SOME REMARKS ON CURRENT MATHEMATICAL PRACTICE

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1. Contemporary mathematics presents the spectator with a formidable collection of results and techniques, most of which appear to lack any connection with objective reality whatsoever. Indeed, a point has been reached where mathematics, of necessity *abstract,* has become so *arcane* that it is difficult even for practicing mathematicians to see where it is headed[1]. As in other areas of scientific activity, production for production's sake has become the mathematician's chief aim, with the result that technical papers of an ever more mystifying nature are proliferating at an ever-increasing rate. Underlying this state of affairs is a formalist ideology which, by encouraging mathematicians to assume a "neutral" attitude to their activity and to subordinate themselves to the imperative of production, has obscured the relationship between mathematics and objective reality and stifled work in the foundations and philosophy of mathematics.

2. The greater part of research activity in turn mathematics is, of course, devoted to the proving of theorems within the established mathematical framework which has emerged in the present century. This framework has three principal features:

(i) its basic constituents are taken to possess a purely *formal* character, i.e. are meaningless in themselves;

(ii) its flexibility accommodates the development of increasingly refined techniques;

(iii) ostensibly, it allows all current mathematical concepts to be expressed within it.

In view of feature (i), the questions of the *meaning* and *use* of the concepts expressed and the results established within this framework become external issues, and so are usually ignored. With the banishment of such questions, internal, technical criteria alone remain for determining the import of a mathematical proposition. This has the effect of rendering mathematics, along with the practice of mathematicians, immune to criticism from the outside. Moreover, the confining of attention to the framework's purely internal, technical features, reinforced by features (ii) and (iii), creates the illusion that it is *absolute.* The insolubility of a problem - that of the cardinality of the continuum, for instance - within the framework thus becomes identified with absolute insolubility[2]. The idea of searching outside it for inspiration is treated as impious and, worse, unprofessional. Activity within the framework shrinks to purely *operational* procedures, and the very practice of mathematics comes to be identified with these procedures. Mathematical concepts themselves are

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[1] Physicists are often critical of mathematical obscurity. A French Nobel Laureate in physics [Alfred Kastler] declared recently that that the unnecessarily exacting mathematical requirements imposed on physics students in French universities was putting them off not just mathematics, but physics as well!

[2] Compare this with the orthodox interpretation of the Heisenberg uncertainty principle in quantum theory, which is believed to provide an "absolute" refutation of causality in the microworld. For a criticism of this view, see D. Bohm, *Causality and Chance in Modern Physics,* pp. 94-103

assimilated to the framework's formal constituents. In particular, the introduction of a new mathematical concept becomes a matter of reducing it to these constituents. If this cannot be done, the concept is rejected.

3. Mathematics is thereby reduced to a bundle of technical operations performed on a collection of fixed formal "neutral" objects from which all intrinsic meaning has been drained. This in turn induces a shift in emphasis from *content* to *production*, from *substance* to *technique.* In this respect contemporary mathematics resembles the world of mass technology, central to which is the production and manipulation of "neutralized" objects (including human beings) within an established *economic* structure. The language of mathematics furnishes the ideal medium for the presentation of technological procedures in abstract "objective" form. Subjects such as military logistics and management "science" achieve both efficiency and respectability when clothed in mathematical formalism[3]. Establishment economics, with its expansionist goals, its obsession with "growth" and plethora of "models" becomes just another chapter in the development of "neutral" mathematics. In cases such as these the unrivalled authority of mathematics has the effect of disguising the true nature of the subjects in question.

4. The abstract-operational character of contemporary mathematics[4] has the effect of transforming it into a kind of rarefied technology, in which the objective is production for its own sake. The resulting struggle to produce forces mathematicians to become more competitive: in order to survive as a mathematician, one must, so to speak, "outprove" one's competitors. "Publish or perish" becomes the order of the day. The elimination of questions of the meaning and purpose of mathematical activity intensifies the competitive struggle. Now, the narrower the field of competition, the fewer the techniques one has to master to succeed (and the fewer one's competitors), so a tendency to *specialize* begins to emerge. As the field of specialization narrows, and its relationship to the whole becomes less and less evident, the specialized activity becomes increasingly *esoteric.* The imperative of technical production places both esoteric specialization and the "expert" practitioner entirely beyond criticism, so much so that mathematicians typically profess their ignorance of the meaning of the word "esotericism" when it is applied to their own professional activity.

5. Professional esotericism among mathematicians has had an adverse effect on the teaching of mathematics. Mathematics is, by and large, taught in an isolationist fashion[5]. Attention to the minutiae of rigour dominates the teaching of mathematics to such a degree as virtually to exclude all mention of the relationship between mathematics and objective reality, the historical origin of mathematical concepts, and even the applicability of mathematics. The student of mathematics typically leaves the classroom or lecture theatre completely mystified, and when he or she succeeds in gaining some grasp of the subject, has at the same time received the demoralizing impression that the creators of mathematics must be intellectual supermen, of unchallengeable authority. The situation is worse still for the philosopher of mathematics, who is looked down on as a kind of "failed" mathematician. It is then hardly surprising that the philosophy of mathematics is typically

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[3] The war analysts at the Pentagon would doubtless be delighted if World War III could be expressed in terms of, say, noncommutative semigroups!

[4] It is interesting to note that certain philosophies, structuralism for instance, have a distinctly operational character.

[5] See M. Thomas and L. Hodgkin *Ideology and Mathematics Education* for a fuller analysis of the current situation in mathematics teaching.

regarded by professional mathematicians as a "dead" subject, at best a closed chapter in the history of mathematics, and in any case a pursuit distinctly inferior to that of mathematics itself. Mathematics is self-justifying, they confidently proclaim; it needs no philosophical input.

6. Despite the supposedly "universal" character of the established mathematical framework, there is no doubt that certain important *internal* problems - such as the problem of the cardinality of the continuum, mentioned above - cannot be resolved within it as it now stands. One might expect this fact to induce mathematicians to abandon the formalist viewpoint held by many but this has not so far occurred[6]. By "formalist" here I mean the position that the formal independence of a proposition justifies its postulation, or that of its negation at will, on a purely pragmatic basis. Thus the question of whether to adopt as an axiom a given independent assertion or its negation assumes an operational form: if the assertion has more fruitful consequences than its negation, it should be adopted; if not, its negation should be adopted. The question of the "truth" of the assertion - i.e. its relation to an informal or objective subject matter simply does not arise for the formalist in this sense. Even as profound a result as the Gödel incompleteness theorem is treated in this exclusively operational fashion[7]. Mathematical logicians use Gödel's theorem to construct nonstandard models of arithmetic, but, by and large, ignore its implications for the established framework.

7. In this brief essay I have touched on certain aspects of contemporary mathematical practice which isolate it from other spheres of human activity. Further analysis along these lines may provide a means of breaking down this isolation, and at the same time, aid in the emergence of an account of the nature of mathematical activity in which the relationship between mathematics and objective reality is assigned its proper place.

Afterword

This is a lightly edited version of an article published in 1972 in the *Proceedings of the Bertrand Russell Memorial Logic Conference,* edited by myself, Julian Cole, Graham Priest, and Alan Slomson. The background to this conference is of some historical interest. It was organized by a small group of logicians, Max Dickmann, Moshe Machover, Alan Slomson, Yoshindo Suzuki , George Wilmers and myself. We were opposed to the military financing of scientific conferences, and in particular to the funding by NATO of conferences in mathematical logic. During the 1960s a number of British logic conferences had received funding from NATO, thus becoming officially identified as "NATO Advanced Study Institutes". The funding of scientific conferences by military organizations such as NATO seems to have gone more or less unquestioned until in 1969 a public protest against such financing was mounted at the NATO supported logic conference held in Manchester. The resulting declaration, which concluded with the phrase "we believe that scientific conferences should not be linked with organizations of this [i.e., Nato's] character" attracted nearly 40 signatures. But this protest was ignored, and early in 1971 it emerged that the organizers of the logic conference to be held in Cambridge that summer had secured NATO funding for it.

[6] Cf. Paul Cohen's 1971 article *Comments on the Foundations of Set Theory*, in which he casts his vote for formalism, if with some reluctance.

[7] Despite the incalculable impact of Gödel's work on technical mathematical logic, his ideas on the philosophy of mathematics have been largely ignored by logicians (with one or two conspicuous exceptions). Ditto for Cantor.

Accordingly our group decided to launch a stronger protest. We thought that maximum impact would be achieved by staging a counter-conference timed to coincide with the Cambridge meeting. In promulgating our meeting we insisted that mathematicians should take seriously the social implications of their activity and that accepting money from military bodies such as NATO is intellectually and morally incompatible with this aim.

We thought it would be natural to dedicate the conference to the memory of Bertrand Russell, who had died, at the age of 98, the previous year. It was felt that Russell, old radical that he was, would have been sympathetic with the anti- military aims of our gatherimg. We approached the Russell Foundation and received its support.

The conference duly went ahead and was held in Uldum, Denmark in August 1971. Of two weeks duration, it attracted 50 or so participants, including the great mathematician Alexander Grothendieck, and all agreed that it was a great success. Its mix of mathematics and socio-political themes was likely unique. The Russell Conference was also modestly successful in achieving our acknowledged goal of preventing future NATO financing of logic conferences: for the next seven years no applications were made for NATO money by logicians.

Let me conclude with some reflections on the essay itself. When it was written, nearly half a century ago, - I was 27 at the time – I was enthralled, along with many young *gauchistes,* with Herbert Marcuse's book *One-Dimensional Man.* Anyone acquainted with that work will recognize the influence it had on both form and content of my essay. I'm happy to acknowledge that influence, since I still find Marcuse's excoriating criticism of mass technologized society exciting and as relevant as it was half a century ago. As for the essay itself, while some of the claims I made now seem to me somewhat exaggerated, the products of youthful radicalism, I believe that its central points remain valid. Witness, for example, the present lamentable state of British universities, in which the imperative of production reigns supreme!

Still, time and tide waits for no one, and it is inevitable that both external conditions and certain of the personal views I then held on the nature of mathematics have been subject to change. One positive external development has been the flowering of work in the philosophy of mathematics, in particular with focussed attention now being paid to the philosophical perspectives of great mathematicians such as Cantor, Poincaré, Gödel, Brouwer, and Weyl. Foundational work in mathematics has been enlivened by the emergence of topos theory and new forms of type theory. The amazing advances in computer science, which have transformed our everyday lives almost beyond recognition, have also permeated mathematics. Not all the effects of the computer revolution can be said to be positive, but through that revolution the isolation of mathematics has surely been lessened.

Regarding my personal views on the nature of mathematics, at the time my essay was written I was a Platonist - and the tenor of my essay clearly indicates this - but I later grew out of it. (In particular, I no longer believe that there is an objectively determined cardinality for the continuum, as implied in paragraph **2.)** I like to joke that Platonism (in mathematics at least) is mild malady of the intellect which, like measles, should be contracted early in life so as to obtain the necessary immunity to protect one's older self. I have come to agree with Saunders Mac Lane (and, I suppose, Wittgenstein) that pure mathematics is concerned with *rule* rather than *truth*: you may call this "formalism" if you like. (Formalism does at least have the merit of offering the weary ex-Platonist a welcome refuge.) A mathematical theorem is *correct*, that is, established by the correct application of rules laid down in advance as opposed to being *true.* Truth requires correspondence with facts, and I don't think there are pure mathematical "facts" as such, except in the sense of, e.g. "it is a fact that '2 + 2 = 4' is a theorem". But this not the case for applied mathematics, because applied mathematics is concerned with objective reality, and there certainly are facts about objective reality which involve mathematical terms such as numbers. Thus the proposition of applied mathematics "2 apples plus 2 apples yields 4 apples" is true, but the proposition "2 + 2 = 4" is simply correct. Of course, mathematicians (and others) routinely say that mathematical propositions are "true", but only the true Platonist would claim that this means more than mere correctness, that there is a pure mathematical "fact" making the proposition literally "true".

I regard mathematical Platonism as arising from a natural effort to supply pure mathematics with an objective subject matter, to enable it to transcend the practice of "mere" rule-following, in some sense to ennoble it. The origins of mathematics lie in its use in applications to the material world -arithmetic in counting and geometry in measuring size. In prehistory there was no "pure" mathematics (except, possibly, the use of counting in early religious ritual, and it is debatable whether even this can be considered "pure"). But it was found that the practice of mathematics, initially in arithmetic, later in geometry, could be distilled into a number of formal rules (such as the laws of arithmetic, or the postulates of geometry) which, when correctly followed, always resulted in true (or, in the case of geometry nearly true) assertions about the real world ("2 apples plus 2 apples yields 4 apples", " the sum of the angles in a triangle is equal to two right angles, etc.) To ancient cultures such as the Egyptians and Babylonians, if there was such a thing as pure mathematics, it was the body of these formal rules. But this changed with the ancient Greeks. The Pythagoreans, who introduced the term "mathematics" meaning "knowledge" asked themselves "what is mathematical knowledge actually about?". They thought that the rules of arithmetic, for example, had to be more than just formal rules, like the rules of a game, but in actuality express truths about an actual subject matter, in this case a realm of objects called numbers. Numbers are not material objects like apples or stones, but they are objects nonetheless and arithmetical propositions express truths about them. Later Plato extended this idea to geometry, supplying it with a nonmaterial, but objective subject mater of ideal geometric objects such as lines, circles and spheres. This is the source of "Platonism" in mathematics. Mathematical Platonists such as G.H. Hardy, Gödel, Roger Penrose and Alain Connes believe, like Plato and the Pythagoreans, that the purpose of mathematics is to establish truths about an independently existing, but nonmaterial realm of objects - for the last century or so these have been called "sets". At the time I wrote the essay I more or less accepted this doctrine, but I've since moved on.

Finally, I'd say that while my views on the philosophy of mathematics have changed since my essay was written, the socio-political convictions I expressed there remain the same. Mathematics may be formal in content but still must be recognized as the product of human activity in the real world. Like everybody else, mathematicians must acknowledge some degree of responsibility for how their work is used.

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