Abstract Hardy inequalities and monotone functions

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We consider an abstract formulation of Hardy operators, exploring their boundedness from an L^p space to an L^q space over general measures with no fixed order on the elements for $p \in [1, \infty)$ and $q = (0, \infty)$. These abstract Hardy operators offer a unified approach to various forms of Hardy inequalities, while also inducing a highly flexible notion of order.

We will define monotone functions in general measure spaces to be compatible with this notion of order. We will find extensions of certain functional analysis constructions involving monotone functions previously confined to the real line. As an application of these tools, we present new results concerning necessary and sufficient conditions for a two-weight Hardy inequality to gold on metric measure spaces.

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