

Split-plot Factorial Multivariate Analysis of Variance

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Purpose

To assess the effects of two or more factors where at least one of the factors is based on Between Subject variation and at least one is based on Within Subject variation. It provides multivariate (Pillais' Trace, Wilk's Lambda, Hotelling's Trace, and Roy's Largest Root) and univariate (F-ratios) tests of the effects of each main effect and each interaction effect. Multivariate analysis of variance has traditionally been employed to control Type I error at nominal rates (i.e., .05) for each effect assessed but this has been found to be inappropriate and alternatives have been proposed (see Gardner & Tremblay, 2006 for a discussion). It also maximizes the possibility of identifying multivariate effects by computing weighted aggregates of the dependent measures that maximally distinguish among levels of the treatment factor(s).

Applications

Used in any analysis of variance context where at least one factor is based on independent samples of respondents (Between Subjects) and one is based on repeated observations on the same subjects (Within Subjects) or on randomized blocks of respondents (i.e., Within Blocks). Examples of the Between Subjects factor might be different treatment conditions (i.e., Treatment1, Treatment 2, Control) or different levels of a categorical individual difference variable (i.e., Gender, Socio-economic Status). Examples of the Within Subjects Factor might be Trials or repeated testings on an outcome variable (pretest, follow-up, post-followup, etc.).

General Rationale

The general rationale is identical to that of the completely randomized factorial multivariate analysis of variance. Weighted aggregates are formed to maximize the effects of the source of variance of interest. The sole difference is in the form of the determinantal equations.

Basic Mathematics

Assuming Factor A is the Between Subjects factor, the Determinantal Equation for the Main Effect of A is:

$$|SS_{S/A}^{-1} SS_A - \lambda I| = 0$$

To determine the weights for each λ , solve the following:

$$[SS_{S/A}^{-1} SS_A - \lambda I][w] = 0$$

with the side condition that:

$$[w^T] MS_{S/A} [w] = 1$$

The corresponding formulae for the multivariate tests of B and AB take the same form,

except that the error term involves the interaction of subjects with the repeated measures factor (i.e., BS/A). For example, the three equations for the AB interaction are:

$$|SS_{BS/A}^{-1} SS_{AB} - \lambda I| = 0$$

$$[SS_{BS/A}^{-1} SS_{AB} - \lambda I][w] = 0$$

$$[w^T] MS_{BS/A} [w] = 1$$

Assumptions

The assumptions are comparable to those presented for the completely randomized factorial multivariate analysis of variance except that it is necessary to consider the added distinction between Within Subjects and Between Subjects effects. There is one further complication. Recall that for the univariate Split-plot factorial design, it is possible to evaluate the Within Subjects effects in terms of multivariate or univariate tests. This can also apply in this context. That is, with one repeated measures factor and more than one dependent variable, this could be considered a doubly multivariate design where both the repeated measures and the various dependent variables could both be considered as multiple dependent measures. This results in multiple tests of significance in the SPSS output. For this presentation, we will focus on the multivariate analysis of variance where the dependent variables are considered the dependent variables, and the repeated measures are treated as levels of a factor. With this restriction, we can distinguish between those assumptions referring to Between Subject Effects and those referring to Within Subject Effects.

1. Between Subject Effects.

A. Independent Observations. Respondents are randomly assigned to the Between Subjects factor, or at least are considered random samples from the population of interest. It is also assumed that scores on the dependent variable are not influenced by other individuals in the treatment condition.

B. Multivariate Normality. This is as discussed under the completely randomized factorial design except that it refers to the distribution of respondents' mean scores for the various groups.

C. Equivalence of Covariance Matrices. This assumption is the same as that discussed under the completely randomized design. One aspect of it is the homogeneity of variance necessary for the univariate tests. In this case, it is that the variances of the subjects' mean scores in each treatment condition are all representative of the same population variance. SPSS Repeated Measures does not evaluate this hypothesis but instead tests homogeneity of variances between groups for each level of the repeated measures factor and each dependent variable.

D. Null hypothesis with respect to the A factor.

2. Within Subjects Effects

A. Independent Observations. As above.

B. Multivariate Normality. As discussed before in the completely randomized design multivariate analysis of variance. In this case, we are considering the distributions of the

different variables within each level of the repeated measures factor.

C. Equivalence of the Covariance Matrices and Circularity of the Pooled Covariance Matrix. For the univariate tests of the B and AB effects, there is the added assumption that the covariance matrices for each dependent variable are equivalent in the various levels of the Between Subjects factor, and that the pooled covariance matrix for each dependent variable satisfies the assumption of circularity.

D. Null Hypotheses. The multivariate and univariate null hypotheses of the B and AB sources of variance.

Running SPSS GLM Repeated Measures

Consider the following data set.

GROUP	T1V1	T1V2	T2V1	T2V2	T3V1	T3V2
1.00	27.00	29.00	29.00	29.00	29.00	27.00
1.00	30.00	33.00	29.00	32.00	30.00	29.00
1.00	34.00	34.00	33.00	36.00	33.00	33.00
1.00	32.00	35.00	30.00	33.00	30.00	28.00
1.00	33.00	30.00	32.00	34.00	30.00	29.00
1.00	28.00	32.00	30.00	32.00	30.00	31.00
2.00	26.00	29.00	28.00	33.00	27.00	31.00
2.00	27.00	28.00	33.00	35.00	26.00	32.00
2.00	25.00	31.00	34.00	36.00	25.00	33.00
2.00	28.00	36.00	31.00	32.00	27.00	31.00
2.00	29.00	37.00	30.00	37.00	26.00	35.00
2.00	31.00	25.00	29.00	35.00	28.00	31.00

This data file presents two dependent variables (V1 and V2) assessed over three time periods for two treatment groups. When using SPSS GLM Repeated Measures, the Within Subjects factor is defined as Time with 3 levels. Under Measures, two are defined var1 and var2. When defining the variables, T1V1, T2V1, and T3V1 are entered as the 3 levels of the repeated measures on the first variable, and the remaining three measures are defined as the 3 levels for the second variable. Group is defined as the Between Groups factor. A number of options are available. Though not shown here, the most useful are descriptives, power, and homogeneity of variance.

The first table of output to be considered is labelled “Multivariate Tests” and for these data appears as follows:

Multivariate Tests^b

Effect			Value	F	Hypothesis df	Error df	Sig.
Between Subjects	Intercept	Pillai's Trace	.998	2628.601 ^a	2.000	9.000	.000
		Wilks' Lambda	.002	2628.601 ^a	2.000	9.000	.000
		Hotelling's Trace	584.134	2628.601 ^a	2.000	9.000	.000
		Roy's Largest Root	584.134	2628.601 ^a	2.000	9.000	.000
	GROUP	Pillai's Trace	.587	6.403 ^a	2.000	9.000	.019
		Wilks' Lambda	.413	6.403 ^a	2.000	9.000	.019
		Hotelling's Trace	1.423	6.403 ^a	2.000	9.000	.019
		Roy's Largest Root	1.423	6.403 ^a	2.000	9.000	.019
Within Subjects	TIME	Pillai's Trace	.917	19.304 ^a	4.000	7.000	.001
		Wilks' Lambda	.083	19.304 ^a	4.000	7.000	.001
		Hotelling's Trace	11.031	19.304 ^a	4.000	7.000	.001
		Roy's Largest Root	11.031	19.304 ^a	4.000	7.000	.001
	TIME * GROUP	Pillai's Trace	.756	5.408 ^a	4.000	7.000	.026
		Wilks' Lambda	.244	5.408 ^a	4.000	7.000	.026
		Hotelling's Trace	3.090	5.408 ^a	4.000	7.000	.026
		Roy's Largest Root	3.090	5.408 ^a	4.000	7.000	.026

a. Exact statistic

b.

Design: Intercept+GROUP

Within Subjects Design: TIME

The Between Subjects Test of Group is the multivariate test of the Between Subjects factor. This effect is significant, $F(2,9) = 6.403$, $p < .019$, indicating that, averaging over the three levels of the repeated measures factor, the two levels of the Between Subjects factor differ significantly from each other on the weighted aggregate of the two variables. The tests of Within Subjects effects are multivariate tests of the univariate effects treating the repeated measures of the two variables as 6 dependent variables. Generally, these last two tests of significance are not considered. They are not the multivariate tests of B and AB as discussed above.

Another table is labelled "Multivariate" and presents tests of significance of the multivariate effects of Time and Time X Group. This describes the multivariate effects of the repeated measures factor and the interaction of this factor with Group. In this case, both effects are significant, suggesting there are multivariate effects for both Time and the Time X Group interaction.

Multivariate^{c,d}

Within Subjects Effect		Value	F	Hypothesis df	Error df	Sig.
TIME	Pillai's Trace	.518	3.492	4.000	40.000	.015
	Wilks' Lambda	.483	4.166 ^a	4.000	38.000	.007
	Hotelling's Trace	1.068	4.805	4.000	36.000	.003
	Roy's Largest Root	1.066	10.663 ^b	2.000	20.000	.001
TIME * GROUP	Pillai's Trace	.538	3.678	4.000	40.000	.012
	Wilks' Lambda	.530	3.544 ^a	4.000	38.000	.015
	Hotelling's Trace	.757	3.405	4.000	36.000	.018
	Roy's Largest Root	.499	4.988 ^b	2.000	20.000	.017

- a. Exact statistic
- b. The statistic is an upper bound on F that yields a lower bound on the significance level.
- c.
 - Design: Intercept+GROUP
 - Within Subjects Design: TIME
- d. Tests are based on averaged variables.

Mauchly's test of sphericity, and the epsilon multipliers are applicable to the *univariate* repeated measures tests of the Time and Time X Groups within Subject effects.

Mauchly's Test of Sphericity

Within Subjects Effect	Measure	Mauchly's W	Approx. Chi-Square	df	Sig.	Epsilon ^a		
						Greenhouse-Geisser	Huynh-Feldt	Lower-bound
TIME	VAR1	.711	3.073	2	.215	.776	.983	.500
	VAR2	.299	10.866	2	.004	.588	.686	.500

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.

- a. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are displayed in the Test Within-Subjects Effects table.
- b.
 - Design: Intercept+GROUP
 - Within Subjects Design: TIME

The following table labelled “Univariate Tests” presents the univariate tests of Time and the Time X Group interaction for each of the dependent variables. Applying the Greenhouse-Geisser epsilon value, significant effects were obtained for Time on var1 and var2 and a significant interaction of Time by Group for var1, but not var2. As a consequence, it would be meaningful to perform post-hoc tests on the main effects of Time on both var1 and var2, and post-hoc simple main effects or interaction effects on var1.

Univariate Tests

Source	Measure		Type III Sum of Squares	df	Mean Square	F	Sig.
TIME	VAR1	Sphericity Assumed	31.500	2	15.750	5.250	.015
		Greenhouse-Geisser	31.500	1.551	20.306	5.250	.024
		Huynh-Feldt	31.500	1.967	16.018	5.250	.015
		Lower-bound	31.500	1.000	31.500	5.250	.045
	VAR2	Sphericity Assumed	51.722	2	25.861	5.595	.012
		Greenhouse-Geisser	51.722	1.176	43.990	5.595	.032
		Huynh-Feldt	51.722	1.372	37.691	5.595	.025
		Lower-bound	51.722	1.000	51.722	5.595	.040
TIME * GROUP	VAR1	Sphericity Assumed	29.167	2	14.583	4.861	.019
		Greenhouse-Geisser	29.167	1.551	18.802	4.861	.030
		Huynh-Feldt	29.167	1.967	14.831	4.861	.020
		Lower-bound	29.167	1.000	29.167	4.861	.052
	VAR2	Sphericity Assumed	25.167	2	12.583	2.722	.090
		Greenhouse-Geisser	25.167	1.176	21.404	2.722	.122
		Huynh-Feldt	25.167	1.372	18.339	2.722	.114
		Lower-bound	25.167	1.000	25.167	2.722	.130
Error(TIME)	VAR1	Sphericity Assumed	60.000	20	3.000		
		Greenhouse-Geisser	60.000	15.512	3.868		
		Huynh-Feldt	60.000	19.665	3.051		
		Lower-bound	60.000	10.000	6.000		
	VAR2	Sphericity Assumed	92.444	20	4.622		
		Greenhouse-Geisser	92.444	11.758	7.862		
		Huynh-Feldt	92.444	13.723	6.737		
		Lower-bound	92.444	10.000	9.244		

The tests of homogeneity of variance are not presented here. If you selected this option two tests are provided, Box’s test of equality of the error covariance matrices (if there is sufficient data), and Levene’s test of homogeneity of variance. Levene’s test is applied to each dependent variable under each level of Time. This does not evaluate the homogeneity of variance assumption for the Group effects for each dependent variable, though it is reasonable to assume that if either variable satisfies the assumption for each level of the treatment condition, the assumption would apply to mean of that variable.

The final table presents the Between Subjects effects at the univariate level. In this analysis, the univariate effect of Group is significant only for Var1. The multivariate effect of Group was significant, thus it is meaningful to investigate the group effect for Var1.

Tests of Between-Subjects Effects

Transformed Variable: Average

Source	Measure	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	VAR1	31152.250	1	31152.250	5386.556	.000
	VAR2	36928.028	1	36928.028	2945.080	.000
GROUP	VAR1	42.250	1	42.250	7.305	.022
	VAR2	12.250	1	12.250	.977	.346
Error	VAR1	57.833	10	5.783		
	VAR2	125.389	10	12.539		

Steps to Follow in Interpreting the Results

Consider the multivariate effects. In the present case, there are significant multivariate effects for Group, Time and Group X Time, permitting evaluation of each of the univariate effects. When attention is directed toward the univariate effects, main effects are observed for Group on variable 1, for Time on variables 1 and 2, and the Time x Group interaction on variable 2. Thus, attention should be directed to post-hoc simple main effects or interaction effects on variable 2, and main effects of Group and Time on variable 1, taking into account the definition of Type I error rate.

Reference

Gardner, R. C. & Tremblay, P.F. (2006). *Essentials of Data Analysis: Statistics and Computer Applications*. London, ON: The University of Western Ontario.