Questionnaire answered in 1986 by John L. Bell
Additional comments added 2002

1. What are your earliest recollections of interest in mathematics?

I recall teaching myself elementary algebra from a textbook at the age of 10. Before this I remember being fascinated by the symbols (e.g. ∫ and Σ) in my father's engineering handbooks. Later when I had learned the basic rules of algebra, I recall my puzzlement at the form of the derivative $\frac{dy}{dx}$: "Why", I wondered, "can't the *'d's* be cancelled in numerator and denominator?"

2. Were you directed towards mathematics by any immediate family influence?

My father, as an engineer, encouraged my youthful scientific interests (and expected me to become a physicist). But my later decision to take up mathematics was, I believe, largely my own.

3. Was your educational progress into mathematics conventional?

I am uncertain as to what the term "conventional" is supposed to signify here. If it means proceeding to mathematics via a scientific background, then the answer in my case is yes. On the other hand I had a somewhat unconventional education in the social sense, attending a variety of schools in different parts of the world, and being as a result largely self-taught.

4. Do you recall particular teachers or texts of seminal importance to you?

The Cambridge mathematician and physicist E.H. Linfoot, with whose family I stayed during school vacations between the ages of 13 and 16, had a large influence on me. Indeed I regarded him as my mentor. As for an important textbook, Kelley's "General Topology" played a major role in weaning me away from physics to mathematics. Bourbaki's *Topologie Generale* and *Theorie des Ensembles* was also influential. I must also mention George Gamow's popular classic *One, Two, Three,…Infinity*, which thrilled me as a kid.

5. In what specialist subjects were you initially trained (or self-educated)? In what areas do you now work?

Initially I studied physics (relativity theory – I attended Fred Hoyle's lecture course on cosmology in Cambridge in 1962). Then I turned to (general) topology, algebra, functional analysis, set theory, and mathematical logic. I now work in topos theory (category theory), set theory, Boolean algebras, philosophy of mathematics and foundations of physics (quantum logic).

<u>Since my official turn to philosophy in 1989, my research interests have come to include the philosophy of Hermann Weyl, constructive mathematics, Hilbert's epsilon symbol, infinitesimals and the continuum, Frege's theorem and type-reducing correspondences.</u>

6. Do you strive to understand new mathematics, not of immediate relevance to your work?

Since my mathematical interests have always tended to the synoptic, I certainly do strive to understand interesting new areas of mathematics, even if they are not of immediate relevance to my work.

7. Are you concerned to obtain a broad overview of mathematics or do you prefer to explore special topics in depth?

I regard the broad overview/special topics dichotomy as an instance of the "unity of opposites". Accordingly, although I am now primarily concerned with the broad overview, I an aware that this can't really be achieved without "exploring special topics in depth".

8. Do you distinguish sharply between original thinking and learning?

No: this is, again, in my view, an instance of the "unity of opposites". In my experience, the most effective way of learning some unfamiliar mathematics is to attempt to think creatively (or at least independently) as soon as possible. In this way, through a kind of creative assimilation, one overcomes the resistance to understanding presented by the unfamiliar, and especially by unfamiliar mathematics.

9. Are you more interested in mathematics *per se* or in its applications. If the latter, to what fields?

I would have to admit that from a formal standpoint I am primarily interested in mathematics *per se.* Nevertheless, in my view mathematics is in essence a vehicle for rendering the structure of the world and our experience of it as intelligible as possible, I believe that the meaning of mathematics must ultimately reside in the fact that it can be applied to domains other than itself. My own major interests in applications of mathematics are (i) its self-application through mathematical logic to its own foundations; (ii) physics.

10. Do you study mathematics books linearly or by sampling? Do you study a topic from multiple sources?

I seem to be congenitally incapable of reading a mathematics book linearly (for a partial explanation see **8**). I usually study a topic from multiple sources, be they books or papers.

11. Are philosophical or foundational issues of interest to you? If so, to what major school are you most attracted?

Philosophical and foundational issues constitute major interests of mine. To which philosophical school am I most attracted? Well, like most mathematicians, I guess, I was initially attracted to Platonism as providing the simplest account of mathematical truth, and having the additional advantage of avoiding what I felt to be a certain cynicism inherent in Formalism. However, like the child's loss of belief in Santa Claus, I came to regard the Platonistic account of mathematical entities as a species of fairy tale, and in any case as engendering insuperable epistemological difficulties. At all events, I believe that mathematics is concerned with *concepts* and *relations* rather than objects.

I have since come to liken Platonism to a disease, which, like measles, must have been undergone in one's youth so as to confer an immunity in later life. I would now describe

<u>my view of foundations of mathematics as *pluralistic*: no unique foundation, rather an interlocking ensemble of "foundations".</u>

12. Do you work primarily as an individual or part of a team?

Like most mathematicians (I would guess), I work primarily as an individual.

13. Are the problems you tackle determined by your work place or can you freely choose them?

I am still in the fortunate position of being able to choose freely the areas into which I venture and the problems I tackle.

14. Do you prefer to study with others or alone? If the former, do you prefer to discuss only the general attack, reserving details for individual work?

I prefer to study in solitude.

15. What proportion of your publications are collaborative?

Only a handful. This fact is not, however, solely the result of wilful solitariness on my part; it also reflects a certain lack of opportunity for collaboration.

16. Are you rigorous about structuring your time for mathematics?
17. Do you prefer to work steadily for a protracted period or in short bursts? Does this depend on the task?

I cannot claim to be rigorous about structuring my work time, which tends to take the form of brief but intense bouts of mathematical activity punctuating extended periods of (mathematical) idleness.

18. Do you work best at a particular time of day?

In my youth I worked "nightclub hours", but latterly, owing in large part to the constraints of parenthood, I have become a diurnal animal.

<u>Although those constraints have long been relaxed, I still find myself a largely diurnal animal.</u>

19. Do you think best while lying down, sitting at a desk, walking, etc?

I think best in a supine position, preferably immersed to the nose in warm water.

20. Are the following physical/environmental characteristics important for your work? (i) diet (ii) sleep (iii) fitness (iv) temperature (v) light (vi) noise (vii) stimulants?

The environmental characteristics important for my work: (a) solitude, (b) an inexhaustible supply of cigarettes, (c) a continuous source of classical music, (d) a functioning fountain pen.

<u>Having finally abandoned smoking, (b) no longer applies. And, sad to say, the item under (d) has been largely supplanted by the computer.</u>

21. On holiday, do you spend some time studying or do you devote the entire time to diversions from mathematics?

On the whole I find it difficult to take a genuine “holiday” from mathematics.

<u>These days I’m finding it a lot easier.</u>

22. What are your favourite leisure activities?

Listening to music, reading, watching, old movies, talking.

<u>Latterly, I have taken to writing my memoirs, but I’m uncertain as to whether to term that a “leisure” activity.</u>

23. When about to investigate or study a new topic, do you prefer to assimilate first or to try your own attack?

For an attempt at answering a related question, see **8.**

24. Do you like to get into a new area by studying special case or do you try to generalize from the outset?

Instinctively, I am driven to generalize, but my mode of ingress to a new area is often the special case.

25. Do you follow problems wherever they lead mathematically, or do you confine yourself to a discipline? Are you problem-driven or technique-driven?

I can’t follow problems wherever they lead mathematically, much as I’d like to, because the trail may lead into certain areas of mathematics (e.g., combinatorics) which neither appeal to me nor for which I have particular aptitude. On the other hand, the idea of following problems (or concepts) wherever they lead appeals very much to me *in principle.* I am, I suppose, more technique- than problem-driven, only I would prefer to use the term *concept-driven*, since in my view both techniques and problems must lead to the elucidation and development of concepts, and should not be regarded as things in themselves.

<u>Two remarks here. There are areas such as geometry or algebraic topology which appeal to me but for which I have no particular aptitude. And I’d now say that, like the chicken and the egg, a problem is a concept’s way of engendering a new concept.</u>

26. Do you write up partial results or accumulate notes until a totality warrants publishing?

I share the passion for completeness found in most mathematicians. On the other hand, I am not by nature patient. These contradictory aspects of my character have impelled me to take up areas of mathematics where complete results can be obtained (and published) comparatively quickly. Above all, I think mathematics *should not be boring.*

27. Do you prefer to concentrate on one problem over a protracted period or on several problems concurrently?

Since I rarely succeed in solving "hard" problems that require protracted concentration, I have found it more rewarding to juggle several (related) problems concurrently, in the hope that at least one of them will pan out.

28. Do discoveries or solutions to problems, entirely different from work on which you are currently engaged, sometimes occur?
29. Have your best ideas occurred as a direct result of deliberate endeavour, or have they arisen spontaneeously?

When a "good" idea occurs to me (a sadly rare occurrence), it seems to arise spontaneously insofar as it is *apparently* not directly derivable from its predecessors. So its appearance is not, so to speak, *locally*, the result of "deliberate endeavour" on my part. On the other hand, the *context* within which said "good" idea arises is usually the ultimate product of a deliberate decision to study a specific topic, to orientate my mind in a prescribed direction. Thus, form a *global* standpoint, spontaneity may be an illusion. As Coleridge said, *a single thought is that which it is from other thoughts, as a wave of the sea takes its form and shape from the waves which precede and follow it.*

30. Have you ever dreamt the solution of a problem?

Would that I had dreamt the solution to a mathematical problem—however trivial! Despite the vividness of my dreams, they yield no proofs. None, at least, are to be found in the *manifest* content of my dreams. The extraction of any such solutions in the *latent* content of dreams suggests the possibility of ascribing a new meaning to the term "mathematical analysis"

But recently I had a dream with a punningly mathematical content. I dreamt that someone asked me whether I still possessed a union card, to which I responded, "I am the union, and, come to think of it, the same could be said by anybody, since each person is the union of his singleton."

31. While holding a central concept in the forefront of your attention, are you aware of how it connects to related ideas in the fringes of your consciousness?

This is a subtle question, to which I think the answer must be in the affirmative. I say "must" because I don't think that a concept—however pure in principle—is ever presented to consciousness in a pure form; it is always accompanied by a penumbra of associations, instances and the like.

32. Do you experience periods of inspiration and enthusiasm alternating with periods of creative incapacity?

Oh yes indeed. The question itself contains a disturbingly exact description of the creative bipolarity to which I, and I suspect many others, are subject.

33. In trying to form an intuition about the truth of a proposition, which of the following is the surest guide: (i) repeated checking of a formal proof; (ii) multiple, convergent partial proofs; (iii) coherence with other results; (iv) applications?

Doesn't intuition about the truth of a proposition *precede* any formal proof? A formal proof has the effect of transforming intuitive likelihood into *certainty.* Thus, once one actually *has* a proof, it must be the surest guide to truth. Indeed, a case has been made that the content of a mathematical proposition is *nothing more than* the assemblage of its possible proofs. In trying to form an intuition concerning the truth of a proposition *before* a proof (or refutation) has been obtained, it seems to me that coherence with other results is generally the surest initial guide. This often leads to a number of multiple, convergent partial proofs, which, if one is fortunate, culminates in a complete proof.

34. Have you any experiences where such intuition was radically misleading?

On occasion the coherence idea I normally employ has failed. Even worse, my conviction as to its validity has sometimes misled me into producing a "proof" of a proposition which later turned to be false. Fortunately for my slender reputation as a mathematician, this has not occurred very frequently. One version of the mathematician's nightmare is that mathematics is built on false propositions to which everybody has given their assent!

35. Are there results which are unintuitive to you, no matter how often you have employed them?

All the mathematical results I have employed with any regularity do seem intuitively clear to me, but this fact probably attests less to the acuity of my insight than to the paucity of the results I employ.

36. Do you employ computers extensively for numerical or symbolic computation? Are computer graphics a ready aid to your intuition?

No.

Of course, like everybody else, I now use a computer extensively for word processing. But they still play no "creative" role in my work.

37. Are geometric and symbolic thinking markedly different for you?
38. What mental imagery do you use to think about mathematical entities and deductions? Is it primarily pictorial, linguistic, kinaesthetic, etc? What mental imagery do you use to relate overall direction of an argument to its details?

Yes, geometric and symbolic thinking are different for me. And here is one reason: I find it very difficult to retain geometric images of any sophistication in my mind's eye, whereas my symbolic or linguistic thinking can be underpinned by a kind of mental subvocalization ($E = mc^2$*: ee equals em see squared*, and the like). The purpose of this is to enable groups of symbols to retain their form throughout the course of a mental argument; I have not succeeded in devising a cerebral mechanism which will render the same service for visual or geometric images. By nature an algebraist rather than a geometer, in my mathematical thinking I tend to rely more on symbolic or linguistic devices than on the rudimentary images my impoverished visual imagination permits me to deploy—typically, amoeboid shapes suitable for illustrating propositions in general topology, or arrow diagrams in category theory. Nevertheless I have always believed that a command of visual imagery is the master key to mathematical creativity. For this reason I always *attempt*, however ineffectually, to use such imagery—if not visual, then at least figurative—in relating the overall structure of a mathematical argument to its details.

39. Can the use of imagery be taught?

The *general use* of imagery can be taught, and is taught, by example. I doubt, however, whether *specific* forms of imagery should be taught. The purpose, after all, of drawing pictures on the blackboard in the course of a lecture, for instance, is to convey the the *idea* of using visual imagery rather than to impose the specific images themselves.

<u>But it now occurs to me that Venn and Feynman diagrams constitute an exception</u>.

40. When thinking, do you conduct an inner dialogue with others? With whom?

Yes, when thinking I often conduct an inner dialogue (*cf.* "mental subvocalization" in **37, 38.**) Disappointingly however, my mental interlocutor turns out to be myself.

41. Do artistic and literary interests help or hinder your mathematical work?

I have always felt that, although mathematics is distinct in some way from art, the underlying reasons for their pursuit are the same. (For what I believe to be the likeliest ultimate reason, see **46.**) So it is hard for me to say whether my artistic and literary interests help or hinder my mathematical work. They "help" it in the sense that I cannot really distinguish the impulse to pursue these interests from the impulse to do mathematics. But they "hinder" it in their essential incompatibility with the tunnel vision required for the creation of "deep" mathematics.

42. Do metaphysical, ethical, or religious questions interest you greatly?

Yes, but this is not the place to enlarge on these matters.

43. Is it important to achieve intellectual integrity between your mathematics and any other aspect of your thinking—political, religious, philosophical or personal?

In my youth I felt it important to a achieve a degree of coherence between my mathematics and my personal and political activity and thinking. As I grow older, however, all that seems to remain is the impulse to achieve some rapport between my mathematical and philosophical thinking. This probably reflects a diminution of active engagement in (professional) mathematics as such.

<u>And indeed I now find myself a member of the philosophy department!</u>

44. Does the non-popular status of mathematics bother you greatly?

Yes it does! It also puzzles me. *Why* is mathematics so unpopular? Contrast, for example, mathematics with music (<u>which I have since come to define as "audible mathematics inducing objectless emotion"</u>). Musical activity involves three overlapping but essentially distinct classes of people: composers, performers and listeners. In (serious) mathematics these three categories are collapsed into one. Why should this be the case? Mathematics may be more cerebral than music, but is not intrinsically more *difficult*, at least in terms of technique. A performing musician puts enormous effort into perfecting his technique, certainly comparable, and in many cases surpassing, the mathematician's effort to master his subject. The difference is that the (performing) musician's struggle to perfect his art is finally crowned with popular success: no matter how difficult it may be to play a Liszt transcendental study, a Paganini caprice, or even a Bach sonata, a "lay" audience will enjoy listening to the piece, despite their lack of the slightest idea of how to play it. Contrast this with mathematics. The beauties of even a comparatively straightforward result such as the Stone-Weierstrass theorem (say) are appreciated only by those who are capable, not necessarily of creating ("composing") it, but at least of recreating ("playing") it. Mathematics is an esoteric art but whether this is so for intrinsic reasons I am not sure. Perhaps historical and social causes must be taken into account.

45. Is the social relevance of your work of prime concern?

See **47.**

46. Has mathematics been, or is it now, in part a refuge from reality?

Someone once observed that chess, like music, like love has the power to make people happy. The same, it seems to me, is true of mathematics, and because of this it has often provided me with a welcome refuge from reality.

<u>The "someone" was Dr. Siegbert Tarrasch, a prominent 19th century chess player, one of the original "grandmasters".</u>

47. Are you prepared to grant an in-depth, follow-up interview?

I should refuse?

<u>Needless to say, it never took place.</u>