Some Remarks on Current Mathematical Practice

by John Bell

1. Contemporary mathematics confronts the spectator with a formidable array of results and techniques, most of which appear to have little or no connection with reality. A point has been reached where mathematics, of necessity abstract, has become so arcane that it is difficult even for practising mathematicians to see where it is going.* As in other areas of scientific activity, production for production's sake has become the mathematician's chief aim, with the result that technical papers of an ever more mystifying nature are proliferating at an enormous rate. Underlying this state of affairs is a formalist ideology which, by encouraging the mathematician to assume a "neutral" attitude toward his activity and to devote himself exclusively to the imperative of production, has obscured the relationship between mathematics and reality and stifled work in the foundations and philosophy of mathematics.

2. The major part of research activity in mathematics is devoted to proving theorems within the established mathematical framework which has made its appearance over the past three or four decades. This framework has three major features:

(i) its basic constituents are officially regarded as being of a purely formal character, i.e. meaningless in themselves;

(ii) it is sufficiently flexible to allow for the development of increasingly refined techniques within it;

(iii) it is, ostensibly, broad enough to enable all current mathematical notions to be expressed within it.

In view of (i), the question of the meaning or use of the notions expressed and the results proved within this framework becomes an external problem, hence usually ignored. Once the questions of meaning and use have been removed from the scene, only internal technical criteria remain for determining the import of a mathematical result. This has the effect of making mathematics immune to criticism from the outside. Furthermore, this confining of attention to purely internal, technical aspects of the framework, together with the reinforcement provided by features (ii) and (iii), creates the impression that is absolute. If a problem is insoluble within the

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* Physicists are frequently critical of mathematical obscurity. A French Nobel laureate in physics declared recently that the unnecessarily exacting mathematical requirements imposed on physics students in French universities was scaring them off not only mathematics, but physics as well!

framework, it is customary to regard it as absolutely insoluble* (the continuum problem in set theory, for instance). The idea of searching outside it for inspiration is regarded as at best impious, and at worst unprofessional. Activity within the framework itself boils down to a purely operational procedure applied to its constituents. Thus mathematical activity itself comes to be identified with operations within this framework, and mathematical concepts with its unchanging objects. In particular, the introduction of new mathematical concepts becomes a matter of reducing them to notions already present in the framework: if such a reduction cannot be effected, the concept is rejected.

3. Under these conditions, mathematics tends to be viewed as a bundle of technical operations performed on a collection of fixed formal objects, from which all intrinsic meaning has been extracted. This in turn induces a shift in emphasis from content to production, from substance to technique. In this respect modern mathematics resembles the world of mass technology, which involves the production and manipulation of "neutralized" objects (including human beings) within an established economic structure. It is certainly true that mathematics provides an excellent system for expressing technological manipulations in abstract "objective" form. For instance, subjects like military logistics and management "science" become both efficient and respectable when clothed in mathematical formalism.** Establishment economics, with its expansionist goals and its plethora of "models" becomes merely another chapter in the growth of "neutral" mathematics. In cases like these the tremendous authority of mathematics has the effect of disguising the true nature of the subjects formalized.

4. The abstract-operational character of modern mathematics*** causes it to assume the form of a kind of rarefied technology, so that the goal of mathematics becomes the technological goal of production for its own sake. The struggle to produce forces mathematicians to become increasingly competitive: in order to survive as a mathematician, one must produce more results than one's competitors. "Publish or perish" becomes the order of the day. The competitive struggle is rendered all the more efficient by the elimination of embarrassing questions of meaning, purpose, etc. Now,

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* Compare this with the orthodox interpretation of the Heisenberg uncertainty principle in quantum theory, which is believed to provide an "absolute" refutation of causality in the small. For a criticism of this interpretation, see D. Bohm, Causality and Chance in Modern Physics, pp. 94-103.

** No doubt the war analysts at the Pentagon would be delighted if World War III could be expressed in terms of, say, non-commutative semi-groups!

*** It is interesting to note that certain philosophies, structuralism for instance, which assume contemporary mathematics as a basic descriptive framework, have a distinctly operational character.

the narrower the field of competition, the fewer the techniques one has to master in order to succeed (and the fewer the competitors), so a tendency to specialize appears. (Of course, I do not mean to claim that the competitive struggle is the sole reason for specialization, only that it is an important factor in its emergence.) As the field of specialization narrows, its connection with the whole becomes less and less evident, so that the specialized activity becomes increasingly esoteric. But the imperative of technical production places both esoteric specialization and the "expert" practitioner entirely beyond criticism, so much so that many mathematicians profess to be ignorant of the meaning of the word "esotericism" when applied to their own activities!

5. Professional esotericism in mathematics has also had an adverse effect on its teaching. In fact mathematics is often taught in an isolationist fashion,* with great respect accorded to the minutiae of rigour, but little or none to the connection between mathematics and reality, or to the historical genesis of mathematical ideas, or even to applications. The mathematics student often leaves the lecture theatre completely mystified, and when he succeeds in gaining some understanding of the subject, he has the demoralizing impression that its creators must be intellectual supermen, of unchallengable authority. The situation is worse still for the philosopher of mathematics, who is frequently regarded as a kind of failed mathematician. Under these conditions it is hardly surprising that the philosophy of mathematics is regarded by many mathematicians as a "dead" subject, a closed chapter in the history of mathematics, and in any case a subject distinctly inferior to mathematics itself. Mathematics is self-justifying, they say, so why bother to develop a philosophy for it?

6. Despite the supposedly "universal" character of the established mathematical framework, there is no doubt that certain important internal problems (the continuum problem for example) are incapable of resolution within it as it now stands. One might expect this to lead mathematicians to abandon their formalist viewpoint in favour of a broader philosophy of mathematics but this has not so far occurred.** As far as the formalist is concerned, the independence of a certain proposition P with respect to a formal system merely means that one can assume P or its negation at will, on a purely pragmatic basis. Thus, again, the question of which assertions to adopt as axioms assumes an operational form: if P yields more consequences than its negation, take P as an axiom. The problem of the "truth" of P (i.e. its relation to an (informal) subject-matter)

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* See M. Thomas and L. Hodgkin, Ideology and Mathematical Education for a fuller analysis of the situation in mathematical teaching.

** Cf. Cohen's article in the Proceedings of the Los Angeles Colloquium on Set Theory, in which he (reluctantly) casts his vote for formalism.

simply does not arise for the formalist. Even as negative a result as the Gödel incompleteness theorem is treated in this exclusively operational fashion.* The logician uses it, quite rightly, to produce non-standard models of arithmetic, but ignores its negative implications for the established framework.

7. In this brief essay I have touched on some of the aspects of contemporary mathematical practice which in my view contribute strongly to its present isolation from other spheres of human activity. Further analysis of these and related problems may yield a method for breaking down this isolation and a conception of the nature of mathematical activity which takes more account of its connection with reality than does the formalist view which now prevails.

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* Despite the incalculable impact of Gödel's work on technical mathematical logic his ideas on the philosophy of mathematics have been almost completely ignored by logicians (with one or two conspicuous exceptions). Similar remarks apply to Cantor's philosophy of mathematics.