my emphasis.

¹⁰⁰ Riemann 1909, 441.

¹⁰¹ *Ibid.*, 568; my emphasis.

‘Waldstein’ sonata, provides a nice example of the connection between active synthesis and function ascriptions.¹⁰² Recall that the tonal function of a chord is its meaning within a key (Riemann 1893, 9). Yet the main key of a movement is not always apparent at the local level. Since we are constantly and actively synthesizing different moments in musical hearing, how we understand these moments depends on how they are grouped. Because of the “peculiar, fragmentary nature” of the Waldstein second movement opening, Riemann warns, there is a risk of incorrectly locating the boundaries between motives (*Motivbegrenzung*). However, once we have heard the whole introductory section (mm. 1-9), we know the meaning of its different parts: “In light of the significance that the wistful, upward-reaching thirty-second-note motive attains in the course of the introduction [...], it would only be correct to see it as already beginning in the first bar.”¹⁰³

INTRODUZIONE.
Adagio molto.

Example 1. Beethoven, *Piano Sonata No. 21 in C major, Op. 53, Second Movement*, mm. 1-10.

Riemann identifies four parts in the opening section: mm. 1-2, mm. 3-4, mm. 5-7, mm. 8-9 (see Example 1). Each of the first two parts (mm. 1-2 and mm. 3-4) is initially described by Riemann as

¹⁰² William Caplin 2002, 688-691 and David Bernstein 2002, 799-800 both discuss Riemann’s analysis of this passage. I am indebted to their accounts. See also Smith 1986.

¹⁰³ Riemann 1920, 30.

moving from subdominant to dominant to tonic, with local cadences in E major and B major, respectively, as shown in Figure 1.¹⁰⁴ (This is not his final analysis; see the next paragraph).

$$\begin{array}{ccc}
 (\overset{\circ}{S} - D_3^7 - T; \dots \overset{\circ}{S} - D^7 - T; \dots \overset{\circ}{S} - D - T; & & \\
 \text{(E-dur)} & \text{(H-dur)} & \text{(F-dur)} \\
 & \text{Sp} - D - T & \\
 & \text{(F-dur)} &
 \end{array}$$

Figure 1. Riemann's preliminary analysis of the opening cadences.

Interpreting the first chord (F-A-C) of the movement as having a subdominant function may seem strained, given that the movement is in F major. But Riemann argues that the actively synthesizing mind of the listener understands the opening four measures—albeit only locally—as two S-D-T progressions in E major and B major, echoing the progressions to G and to F at the beginning of the first movement. Thus, instead of being understood as a tonic, the first chord (F-A-C) is ascribed a subdominant function; viz., it is interpreted as a leading-tone change (from E to F) of the minor subdominant (A-C-E) in E major.¹⁰⁵ This may seem a stretch, but according to Riemann, the listener is forced into this understanding, at least locally, given the motivic context and the echo of the opening movement. This subdominant *Leittonwechselklang* prepares a chord understood as a dominant seventh in E major (B-D#-F#-A) with a flatted fifth and missing root, which then resolves to a tonic E major chord; the first two bars are then heard as S-D-T in E major. Thus, the synthesizing mind is always actively engaged in ascribing functions to chords based on their context—on the fly, as it were.

This 'on-the-fly' analysis fails, however, when the broader context is taken into account. Because the opening and closing chords of the introductory period (mm. 1-9) are both F-A-C, and

¹⁰⁴ *Ibid.*, 30.

¹⁰⁵ For an overview of Riemann's function system, see Rehding 2003, 188-189. The function in question is labeled as 'S' with a '>' strikethrough (Riemann 1920, 30).

given the chromatically descending bassline from F2-C2 (mm. 1-8), the “whole period is ultimately understandable only as a gigantic cadence [*Riesenkadenz*] in F-major,” with the first two cadences acting merely as detours.¹⁰⁶

Introduzione.
I. *Adagio molto.*

The figure shows three staves of musical notation. Below the staves, Riemann's functional analysis is indicated by chord symbols and numbers in parentheses. The first staff has annotations: T, =⁰Sp, D, (2), D, ..³=⁰S. The second staff has: .. D, (4), D, ..³=⁰D, D³, (6). The third staff has: D, .. (7)Tp, .. (7)Sp, D, (8)T, ..

Figure 2. Riemann's global functional analysis.

As Figure 2 shows¹⁰⁷, Riemann's global functional analysis treats the chords of mm. 1-4 differently: for instance, he understands the E-G#-B chord in the second bar as a dominant in A-minor, approached by the F-A-C chord as a minor subdominant parallel and the F-A-D# chord as a dominant-of-the-dominant (or Italian sixth to us).¹⁰⁸ This new functional ascription, dominant in A minor, is much closer to how a modern theorist would interpret

¹⁰⁶ *Ibid.*, 31.

¹⁰⁷ *Ibid.*, 32.

¹⁰⁸ *Ibid.*, 32.

the passage than his original ascription (tonic in E major), and as with the local case, the synthesizing activity of the mind forces a particular understanding upon the listener.¹⁰⁹ Theorists today might argue that many understandings are often possible, but Riemann thought that a specific understanding stemmed inevitably from the logic of *Musikbören*. Nevertheless, for Riemann, logical synthesis in musical hearing is scale-dependent: the tonal function of chords depends on the active synthesis of moments over a particular range of time, i.e., on whether the relevant context is local or global.

The main lesson of the Waldstein example is that, in Riemann's work, tonal functions are not simply there in the music independent of the hearer. It is the continuous synthetic activity of the mind in musical hearing that, by organizing the sensory input, gives chords their respective functions. In the course of listening and in the course of analysis, the subject may ascribe different local and global functions to the same chord, as Riemann does for the opening bars of the second movement of the Waldstein sonata. In this light, Riemann's definition of harmonic theory as "[t]he theory of the meaning of harmonies (chords), i.e., the explanation of the thought-processes in musical hearing" is less surprising.¹¹⁰ The meaning of a chord is its function, and this meaning is inseparable from the thought-processes involved in musical experience. This does not mean that Riemann makes tonal functions dependent on the personal idiosyncrasies of the listener. Like Kant and Drobisch before him, he believed that it was only the spontaneous, logical activity of the subject that guaranteed objectivity. Although some modern theorists view analysis and listening as separate activities, the one objective and the other subjective, Riemann suggests that this separation is artificial.

The epistemological presuppositions of Riemann's theory of tonal functions, whatever we think of the theory itself, are still relevant today. For instance, Eytan Agmon suggests that "many modern accounts of traditional harmony [...] incorporate one

¹⁰⁹ I am indebted to Peter Kupfer for helping me understand how a modern theorist would understand this passage, and to Michael Callahan, for clarifying Riemann's analysis of this passage.

¹¹⁰ Riemann 1909, 568.

version or another of functionalism as an essential component.”¹¹¹ However, he defines harmonic functionalism as simply the claim that the triads I, IV, and V embody the essence of the functional categories of tonic, subdominant, and dominant. Although Agmon traces this view to Riemann, it is decidedly non-Riemannian, emphasizing only relations in the music and not the mental activity involved in experiencing those relations.¹¹² Agmon’s approach to functional harmony, however, is explicitly engaged with cognitive science and theories of perception, and it is here that the historical perspective is helpful. Too often, function theory is discussed, and linked directly to Riemann, without any sense of its epistemological underpinnings. As David Kopp has suggested, the term ‘function’ has “grown uncommonly vague through use.”¹¹³ The above reconstruction of Riemann’s epistemological position, and its implications for his theory of tonal functions, demonstrates one way in which a whole theory of musical experience can lie behind an apparently neutral analytical tool. And while we may not agree with the epistemology that undergirds Riemann’s functionalism, we now at least understand it, and can decide which parts of it to accept and which to reject. The lesson of Riemann’s work is that if we separate music psychology and cognitive science from harmonic analysis, the presuppositions of that analysis retreat into the background and disappear. Riemann shows us why how we analyze music always relates in important ways to how we hear music. His great epistemological contribution is still fascinating: tonal functions and harmonic relations do not sit passively on the page; they are determined by the active spontaneity of the human mind.

¹¹¹ Agmon 1995, 197.

¹¹² David Kopp 1995 gives a nice overview of various historical definitions of function, including Riemann’s, but does not mention the importance (for Riemann) of mental activity.

¹¹³ *Ibid.*, 1.

Conclusion

This paper can be read as a historical response to the demand of Cook for more reflection on the “epistemological underpinnings” of music theory.¹¹⁴ It is often difficult, given the formal methods employed by many modern theorists, to unearth their basic presuppositions. Thus, I have taken a historical approach. In exploring the philosophical basis of the harmonic theories of our predecessors, theories that have importantly shaped our own, we can open up a debate about the very nature of the analytical categories we employ. This is not to say that all theorists who discuss harmonic function today are automatically dependent on Riemann’s own epistemological views. But with these views as a starting point, we can discuss what exactly modern theorists mean by the term ‘function.’¹¹⁵ Many theorists believe that the listener plays an active role in musical hearing, and that chords have different functions in different contexts; but they do not necessarily connect these two beliefs. Riemann shows us that this connection is possible, and in fact natural. Musical hearing and musical analysis are not separate processes, and reflection on the nature of *Musikhören* through history allows us to frame questions about the different roles of brain, mind, history, and culture in the reception of music, providing conceptual tools for those who are attempting to link cognitive science and music.¹¹⁶ And even Riemann, in the end, acknowledged the influence of history:

The continuous operation of the productive as well as the receptive artistic imagination with naturally given and historically changing categories, which strips artistic creation of any arbitrariness and turns it into a logically necessary obligation [*Müssen*], is a fact of our psychical life [...].¹¹⁷

¹¹⁴ Cook 2002.

¹¹⁵ Complicating the story in Kopp 1995.

¹¹⁶ Such as Zbikowski 2002.

¹¹⁷ Riemann 1919, iii.

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