Limited Enforcement and Efficient Interbank Arrangements*

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Abstract

A nontrivial interaction between asset and liquidity risk plays a crucial role in shaping optimal banking arrangements when enforcement is limited. Liquidity shocks are essential for the provision of insurance between banks against asset shocks as they mitigate the enforcement problem associated with interbank insurance arrangements. Since investment allocations depend on the joint distribution of shocks, enforcement problems lead to endogenous *aggregate* uncertainty. Paradoxically, a negative correlation between liquidity and asset shocks ameliorates enforcement limitations and facilitates interbank cooperation. Since liquidity markets cannot fully exploit this correlation, liquidity between banks is best provided in a centralized fashion.

Keywords: Collateral Shocks; Endogenous Uncertainty; Liquidity Shocks; Interbank Relations; Markets vs. Relationships

JEL Classifications: G21, D80

1 Introduction

Models of banking relationships typically analyze the implications of one of two types of uncertainty associated with banking: uncertain liquidity demands or asset risk arising from default on loans. An open question is whether interbank relationships are affected by the interaction between liquidity and asset shocks. Can banks insure mutually against their asset risk? What are the implications for the efficient provision of liquidity between banks? Should interbank liquidity be provided via a decentralized competitive market or directly allocated within "banking clubs"?

To address these questions, we study an environment with both asset and liquidity shocks and limited enforcement. At the core of our model is the basic investment financing problem of [5] and [8]. To finance projects, borrowers seek funds from lenders who are endowed with an investment good, but who do not have direct access to profitable investment opportunities. Stochastic liquidity needs for lenders are modelled as preference shocks, which generate a deposit feature with the option of early withdrawal of funds.

To incorporate asset risk, we follow [13], and assume that borrowers are endowed with a risky collateral good. This collateral good is essential for borrowing, as the repayment of loans can only be enforced via the threat of seizure of the borrowers collateral. This feature leads to financing via collateralized debt and non-performing loans due to strategic default by borrowers.

The two frictions discussed above give rise to a contract that we interpret as a bank. To study interactions between banks, we replicate our basic environment and assume that each group of lenders and borrowers is spatially separated. This feature captures the observation that bank lending and deposit taking tends to be regionally or functionally specialized. This leads to a lack of ex-ante diversification, and hence creates gains from ex-post risk sharing between banks. Contracts which seek to take advantage of these opportunities are complicated by the inability of individual banks to commit to prior agreements. As a result, these risk sharing arrangements have to be self-enforcing.

We find that the interaction between asset and liquidity risk plays a crucial role in shaping optimal banking arrangements when enforcement is limited. Although liquidity shocks do not provide a motivation for insurance among banks, the threat of exclusion from liquidity markets plays an essential role in offsetting enforcement limitations. In the absence of liquidity shocks, limited enforcement implies that banks do not provide insurance to one another against collateral shocks. With liquidity shocks, the enforcement problem is partially mitigated, so that banks can enter into self-enforcing agreements to insure one another. In other words, the possible exclusion from interbank lending serves as a threat to induce ex-post participation in insurance arrangements.

The extent to which insurance is provided depends critically upon the realization of the joint distribution of asset and liquidity shocks across banks. Paradoxically, a negative correlation between liquidity and asset shocks ameliorates enforcement limitations and facilitates cooperation between banks. Moreover, since the level of investment depends upon the joint distribution of shocks, our model generates endogenous aggregate uncertainty. Thus this paper also provides a contribution to the theoretical literature on limited enforcement, as it illustrates how the interaction of purely idiosyncratic shocks in the presence of enforcement limitations can generate aggregate uncertainty.

The second main finding of our paper is that the organization of banks into "clubs" for the provision of liquidity needs Pareto dominates a competitive market for liquidity. The problem with a market based system is that it prevents the appropriate set of insurance transfers from being allocated which lowers the total level of investment in the economy.

The intuition for these results is simple. In the model, investment projects are always socially optimal since they yield a positive net return. However, due to the default problem associated with debt contracts, for some realizations of the asset shock lenders will terminate all investment projects on an island since they are unable to recover their investment. On islands with good asset shocks, liquidity demands lead to the premature termination of investments. The optimal contract seeks to mitigate these two forces. First, it seeks to allocate liquidity efficiently to islands with good asset shocks. Secondly, it must allocate insurance payments (investment "subsidies") so as to fund productive projects which lenders would otherwise terminate since they cannot recover their investments. Since there is limited enforcement of inter-island insurance contracts, funds for these insurance payments are extracted from islands with a good asset shock through the threat of exclusion from liquidity provision. The extent of asset insurance varies with the realizations of the shocks since the cost of exclusion varies with the joint distribution of shocks. Similarly, anonymous liquidity markets are worse than "banking clubs" since asset insurance is not allocated so as to maximize the number of subsidized projects.

These results suggest that interactions between liquidity and asset shocks should not be neglected when studying interbank relationships. However, the theoretical literature has concentrated mainly on two broad areas. One branch has focused on the role of interbank markets in dealing with liquidity shocks and investigates whether these markets can achieve an optimal diversification of liquidity risk ([3], [4], [7]). A second branch has investigated the potential for contagious effects in the banking sector via links between banks and discusses optimal regulatory responses ([1], [11], [15]).

Our paper differs from these contributions as we analyze a model where both stochastic liquidity needs and co-insurance against asset risk *interact* to shape interbank relationships. Closest in spirit to our work is [9] (and their related work). They also examine an environment with liquidity and asset shocks. Their definition of a bank is similar to ours, as they define a bank as a technology that can run a project abandoned by an entrepreneur more efficiently than any other agent. We define a bank as bank as a technology that can seize collateral. However, their focus is fundamentally different than ours. They argue that asset shocks to banks' loan portfolios can lead to liquidity shortages that trigger further bank failures. In contrast to their emphasis on contagion, we focus on the nature of interbank relationships in an environment with both types of shocks and limited interbank enforcement.

The remainder of the paper is organized as follows. The next section sets out the environment, concentrating on the contractual framework and the enforcement frictions. We proceed in section 3 by formally setting up the problem and defining the concept of a contract. Section 4 contains the main analysis: optimal contracts are characterized and the interaction between both types of shocks investigated. In section 5, we compare liquidity provision through a market based system with a banking club system. Section 6 discusses the implications of our model and outlines some empirical examples which match our results. All proofs are relegated to the appendix.

2 The Environment

The economy consists of $N \geq 3$ identical islands. On each island there is a continuum of two types of agents, denoted as borrowers and lenders, each having unit mass. The economy lasts for a single period divided into three stages labelled $t = 0, 1, 2$.

2.1 Lenders

Lenders on each island i are endowed with a single unit of a consumption/investment good and a storage technology which yields a gross return of one. Let $\tilde{\theta}_i$ be a random variable with support $\Theta \subset (0,1)$ and denote a realization of $\tilde{\theta}_i$ by θ_i. Lenders can be of two types: whereas type 1 lenders consume only at $t = 1$, lenders of type 2 consume exclusively at $t = 2$. Preferences are state-dependent on $\tilde{\theta}_i$. The random variable $\tilde{\theta}_i$ represents an island-specific liquidity shock as in [8]. One can interpret θ_i as the fraction of lenders on island i having to consume at $t = 1$. Formally, preferences of lenders are defined by:

$$u_L(c^1_{L,i}, c^2_{L,i}; \theta_i) = \begin{cases} c^1_{L,i} & \text{if type 1 and realization is } \theta_i \\ c^2_{L,i} & \text{if type 2 and realization is } \theta_i \end{cases},$$

where $c^t_{L,i}$ is the amount consumed by lenders at t. Ex-ante preferences are given by $E[u_L(c^1_{L,i}, c^2_{L,i}; \theta_i)] \equiv E_{\tilde{\theta}_i}[\tilde{\theta}_i c^1_{L,i} + (1 - \tilde{\theta}_i) c^2_{L,i}]$.

2.2 Borrowers

Borrowers are endowed with an investment project and a unit of an indivisible collateral good. To initiate the project an amount $x > 0$ of the consumption good must be invested at $t = 0$. Projects are of a long-term nature. Once funds are withdrawn from a project, the project is stopped. The project payoff is given by $\min\{Rx, R\}$ units of the consumption good at $t = 2$, where $R > 1$ is deterministic.

The collateral good is agent-specific: only the specific borrower endowed with the collateral good derives utility from it. Let ν be a random variable with support $\{0, V\}$, representing a shock to the value of the borrower's collateral good. The distribution of ν for all borrowers on island i is characterized by the random variable $\tilde{\pi}_i$ having support $\Pi \subset (0, 1)$, where π_i is a particular realization of $\tilde{\pi}_i$. The realization π_i is the fraction of borrowers with valuable collateral

Borrowers consume only at $t = 2$. Denoting the borrowers consumption of the good as $c^{\nu}_{B,i}$, preferences are represented by:

$$u_B(c^V_{B,i}, c^0_{B,i}, \nu; \pi_i) = \begin{cases} c^V_{B,i} + V\delta_i & \text{if } \nu = V \text{ and realization is } \pi_i \\ c^0_{B,i} & \text{if } \nu = 0 \text{ and realization is } \pi_i \end{cases},$$

where $\delta_i \in \{0, 1\}$ expresses ownership of the collateral good. Ex-ante preferences are represented by $E[u_B(c^V_{B,i}, c^0_{B,i}; \pi_i)] \equiv E_{\tilde{\pi}_i}[\tilde{\pi}_i(c^V_{B,i} + V\delta_i) + (1 - \tilde{\pi}_i)c^0_{B,i}]$.

2.3 Uncertainty

The underlying probability space can be interpreted as an urn experiment. Consider an urn containing N balls each labelled with a different value of

$\pi \in (0,1)$. A second urn contains N balls, each labelled with a different value of $\theta \in (0,1)$. Let $\Theta = \{\theta^1, \ldots, \theta^j, \ldots, \theta^N\}$ and $\Pi = \{\pi^1, \ldots, \pi^j, \ldots, \pi^N\}$ such that $\theta^j \neq \theta^k$ and $\pi^j \neq \pi^k$ for all $j \neq k$. For each island, one ball is drawn independently from each urn without replacement. A realization of the two shocks across islands is a specific assignment of draws from the two urns to the N islands.

Let $S \equiv \Pi \times \Theta$ be the set of states of nature. It consists of all possible assignments of N elements of Θ to the N elements in Π. A state $s \equiv \pi \times \theta$ is an N-vector of ordered pairs (π^j, θ^k), where the ith component represents the realization of $\tilde{\theta}_i$ and $\tilde{\pi}_i$ for island i. A state $\pi \times \theta$ for the economy is thus given by a particular realization of both shocks across islands. There are $N!$ elements in $\Pi \times \Theta$. We define a probability measure $\mathcal{U}$ over $\Pi \times \Theta$ by putting equal mass $1/N!$ on each element of $\Pi \times \Theta$. The probability space $\{S, \mathcal{F}, \mathcal{U}\}$ represents the uncertainty for the economy, where $\mathcal{F}$ is the σ-algebra consisting of all subsets of $\Pi \times \Theta$.[1]

2.4 Enforceability

Lenders and borrowers enter into contracts to finance projects. These contracts are subject to several restrictions on enforcement. Lenders have the option of investing in either the storage technology or the investment project, and of withdrawing their funds from the investment project at $t = 1$. Hence, the contract has to insure their participation at all stages. Second, borrowers on each island have the option of not repaying the lender and consuming

[1]Modelling uncertainty in this fashion simplifies the analysis. All the results carry over to more general formulations of uncertainty.

all the project returns. Therefore, the contracting parties face an *ex-post* *intra-island* participation constraint on the borrowing side. Since we assume that the collateral good is subject to seizure by the lender, default or non-participation is potentially costly for borrowers. Finally, if the contract specifies any *inter-island* transfers, it must be the case that each island *as a whole*, i.e. the sum of its lenders and borrowers has an incentive to honor the contract *ex-post* (*inter-island* participation). We assume that each island makes its decision on honoring the contract at the beginning of $t = 1$.

2.5 Timing

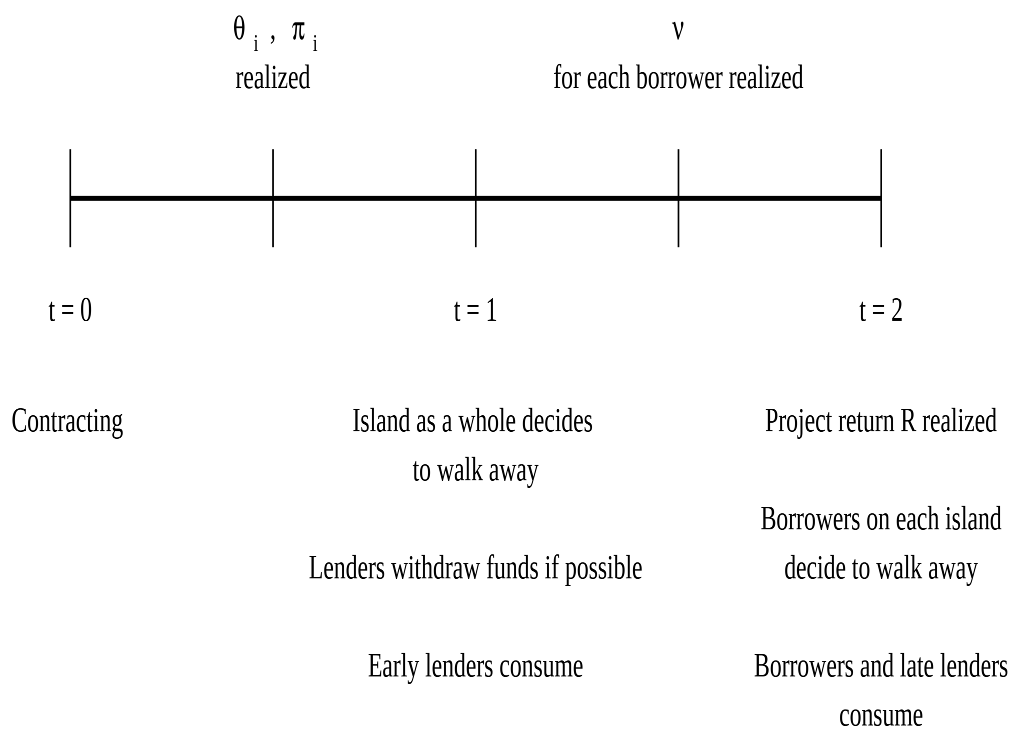
All information is publicly known by all agents at the stage it is realized – there is *no private information*. We denote the initial and final investment decision by lenders on island i by x_i^0 and $x_i^1(\pi \times \theta)$, respectively. Note that lenders on island i can finance only projects on their own island. Hence, there is a lack of ex-ante diversification with respect to investment. The timing is as follows:

Stage 0: Lenders and borrowers agree on a contract. Lenders advance x_i^0 to borrowers on their island.

Stage 1: For each island the shocks $\tilde{\pi}_i$ and $\tilde{\theta}_i$ are realized and each individual lender learns his/her type. While the fraction of borrowers with valuable collateral is known at this stage, the individual realization for each specific borrower is *not*. Islands as a whole decide whether to exit the contract. Then, lenders make withdrawals $x_i^0 - x_i^1(\pi \times \theta)$ conditional on the realizations of the shocks and consume or store them for consumption in stage 2.

Stage 2: The realization of ν becomes publicly known for each individual borrower. Borrowers on each island decide whether to walk away or not. Project returns are realized and consumption takes place.

Figure I: Timing of Events



3 The Contracting Problem

3.1 Contracts

The first objective is to characterize the efficient allocation of investment funds across islands. We study an optimal contracting problem under uncertainty subject to enforcement limitations. Contracts specify state-dependent allocations and investment levels across all N islands and are written before the random variables $\tilde{\pi}_i$ and $\tilde{\theta}_i$ are realized. The set of possible contracts are constrained by the enforcement frictions arising in both the borrower-lender and the inter-island relationships.

Definition 3.1. *An inter-island contract specifies*

- *investment levels* $\{x_i^0, x_i^1(\pi \times \theta)\}_{i=1}^N$
- *allocations* $\left\{c_{B,i}^V(\pi \times \theta), c_{B,i}^0(\pi \times \theta), c_{L,i}^1(\pi \times \theta), c_{L,i}^2(\pi \times \theta)\right\}_{i=1}^N$
- *ownership of collateral for borrowers* $\left\{\delta_i^V(\pi \times \theta), \delta_i^0(\pi \times \theta)\right\}_{i=1}^N$*, where* $\delta_i^\nu(\cdot) \in \{0, 1\}$

for all $\pi \times \theta \in \Pi \times \Theta$*.*

The functions $\delta_i^\nu(\cdot) \in \{0, 1\}$ specify whether the borrower retains title to the collateral. As specified earlier, borrowers cannot consume the collateral good if they do not retain title to it.

When solving for an optimal inter-island contract, we look at a Pareto problem that characterizes constrained efficient investment levels and allocations. Linearity of the utility functions implies that we can maximize total surplus from investment. We assume that all surplus accrues to borrowers.

The objective function is to maximize the sum of borrowers' utility across islands.

$$E_{\Pi\times\Theta}\left[\sum_{i=1}^{N}\pi_i\left[c_{B,i}^V(\pi\times\theta)+\delta_i^V(\pi\times\theta)V\right]+(1-\pi_i)c_{B,i}^0(\pi\times\theta)\right]. \tag{3.1}$$

Lenders have the option of withdrawing their funds and utilizing the storage technology. Thus, any contract must satisfy the following two individual rationality constraints for every island i:

$$c_{L,i}^1(\pi\times\theta)\geq 1 \tag{3.2}$$

$$c_{L,i}^2(\pi\times\theta)\geq 1. \tag{3.3}$$

Borrowers on each island have the option of walking away at the cost of forfeiting their collateral good. Hence, we need a participation constraint for $t=1$ and two different ones – one for each realization of ν – to take into account participation of borrowers in $t=2$:

$$\pi_i[c_{B,i}^V(\pi\times\theta)+\delta_i^V(\pi\times\theta)V]+(1-\pi_i)c_{B,i}^0(\pi\times\theta)\geq Rx_i^0 \tag{3.4}$$

$$c_{B,i}^V(\pi\times\theta)+\delta_i^V(\pi\times\theta)V\geq Rx_i^1(\pi\times\theta) \tag{3.5}$$

$$c_{B,i}^0(\pi\times\theta)\geq Rx_i^1(\pi\times\theta). \tag{3.6}$$

The first constraint compares the expected value of honoring withdrawal requests at $t=1$ with the value of continuing investment at the original level.[2] At $t=2$, if collateral has value V, the borrower weighs the value of

[2] It is clear that lenders would seize collateral in the case of borrowers not honoring the lenders' withdrawal requests. For notational clarity we do not include this decision in the specification of the constraint, since we will rule out early default by borrowers.

consumption under the contract and collateral ownership against defaulting and consuming the project returns. Clearly, if borrowers have valueless collateral, they will always default unless they receive the entire project return.

Contracts must satisfy resource feasibility at $t = 1$ and $t = 2$. The resource constraints for each stage are aggregate constraints across islands and do not have to hold for each island separately:

$$\sum_{i=1}^{N} \theta_i c_{L,i}^1(\pi \times \theta) \leq \sum_{i=1}^{N} (1 - x_i^1(\pi \times \theta)) \tag{3.7}$$

$$\begin{aligned} &\sum_{i=1}^{N} \left[\pi_i c_{B,i}^V(\pi \times \theta) + (1 - \pi_i) c_{B,i}^0(\pi \times \theta)\right] + \sum_{i=1}^{N} (1 - \theta_i) c_{L,i}^2(\pi \times \theta) \\ \leq \quad &\sum_{i=1}^{N} R x_i^1(\pi \times \theta) + \left[\sum_{i=1}^{N} \left(1 - x_i^1(\pi \times \theta)\right) - \sum_{i=1}^{N} \theta_i c_{L,i}^1(\pi \times \theta)\right]. \end{aligned} \tag{3.8}$$

A key ingredient of our model is the inter-island enforcement restriction. Each island has the option of not honoring the contract at $t = 1$ and resorting to an island-specific contract which is defined formally below. Let W_i^A be the value of this island-specific contract for island i at $t = 1$ and denote the value of the contract for island i by W_i^C. The inter-island participation constraint is:

$$W_i^C(\pi \times \theta) \geq W_i^A(\pi \times \theta), \tag{3.9}$$

where $W_i^C \equiv \theta_i c_{L,i}^1 + (1 - \theta_i) c_{L,i}^2 + \pi_i \left[c_{B,i}^V + \delta_i^V V\right] + (1 - \pi_i) c_{B,i}^0$. The final constraints are restrictions on intermediate investment levels and non-negativity restrictions

$$0 \leq x_i^1(\pi \times \theta) \leq x_i^0, \tag{3.10}$$

$$c^{\nu}_{B,i}(\cdot) \geq 0, 1 \geq x^0_i \geq 0. \tag{3.11}$$

The next definition identifies optimal contracts with the solution of a constrained Pareto-problem.

Definition 3.2. *An optimal* <u>*inter-island*</u> *contract is a contract that solves the following maximization problem* ($\mathcal{P}1$)*:*

$$\max \textit{ 3.1 s.to 3.7, 3.8 and s.to. 3.2 - 3.6, 3.9 - 3.11 for all islands } i = 1, \ldots, N.$$

The value of leaving the contract at $t = 1$ remains to be determined. We assume that after leaving the inter-island contract, island i implements the best possible island-specific contract *given* the liquidity and asset shock that has occurred. This island-specific contract for island i is defined by an inter-island contract with $N = 1$. The value of walking away from the inter-island contract, W^A_i, is given by the sum of consumption allocations specified in the optimal island-specific contract for the given values of π_i and θ_i. The next definition clarifies this concept.

Definition 3.3. *Let* $N = 1$*. An optimal* <u>*island-specific*</u> *contract for island* i *is a contract that solves the following maximization problem* ($\mathcal{P}2$)*:*

$$\max E_{\Pi\times\Theta}\left[\pi_i\left(c^V_{B,i}(\pi\times\theta) + \delta^V_i(\pi\times\theta)V\right) + (1-\pi_i)c^0_{B,i}(\pi\times\theta)\right]$$
$$\textit{s.to 3.2 - 3.8, 3.10, 3.11 for island } i.$$

For comparison purposes, we also consider optimal inter-island contracts when islands do *not* have the option of walking away from the contract at $t = 1$. We refer to this as full inter-island enforcement, which is essentially the maximization problem ($\mathcal{P}1$) without constraint (3.9).

Definition 3.4. *An optimal* <u>*inter-island*</u> *contract with* <u>*full inter-island enforcement*</u> *is a contract that solves the following maximization problem* ($\mathcal{P}3$)*:*

max *3.1 s.to 3.7, 3.8 and s.to. 3.2 - 3.6, 3.10, 3.11 for all islands* $i = 1, \ldots, N$.

3.2 Banks as Optimal Contracts

The optimal contract characterized in the next section captures several key features which are associated in the theoretical literature with the institution of a bank. It explicitly contains both a deposit and a debt contract feature. The deposit contract feature arises from the fact that any optimal contract provides lenders with the option to withdraw funds upon demand. This withdrawal can be made either to meet personal liquidity needs or to liquidate investment on the island in response to information about negative asset shocks.[3] The asset side of the contract resembles collateralized debt. Even though the debt feature of the contract is not obvious, a simple and for our purposes inessential extension would make this feature explicit.[4]

We interpret the optimal contract as an optimal banking arrangement. The contract specification for each island is identified as an island-specific bank. Thus, possible inter-island transfers occur between island-specific

[3] The formulation of the participation constraints rules out suspension schemes. Therefore, the contract has both a deposit and a demandable debt feature. [6] emphasize that demandable debt is the crucial feature that characterizes banks. They also provide evidence that suspension schemes were only used by the banking system as a whole, thus empirically supporting this particular feature.

[4] [13] uses asymmetric information with respect to the collateral value to generate collateralized debt as the optimal contract.

banks. The inter-island enforcement restriction, (3.9), reflects the ability of individual banks to exit interbank arrangements. To further identify specifics of these interbank arrangements we define two concepts: Insurance and liquidity provision.

Definition 3.5. *An* <u>*insurance system*</u> *is a transfer scheme* $\{T_i\}_{i=1}^N$, $T_i : \Pi \times \Theta \longrightarrow I\!R$, *for all* $i = 1, 2, \ldots, N$ *s.th.*

1. $\exists\ \hat{\pi} \in [0, 1]$, *s.th.* $T_i \geq 0$ *for* $\pi_i < \hat{\pi}$ *and* $T_i < 0$ *for some* $\pi_i > \hat{\pi}$
2. $\sum_{i=1}^N T_i \leq 0$.

Insurance systems involve transfers from solvent to insolvent banks (where $\hat{\pi}$ determines the solvency of banks), subject to feasibility restrictions. Transfers provide insurance against shocks to banks' asset portfolios. Since transfers, however, have to satisfy the enforcement and feasibility constraints of the inter-island contracting problem, the scheme can be interpreted as a voluntary scheme organized by the banks themselves. Net transfers between banks have to be non-positive, which rules out systems that depend on the provision of funds to the banking system from outside.

Definition 3.6. *A contract features* <u>*liquidity provision*</u> *if* $\exists\ i$ *s.th.* $x_i^1(\pi \times \theta) > 1 - \theta_i$ *for some* $i = 1, \ldots, N$.

This definition formalizes the idea of short-term lending between banks to meet liquidity needs. Liquidity provision enables banks receiving funds to avoid termination of some projects due to withdrawals by type 1 lenders. Banks receiving liquidity transfer the claims to the collateral that has been pledged to other islands. Thus liquidity loans consist in the model are analogous to securitization. This guarantees participation of islands at $t = 2$.

4 Optimal Contracts

4.1 Optimal Island-Specific Contracts

We begin our analysis of efficient arrangements among islands by studying the optimal contract for a single island. This problem is useful for two reasons. First, it determines the value of the walk away option of an island. Second, it illustrates the driving forces of the relationship between borrowers and lenders on an island. Prior to characterizing the optimal contract, we introduce the following assumption which we maintain throughout the paper.

Assumption 4.1. $\min_{\pi_i \in \Pi} \pi_i V \geq R$.

Assumption 4.1 implies that all liquidation requests by lenders at $t = 1$ will be honored, since the expected cost of defaulting exceeds the expected gain for a borrower. Hence, lenders on all islands initially invest all their funds ($x_i^0 = 1$).

It is immediate from the participation constraints for borrowers, (3.5) and (3.6), that the only way to prevent default is to seize collateral if borrowers default at $t = 2$. This allows lenders to recover their loans to borrowers with good collateral. Assigning ownership rights of collateral to borrowers, ($\delta_i^V(\pi \times \theta) = 1$), is therefore optimal independent of initial investment.

Proposition 4.2. *An optimal island-specific contract for island i specifies investment levels*

$$x_i^1(\pi \times \theta) = \begin{cases} 1 - \theta_i & \textit{if } \pi_i \geq \bar{\pi} \\ 0 & \textit{if } \pi_i < \bar{\pi} \end{cases},$$

where $\bar{\pi} = 1/R$. *Optimal consumption levels for island* i *are given by*

$$c^t_{L,i}(\pi \times \theta) = 1 \textit{ for } t = 1, 2,$$

$$c^V_{B,i}(\pi \times \theta) = \begin{cases} \frac{1}{\pi_i}(1-\theta_i)(\pi_i R - 1) & \textit{if } \pi_i \geq \bar{\pi} \\ 0 & \textit{if } \pi_i < \bar{\pi} \end{cases}$$

and

$$c^0_{B,i}(\pi \times \theta) = \begin{cases} R(1-\theta_i) & \textit{if } \pi_i \geq \bar{\pi} \\ 0 & \textit{if } \pi_i < \bar{\pi} \end{cases}.$$

This proposition highlights the fact that the realized value of the collateral shock π_i determines whether lenders leave their funds in the investment project or liquidate them. This decision is characterized by a cut-off rule: If the collateral shock is "good" ($\pi_i > \bar{\pi}$), funds are withdrawn solely for liquidity purposes. All remaining funds stay invested at $t = 1$. Conversely, if there is a bad realization of the collateral shock, all projects are liquidated.

This outcome is socially suboptimal since projects yield a higher social rate of return than the storage technology. The inefficiency arises from the inability of a sufficient number of borrowers to commit to the repayment of their loans in the presence of an island wide collateral shock.

The interaction between liquidity and asset shocks is trivial in the single island case. Liquidity shocks reduce overall investment leading to lower output and consumption by borrowers. However, they do not influence the decision of whether to liquidate projects at $t = 1$. This decision depends solely on the average rate of performing loans π_i.

4.2 Optimal Inter-Island Contracts with Full Inter-Island Enforcement

We now characterize the first best arrangement. To proceed we define three constant integers, M, L and K.[5] The constant M specifies the number of islands having a sufficiently high average rate of performing loans. The integer L stands for the overall liquidity needs in the economy. Finally, K will summarize the maximal extent of insurance against the asset shock in the absence of liquidity needs. Its definition relies on the fact that project returns can only be extracted – and used for transfers – from borrowers with valuable collateral.

Assumption 4.3. *Let $\pi_1 < \cdots < \pi_N$. Assume there exist $\{M, L, K\} \in \mathbb{N}^3$ where $K \geq L$ s.th.*

1. $M = \#\{\pi_i | \pi_i R \geq 1\}$
2. $L = E(\theta_i)N = \sum_{i=1}^{N} \theta_i$
3. $\sum_{\{i|i>K\}}(\pi_i R - 1) = 0.$

The assumption that $K \geq L$ restricts our attention to the case where the demand for liquidity does not rule out an insurance system.[6] The next result describes the first-best contract with full enforcement between islands.

[5] While this restricts parameter values, dropping this assumption would change neither the analysis nor our results and would involve an immense cost of additional notation.

[6] A complete characterization of the optimal contract including the case where $L > K$, is provided in [14]. In that case, the demand for liquidity is so large that there is only limited scope or even no scope for an insurance system.

Proposition 4.4. *Investment at $t = 1$ for the optimal contract with full inter-island enforcement is independent of the joint distribution $\pi \times \theta$. Optimal total investment is given by*

$$\sum_{i=1}^{N} x_i^1(\pi \times \theta) = N - K$$

with investment on the individual islands characterized by

$$x_i^1(\pi \times \theta) = \begin{cases} 1 & \text{if } \pi_i > \pi_K \\ 0 & \text{if } \pi_i \leq \pi_K . \end{cases}$$

Since total welfare is strictly increasing in the number of projects funded, it is socially optimal to fund as many projects as possible. The optimal contract liquidates islands with the worst collateral shocks first to pay early withdrawers. This allows the continuation of investment on islands with the most favorable collateral shock or, equivalently, with the highest return that can be extracted on average from borrowers. A maximum amount of transfers is then available to subsidize projects on other islands that would otherwise not have been undertaken.

Observe that only the distribution of collateral shocks across islands is important for the allocation of investment funds. The key advantages of an aggregate contract including all islands is that one is able to transfer resources from islands with high collateral values to islands with bad realizations of the collateral shock *independent* of the realizations of the liquidity shock. Hence, the economy is not exposed to aggregate uncertainty, since the overall investment levels do not depend on the joint distribution of the shocks.

Liquidity provision is an essential feature of the contract, as it allows the insurance scheme to operate independently of the distribution of liquidity shocks across islands. Without this feature, an insurance scheme would have to take into account that islands are hit by different liquidity shocks, potentially reducing the magnitude of insurance.

As in the island-specific contract, there is a trivial interaction between liquidity and collateral shocks. Liquidity shocks are inessential for the possibility of an insurance scheme. Since $K \geq L$, their realized value only influences the aggregate level of investment. In the next section we show that with limited enforcement between islands, both the realized joint distribution of the shocks and the presence of liquidity shocks determine the extent of insurance.

4.3 Optimal Inter-Island Contracts

In this section, we analyze optimal inter-island contracts with limited enforcement of inter-island transfers. The key finding is that, in the presence of limited inter-island enforcement, a non-trivial interaction between collateral and liquidity shocks allows for insurance and liquidity provision among banks with the optimal allocation of investment depending on the joint distribution of shocks.

Since there is no ex-post enforcement of inter-island transfers, individual islands will honor the contract only if they receive at least the ex-post value of autarky. For islands with realizations of π_i such that $\pi_i R < 1$ it is always optimal to stay within the contractual arrangement, since any contract implicitly specifies non-negative transfers for these islands. For all other islands, however, the ex-post participation constraint cannot be neglected.

Suppose an island i with $\pi_i R \geq 1$ exits an inter-island contract at $t = 1$ before withdrawals are announced. The best it can do on its own is to implement the optimal island-specific contract as of $t = 1$, i.e. after the liquidity and the collateral shock have been realized. If $x_i^0 = 1$, the value of autarky, $W_i^A(\pi \times \theta)$, for this island is the overall value of the optimal island-specific contract:

$$W_i^A = R(1 - \theta_i) + \theta_i + V\pi_i. \tag{4.1}$$

Only borrowers with valuable collateral can be taxed to finance insurance transfers. Using Proposition 4.2, total utility for borrowers with valuable collateral after transfers is given by

$$\pi_i(c_{B,i}^V(\pi \times \theta) + V) = (\pi_i R - 1)x_i^1(\pi \times \theta) + T_i(\pi \times \theta) + \pi_i V, \tag{4.2}$$

where π_i denotes the measure of borrowers with valuable collateral on i and $T_i(\cdot)$ is the total transfer extracted from island i. The value of an optimal inter-island contract W_i^C is

$$\begin{aligned} W_i^C &= 1 + (\pi_i R - 1)x_i^1(\pi \times \theta) + T_i(\pi \times \theta) + \pi_i V + R(1 - \pi_i)x_i^1(\pi \times \theta) \\ &= (1 - x_i^1(\pi \times \theta)) + Rx_i^1(\pi \times \theta) + T_i(\pi \times \theta) + \pi_i V. \end{aligned} \tag{4.3}$$

Using (4.1) and (4.3), the inter-island enforcement constraint (3.9) can be rewritten as

$$-T_i(\pi \times \theta) \leq \theta_i(R - 1) + (x_i^1(\pi \times \theta) - 1)(R - 1).$$

The extent of insurance is also limited by the fact that enforcement of the contract with *each* borrower on an island is limited. This implies that no more than $x_i^1(\pi \times \theta)(\pi_i R - 1)/\pi_i$ per borrower with valuable collateral can be extracted. The next lemma summarizes this discussion and characterizes the ex-post inter-island participation constraint (3.9) in terms of the feasible transfer $T_i(\cdot)$ that can be imposed by an optimal inter-island contract.

Lemma 4.5. *Insurance is feasible for an inter-island contract if*

$$-T_i(\pi \times \theta) \leq \min\{\theta_i(R-1) + (x_i^1(\pi \times \theta) - 1)(R-1), (\pi_i R - 1)x_i^1(\pi \times \theta)\}$$

for any i s.th. $\pi_i > \bar{\pi}$ and $T_i(\pi \times \theta) \geq 0$ otherwise.

Define $\bar{T}_i(\pi \times \theta) = (-1)\min\{\theta_i(R-1), (\pi_i R - 1)\}$. The next result characterizes total investment for the optimal inter-island contract in the presence of enforcement problems among islands.

Proposition 4.6. *Overall investment at $t = 1$ for the optimal inter-island contract depends on the joint distribution $\pi \times \theta$ and is given by*

$$\sum_{i=1}^{N} x_i^1(\pi \times \theta) \leq N - K. \tag{4.4}$$

Investment on individual islands at $t = 1$ is uniquely characterized by

$$x_i^1(\pi \times \theta) = \begin{cases} 1 & \text{if } \pi_i > \tilde{\pi} \\ \tilde{x} & \text{if } \pi_i = \tilde{\pi} \\ 0 & \text{if } \pi_i < \tilde{\pi} \end{cases},$$

such that $\bar{\pi} \geq \tilde{\pi} \geq \pi_K$ and $\tilde{x} \in [0,1]$ satisfy

$$\sum_{i>K}^{N-M} \left[(\pi_i R - 1)x_i^1(\pi \times \theta)\right] = \sum_{i>N-M}^{N} \bar{T}_i(\pi \times \theta). \tag{4.5}$$

Furthermore, $\sum_{i=1}^{N} x_i^1(\pi \times \theta) < N - K$ if and only if $\theta_i < \frac{\pi_i R - 1}{R-1}$ for some i such that $\pi_i > \bar{\pi}$.

Corollary 4.7. *If* $\max_{\theta_i \in \Theta} \theta_i \to 0$, $x_i^1(\pi \times \theta) \to 0$ *for all* i *s.th.* $\pi_i < \bar{\pi}$.

While insurance and liquidity provision are still feasible, the proposition illustrates that the crucial factor for investment in the optimal inter-island contract is now the specific realization of the joint distribution $\pi \times \theta$.[7] Whereas the distribution of asset shocks still determines which islands keep their funds invested, the amount of transfers depends on the size of the liquidity shock for islands with a good asset shock. Unlike in Proposition 4.4 the cut-off point for investment at $t = 1$, however, now depends on the joint distribution of shocks.

If for some island i with $\pi_i > \bar{\pi}$ the liquidity shock is too small relative to the collateral shock, the island has an incentive to leave any contract that imposes transfer payments that are too high. Hence, overall transfers to islands with adverse collateral shocks are too small to achieve full investment. The unique cut-off point and, hence, the total amount of investment depends on the maximal transfer compatible with inter-island enforcement, which in turn is a function of the joint distribution $\pi \times \theta$.

It is worth noting that the optimal contract separates the mechanisms for liquidity provision and insurance against asset shocks. All islands with a sufficiently high rate of performing loans receive sufficient liquidity from other islands to keep all projects alive. However, insurance transfers vary with the joint distribution of shocks due to limited inter-island enforcement. This leads to endogenous aggregate uncertainty, even though the economy

[7] Another factor that matters is the overall size of liquidity needs in the economy and, hence, the importance of overall liquidity needs relative to collateral risk. This is not relevant here since we assume that $K \geq L$ ([14]).

as a whole is exposed solely to idiosyncratic asset and liquidity uncertainty.

Corollary 4.7 demonstrates that liquidity shocks serve as a "threat point" to keep islands from leaving the contract. As long as there are strictly positive liquidity shocks, some insurance against collateral shocks may be possible even if enforcement between islands is limited. In the absence of liquidity shocks ($\theta_i = 0$ for all i), however, limited enforcement among islands results in zero inter-island transfers: $\bar{T}_i = 0$ for all i and feasible $\{x_i^1(\pi \times \theta)\}_{i=1}^N$.

This stands in sharp contrast to Proposition 4.4. With full enforcement between islands, liquidity needs are inessential for the feasibility of insurance. Here one can always extract all surplus from borrowers with valuable collateral. Hence, the maximum amount of transfers can achieve full investment for all i s.th. $\pi_i > \pi_K$ independent of the liquidity shock.

Paradoxically, if inter-island enforcement is limited, a realized negative correlation between both shocks helps to increase investment. This is due to the fact that the higher the liquidity needs on islands with good collateral shocks, the more transfers are available for insurance or, equivalently, for any $\pi_i \geq \bar{\pi}$, $\frac{\partial |\bar{T}_i|}{\partial \theta_i} \geq 0$. Provided liquidity needs are high enough on all islands with $\pi_i \geq \bar{\pi}$, optimal contracts with and without full inter-island enforcement coincide.

5 Competitive Interbank Markets

We turn now to our second question concerning the relative efficiency of interbank liquidity markets versus closed interbank relationships such as "banking clubs". In this section we compare the optimal contract between banks described above with a decentralized market for interbank loans.

5.1 Market Structure

We define a market based system as one where individual islands or banks (i) take the interest rate as given and (ii) demand or supply funds on a decentralized market for interbank loans at $t = 1$ once both shocks have been realized. We assume that the information concerning the realized asset shocks for each island is public.

After the asset and liquidity shocks are realized, an island has then to decide whether to borrow or to lend on the interbank market at the gross interest rate R_{IB} and whether to continue funding projects of borrowers on the island. When making this decision, it has to honor its obligations to lenders that want to consume at $t = 1$ and ensure the participation of lenders that want to consume at $t = 2$. The participation constraints of borrowers remain unchanged.

Let z_i denote the funds supplied on the interbank market by bank i. If $z_i \geq 0$ bank i supplies funds on the interbank market, while $z_i \leq 0$ denotes borrowing. The total investment on island i at $t = 1$ is still denoted x_i^1. These funds can be obtained from funds originally deposited on island i or through interbank borrowing.

Taking into account withdrawals by early lenders, total resources at $t = 2$ available for consumption are equal to

$$Rx_i^1 + R_{IB}z_i + [(1 - \theta_i) - z_i - x_i^1] \tag{5.1}$$

where the first term describes the gross return from continuing projects of borrowers whereas the second term refers to proceeds from interbank loans. Since the last term describes the remaining funds not used for investment and hence stored, we require

$$1 - \theta_i \geq z_i - x_i^1. \tag{5.2}$$

The resource constraint is given by

$$Rx_i^1 + R_{IB}z_i + [(1-\theta_i) - z_i - x_i^1] = \pi_i c_{B,i}^V + (1-\pi_i)c_{B,i}^0 + (1-\theta_i)c_{L,i}^2. \tag{5.3}$$

Since the asset shocks are publicly known when trading takes place on interbank markets, each island faces a borrowing constraint on the interbank market that conditions on the net value of a banks' assets. Specifically, we require that the costs of a loan (principal plus interest) are covered by the value of assets *after* depositors have been paid back. This borrowing constraint is endogenous as it is determined by the equilibrium interbank interest rate R_{IB}, and is formally expressed by

$$\pi_i Rx_i^1 + [(1-\theta_i) - z_i - x_i^1] - (1-\theta_i) \geq -z_i R_{IB}. \tag{5.4}$$

It is apparent that the borrowing constraint is determined by the value of a bank's assets. These are equivalent to the amount that can be extracted from borrowers with valuable collateral.

5.2 Interbank Market Equilibrium

The objective function of island i is to maximizes the utility of its borrowers' subject to resource and participation constraints. This problem is identical to an island-specific contract described earlier except that at $t = 1$ the island supplies or demands funds on the interbank market. We show in the Appendix that it is possible to find the optimal net supply of an island on the interbank market by solving problem ($\mathcal{P}4$):

$$\max_{(x_i^1, z_i)} \pi_i V + (R-1)x_i^1 + (R_{IB}-1)z_i$$
$$\text{subject to}$$
$$(\pi_i R - 1)x_i^1 + (R_{IB}-1)z_i \geq 0$$
$$x_i^1 \in [0,1]$$
$$1 - \theta_i \geq x_i^1 + z_i.$$

Islands maximize the total return of their investment taking into their three options of investment while being constrained in their borrowing. Having defined the net supply of a single island for a given interbank rate R_{IB} we turn next to the concept of interbank market equilibrium.

Definition 5.1. *An interbank market equilibrium at $t = 1$ is an interest rate $\hat{R}_{IB}$ and investment levels $\{\hat{x}_i^1, \hat{z}_i\}_{i=1}^N$ such that*

1. *given $\hat{R}_{IB}$, for every island i, $(\hat{x}_i^1, \hat{z}_i)$ solve ($\mathcal{P}4$)*
2. *the liquidity market clears: $\sum_{i=1}^N \hat{z}_i = 0$.*

It is clear that any interbank rate above R or below 1 cannot be an equilibrium because the market would never clear. Hence, $R_{IB} \in [1, R]$.

5.3 Inefficiency of Equilibrium

The allocation of an island's funds depends on two factors. First, the return on projects *after* default, $\pi_i R$, determines whether an island borrows or lends on the interbank market. Second, depending on the interbank rate R_{IB} islands might be constrained in their borrowing.

Profitable islands ($\pi_i R > 1$) will borrow for liquidity needs while all other islands supply funds. The reason is that islands with a high fraction of performing loans ($\pi_i R > 1$) can secure interbank loans by securitizing their loans. Islands where $\pi_i R > 1$ will borrow to finance all their projects, unless the interbank interest rate exceeds the return on an island ($R_{IB} > \pi_i R$), *and* liquidity needs are so high that the borrowing constraint (5.4) binds.

Other islands lend, but use the return on this lending to subsidize some projects on their island. The borrowing constraint *always* binds for these islands. In effect, equation (5.4) is a solvency constraint for the island. Since lenders have to receive their deposits back, the receipts from interbank lending determine the measure of projects that can be financed. These arguments are summarized in the proposition below.

Proposition 5.2. *1. If $\pi_i R \geq R_{IB}$, $x_i^1 = 1$ and $z_i = -\theta_i$.*

2. Let $R_{IB} > \pi_i R \geq 1$. If $\theta_i \leq \frac{\pi_i R - 1}{R_{IB} - 1}$, $x_i^1 = 1$ and $z_i = -\theta_i$. Otherwise, $x_i^1 = (1-\theta_i)\frac{R_{IB}-1}{R_{IB}-\pi_i R}$ and $z_i = (1-\theta_i)\frac{1-\pi_i R}{R_{IB}-\pi_i R}$.

3. Let $\pi_i R < 1$. Then $x_i^1 = (1-\theta_i)\frac{R_{IB}-1}{R_{IB}-\pi_i R}$ and $z_i = (1-\theta_i)\frac{(1-\pi_i R)}{R_{IB}-\pi_i R}$.

While demand is decreasing in R_{IB}, it is striking that supply is also decreasing in R_{IB}. This is due to the fact that islands want to use the receipts to subsidize projects with a social return of R. Since increases in R_{IB} increase the amount available for an island to subsidize projects or, equivalently, relax the solvency constraint (5.4), the liquidity supply curve is downward sloping.

We turn now to the question of whether interbank markets can allocate investment funds as efficiently as the optimal contract. Since utility is linear we can answer these questions by comparing equilibrium net transfers with the net transfers specified in the optimal contract.

When liquidity needs do not interfere with insurance ($K \geq L$), Proposition 5.2 implies that there will always be an excess supply of funds in the liquidity market unless $R_{IB} = 1$. At this interbank rate, however, transfers are zero and hence islands that supply funds can not subsidize any projects with the returns from their lending. This leads to the following characterization of equilibrium.

Proposition 5.3. *The unique equilibrium is given by the interest rate* $R^*_{IB} = 1$ *and investment levels at* $t = 1$ *are equal to* $x^1_i = 1$ *for* $\pi_i R \geq 1$ *and* $x^1_i = 0$ *otherwise.*

Corollary 5.4. *If* $K \geq L$*, the optimal contract strictly dominates the equilibrium outcome at* $t = 1$ *in welfare independent of the realized joint distribution of shocks.*

Interbank markets can efficiently allocate liquidity across islands or banks, but they are unable to provide insurance against asset shocks. When aggregate liquidity needs are low relative to the supply of funds, competition drives down the interbank rate to 1. To the contrary, the optimal contract conditions in a peculiar way on the joint distribution of shocks thereby effectively charging differential interest rates larger than 1 for liquid funds. It is then possible within the optimal contract to always extract at least some excess returns from profitable islands.[8]

[8] When $N - M \geq L > K$, some insurance is feasible. Liquidity markets, however,

6 Discussion

This paper shows that, when debt contracts are subject to limited enforcement, there is an interesting interaction between liquidity and collateral shocks. In effect, the optimal contract uses the threat of exclusion from liquidity lending to induce banks to provide mutual interbank asset insurance. This illustrates our main message that interbank arrangements for asset insurance can be supported by the need for interbank loans to meet stochastic liquidity demands.

These results can help us understand some examples of banks entering into informal (and sometimes formal) arrangements to provide mutual insurance against shocks affecting their assets and liabilities. These interbank relationships typically feature some form of limited enforcement as banks are free to exit. For example, the arrangements between cooperative banks in Germany appear to match the story of the model. Cooperative banks in Germany are arranged in groups centered about larger banks ([10]). This larger bank coordinates the liquidity based trades of the smaller banks. In addition, these clubs explicitly agree to provide insurance against asset shocks to one another. We interpret this commitment to provide insurance as being credible in part due to the fact that leaving the club would considerably aggravate access to liquidity.

achieve a less efficient distribution of funds across islands. Whether the distribution is *strictly* less efficient depends on the realized joint distribution of shocks. Finally, for $L > N - M$ the optimal contract always finances the maximum number of projects $N - L$. In this case, the interbank market outcome and the optimal contract are identical. Since liquidity problems are extremely severe, insurance is not an issue. For details see [14].

There are also historical examples which match the qualitative predictions of our model. [12] documents that during periods of high liquidity needs, American clearinghouses in the National Banking era would provide guarantees of members assets. [16] documents an example of a group of Canadian banks stepping in to "take-over" an insolvent bank and thus maintain its ongoing operations. In both of these examples, there was active liquidity provision between banks that preceded the provision of "insurance" to one another.

Our second main result is that "banking clubs" are preferred to anonymous interbank liquidity markets. This result is particularly interesting given that [2] have argued that non-market arrangements can yield superior risk sharing than market based arrangements. Our paper provides another example of the superiority of non-market arrangements for insurance.

Another important result is that the optimal contract generates endogenous aggregate uncertainty. The level of interbank transfers – and hence the level of investment – depends upon the realized joint distribution of asset and liquidity shocks. While the paper contains several specific features associated with the banking literature, the implications for multi-dimensional risk sharing are general. Hence, this result will generalize to models with two distinct sources of uncertainty where the threat of exclusion from trades that mitigate one source of risks can be used as a "leverage" to induce or increase risk sharing along the second dimension.

Finally, several short remarks concerning our assumptions and theoretical approach are in order. A key assumption is that projects are inherently "good". This assumption is reasonable for many projects where entrepreneurs post assets (i.e. personal homes) as collateral for projects which do not directly use the collateral. In such a world, shocks to the value of collateral lead to a reduction in loans by local banks. However, these shocks do not directly affect the return on the projects. Moreover, introducing stochastic returns into the model does not influence the results since the cut-off values are still defined over the average rate of profitable projects on an island which is now adjusted for the realization of the return of a project. A second key assumption is that banks use the proceeds from the interbank markets to subsidize projects on their own islands. This assumption may best describe "cooperative banks" which place weight on the economic performance of the region in which they are active.

7 Appendix

Proof of Proposition 4.2

Proof. Since the collateral good is valued only by borrowers, setting $\delta_i^V(\pi \times \theta) = 1$ relaxes the borrowers' participation constraints (3.4) and (3.5) independently of x_i^0 and $x_i^1(\pi \times \theta)$ and, hence, is optimal. By Assumption 4.1 the borrowers' participation constraint (3.4) does, then, not bind for any $x_i^0 \in [0, 1]$. Hence, lenders are always able to withdraw their funds at stage $t = 1$ after they observe the realization of $\tilde{\pi}_i$ and $\tilde{\theta}_i$. Since $x_i^0 = 1$ initiates the maximum number of potentially profitable projects, it is optimal.

note that all the constraints of problem $\mathcal{P}2$ are specified as functions of the state $\pi \times \theta$. Thus, the problem for island i can be decomposed into a separate problem for each island-specific realization of $\tilde{\pi}_i$ and $\tilde{\theta}_i$. To satisfy the participation constraint (3.2), a fraction θ_i of investment has to be withdrawn at stage $t = 1$. Thus, $x_i^1(\pi \times \theta) \leq 1 - \theta_i$, which implies that for $N = 1$ equation (3.7) holds.

By Assumption 4.1, $V > R$ and equation (3.5) never binds. Let equations (3.2), (3.3) and (3.6) hold with equality. For $N = 1$, the feasibility constraint for $t = 2$ is then given by

$$(1 - \pi_i)Rx_i^1(\pi \times \theta) + \pi_i c_{B,i}^V(\pi \times \theta) \leq Rx_i^1(\pi \times \theta) - x_i^1(\pi \times \theta) \tag{7.1}$$

or

$$\pi_i c_{B,i}^V(\pi \times \theta) \leq (\pi_i R - 1)x_i^1(\pi \times \theta). \tag{7.2}$$

Suppose now that $\pi_i R < 1$. Then $x_i^1(\pi \times \theta) \in (0, 1 - \theta_i]$ violates non-negativity of $c_{B,i}^V(\pi \times \theta)$. Hence, $x_i^1(\pi \times \theta) = 0$.

If $\pi_i R \geq 1$, every $x_i^1(\pi \times \theta) \in [0, 1 - \theta_i]$ is feasible. Since the objective function is strictly increasing in $c_{B,i}^V(\pi \times \theta)$ and $c_{B,i}^0(\pi \times \theta)$, which are increasing in $x_i^1(\pi \times \theta)$, it is optimal to set $x_i^1(\pi \times \theta) = 1 - \theta_i$. □

Proof of Proposition 4.4

Proof. By Assumption 4.1 it follows that $\delta_i^V(\pi \times \theta) = 1$ and $x_i^0 = 1$ for all $i = 1, \ldots, N$.

Let $K \geq L$. To verify feasibility of investment levels, set $c_{L,i}^t(\pi \times \theta) = 1$, for $t = 1, 2$, $c_{B,i}^0(\pi \times \theta) = Rx_i^1(\pi \times \theta)$ and substitute the investment functions into equations (3.7) and (3.8) to obtain

$$\sum_{i=1}^{N} \theta_i \leq K \tag{7.3}$$

and

$$\sum_{i=1}^{N} \pi_i c_{B,i}^{V}(\pi \times \theta) + \sum_{i>K}^{N} (1-\pi_i) + \sum_{i=1}^{N} (1-\theta_i) \leq \sum_{i>K}^{N} R - \left[\sum_{i=1}^{N} \theta_i - K \right]. \tag{7.4}$$

Since $E(\theta_i)N = L \leq K$ equation (7.3) is satisfied and equation (7.4) can be rewritten as

$$\sum_{i=1}^{N} \pi_i c_{B,i}^{V}(\pi \times \theta) \leq \sum_{i>K}^{N} (\pi_i R - 1)\,. \tag{7.5}$$

The definition of K implies $\sum_{i>K}^{N} (\pi_i R - 1) = 0$, and, hence, feasibility at $t = 2$ is satisfied with $c_{B,i}^{V}(\pi \times \theta) = 0$ for all $i = 1, \ldots, N$.

Next, we have to verify the optimality of the contract. Increasing $c_{B,i}^{V}(\pi \times \theta)$ for any i - or reducing $x_i^1(\pi \times \theta)$ for some island i - without increasing investment for some other island reduces total investment, and, hence, also welfare. Let $\epsilon > 0$ and define a new investment function by

$$\tilde{x}_i^1(\pi \times \theta) = \begin{cases} 1-\epsilon & \text{for some n s.th. } \pi_n \geq \pi_K \\ \epsilon & \text{for some m s.th. } \pi_m < \pi_K \\ x_i^1(\pi \times \theta) & \text{otherwise} \end{cases} .$$

The slack for the resource constraint (3.7) is unaltered. The right-hand side of equation (7.9) is now given by

$$\sum_{i>K, i \neq n,m}^{N} (\pi_i R - 1) + (1-\epsilon)(\pi_n R - 1) + \epsilon(\pi_m R - 1) < 0. \tag{7.6}$$

Hence, for some i, $c^V_{B,i}(\pi \times \theta) < 0$. Therefore, to satisfy feasibility investment has to be reduced further for some island i with $\bar{\pi} > \pi_i > \pi_K$. This reduces overall investment and, hence, the value of the objective function. $\square$

Proof of Proposition 4.6

Proof. It is clear that $\delta^V_i(\pi \times \theta) = 1$ and $x^0_i = 1$ for all $i = 1, \ldots, N$. Since the set of feasible investment levels of problem $\mathcal{P}1$ is a subset of those of problem $\mathcal{P}3$, equation 4.4 follows then immediately from Proposition 4.4. Dependence of overall investment on the joint distribution is a direct consequence of the optimal investment levels which we establish below.

Suppose $K \geq L$. It is clear from Proposition 4.4 that $x^1_i(\pi \times \theta) = 1$ for all i s.th. $\pi_i \geq \bar{\pi}$ and $x^1_i(\pi \times \theta) = 0$ for all i s.th. $\pi_i \leq \pi_K$. Since the objective function is increasing in the number of projects funded, it is optimal to fund as many projects on islands with $\pi_K < \pi_i < \bar{\pi}$ as possible. The possibility of funding projects on these islands depends on the maximal amount of transfers that can be extracted from islands with $\pi_i \geq \bar{\pi}$. It follows - by an identical argument to that used in the proof of Proposition 4.4 - that equation 4.5 holds with equality for a unique $\tilde{\pi}$ and $\tilde{x}_i$.

Suppose $\exists$ some i s.th. $\pi_i > \bar{\pi}$ and $\theta_i < \frac{\pi_i R - 1}{R-1}$. Then, for any feasible $\{T_i\}_{i=1}^N$,

$$-T_i(\pi \times \theta) \leq \theta_i(R-1) < (\pi_i R - 1) \tag{7.7}$$

and

$$\sum_{i>K}^{N-M} (\pi_i R - 1) < \sum_{i>N-M}^{N} T_i(\pi \times \theta). \tag{7.8}$$

Hence, to satisfy equation 4.5, $x_i^1(\pi \times \theta) < 1$ for some i s.th. $\pi_K < \pi_i < \bar{\pi}$. Therefore, $\sum_{i=1}^N x_i^1(\pi \times \theta) < N - L$.

Conversely, if $\theta_i \geq \frac{\pi_i R-1}{R-1}$ for all $\pi_i > \bar{\pi}$, the transfer scheme of Proposition 4.4 is feasible and achieves $\sum_{i=1}^N x_i^1(\pi \times \theta) = N - L$. $\square$

Derivation of Problem $(\mathcal{P}4)$**:**

Using Assumption 4.1, it is straightforward to show that it is still optimal for island i to choose $x_i^0 = 1$. Since early lenders have to be paid back their investment irrespective of R_{IB}, the problem of an individual bank that has access to an interbank loan market at $t = 1$ can then be reduced to

$$\max_{x_i^1, z_i} \pi_i[c_{B,i}^V + V] + (1 - \pi_i)c_{B,i}^0$$

subject to

$$\begin{aligned}
&Rx_i^1 + R_{IB}z_i + [(1-\theta_i) - z_i - x_i^1] = \\
&\qquad \pi_i c_{B,i}^V + (1-\pi_i)c_{B,i}^0 + (1-\theta_i)c_{L,i}^2 \\
&\pi_i R x_i^1 + [(1-\theta_i) - z_i - x_i^1] - (1-\theta_i) \geq -z_i R_{IB} \\
&c_{L,i}^2 \geq 1 \\
&c_{B,i}^V + V \geq R x_1^i \\
&c_{B,i}^0 \geq R x_i^1 \\
&1 - \theta_i \geq z_i - x_i^1 \\
&x_i^1 \in [0, 1].
\end{aligned}$$

¿From earlier arguments, we have $c_L^2 = 1$, $c_{B,i}^0 = Rx_i^1$ and that the participation constraint for borrowers with valuable collateral never binds (cf.

Assumption 4.1 and $\delta_i = 1$). Using these results one can solve the resource constraint for $c^V_{B,i}$. By the solvency constraint (5.4), $c^V_{B,i} \geq 0$. Thus, one can rewrite the objective function using the expressions for $c^0_{B,i}$ and $c^V_{B,i}$ to obtain the objective function for ($\mathcal{P}4$).

Proof of Proposition 5.2:

Proof. Let $\pi_i R \geq R_{IB} \geq 1$. It is optimal to set x^1_i, since $R > R_{IB}$ and the solvency constraint does not bind.
Let $R_{IB} > \pi_i R \geq 1$. Again $x^1_i = 1$ is the solution as long as the solvency constraint doesn't bind. If $\theta_i \leq \frac{\pi_i R - 1}{R_{IB} - 1}$ it does not bind. Otherwise it will be binding. Solving the constraint for z as a function of x^1_i we obtain

$$z_i = \frac{1 - \pi_i R}{R_{IB} - 1} x^1_i.$$

Since all funds will be invested ($R_{IB} > 1$), using $x^1_i + z_i = 1 - \theta_i$ we obtain the result.
Finally, let $R_{IB} \geq 1 > \pi_i R$. Then the solvency constraint will always be binding and $x^1_i > 0$ only if $z_i > 0$. Using the solvency constraint and the fact that all funds will be invested, we again obtain the result. □

Proof of Proposition 5.3:

Lemma: Transfers in the optimal contract are at least as high as in equilibrium.

Proof. Let $K \geq L$. Denote the equilibrium interest rate by R^*_{IB}. From Proposition 4.6 we know that the optimal contract specifies

$$\min\{\theta_i(R-1); \pi_i R - 1\}$$

as transfers for islands with $\pi_i R - 1$.
If $\pi_i R > R^*_{IB}$, island i transfers

$$\theta_i(R^*_{IB} - 1) \leq \min\{\theta_i(R-1); \pi_i R - 1\} \tag{7.9}$$

resources in equilibrium.
Suppose now that $R^*_{IB} \geq \pi_i R > 1$ and $\theta_i \leq \frac{\pi_i R - 1}{R^*_{IB} - 1}$. Then inequality (7.9) is clearly satisfied.
If $R^*_{IB} \geq \pi_i R > 1$ and $\theta_i > \frac{\pi_i R - 1}{R^*_{IB} - 1}$, we only have to verify that $z_i(R^*_{IB} - 1) \leq \pi_i R - 1$. This follows since transfers in equilibrium are given by

$$(1 - \theta_i)(R^*_{IB} - 1)\frac{\pi_i R - 1}{R^*_{IB} - \pi_i R} \tag{7.10}$$

and

$$\theta_i > \frac{\pi_i R - 1}{R^*_{IB} - 1}. \tag{7.11}$$

Hence, transfers in equilibrium are less than transfers in the optimal contract.

□

We turn now to the proof of the proposition.

Proof. Let $R_{IB} = 1$. Then, if $\pi_i R < 1$ for island i, $z_i \in [0, 1 - \theta_i]$. Clearly, since $N - M \geq L$, there exists $\{z_i\}_{i=1}^N$ such that $\sum_{i=1}^N z_i = 0$. Hence, there exists an equilibrium with R_{IB}.

We show next that there does not exist any other equilibrium. From the optimal decisions of each island, it is clear that $x_i^1 + z = (1 - \theta_i)$. Thus, for all $R_{IB}^* \in [R, 1)$, we have in equilibrium that

$$\sum_{i=1}^{N} x_i^1 = \sum_{i=1}^{N} x_i^1 + \sum_{i=1}^{N} z_i = \sum_{i=1}^{N}(x_i^1 + z_i) = \sum_{i=1}^{N}(1 - \theta_i) = N - L. \quad (7.12)$$

Since $K > L$, total investment in the optimal contract is at most $N - K < N - L$. Since transfers are less in equilibrium than in the optimal contract, $N - L$ cannot be feasible.

Suppose now that $K = L$. If for some island i with $\pi_i R > 1$, $\theta_i < \frac{\pi_i R - 1}{R - 1}$, total investment in the optimal contract is less than $N - L$. By the lemma, there cannot be an equilibrium with $R_{IB} > 1$.

Suppose that for all i such that $\pi_i R > 1$, $\theta_i \geq \frac{\pi_i R - 1}{R - 1}$. If for some i with $\pi_i R > 1$, $\pi_i R \geq R_{IB}$, transfers cannot be maximal since

$$\theta_i(R_{IB} - 1) \leq \theta_i(\pi_i R - 1) < \pi_i R - 1.$$

Hence, for any equilibrium, $R_{IB} > \pi_{max} R$, since otherwise total investment is strictly below $N - L$.

If $\theta_i > \frac{\pi_i R - 1}{R_{IB} - 1}$, transfers are given by

$$(1 - \theta_i)(R_{IB} - 1)\frac{\pi_i R - 1}{R_{IB} - \pi_i R} < \pi_i R - 1.$$

Since for any equilibrium we need total investment $N - L$, we need $\theta_i(R_{IB} - 1) = \pi_i R - 1$ for all islands i such that $\pi_i R > 1$. For any $R_{IB} \in [R, \pi_{max})$, this implies that investment on those islands is $x_i^1 = 1$. Thus,

$$
\begin{aligned}
\sum_{i=1}^{N-M}(1-\theta_i)\frac{R_{IB}-1}{R_{IB}-\pi_i R}+\sum_{i>N-M}^{N}x_i^1 &< \sum_{i=1}^{N-M}(1-\theta_i)\frac{R-1}{R-\pi_i R}+\sum_{i=1}^{N}x_i^1 \\
&< \sum_{i=1}^{N-M}(1-\theta_i)\frac{R-1}{R-\pi_i R}+M \\
&< N-M-K+M=N-K=N-L
\end{aligned}
$$

where the last inequality follows from the fact that all islands with $\pi_i < \pi_K$ finance some projects which is not efficient. This is a contradiction to the fact that in equilibrium total investment is equal to $N-L$. Hence, $R_{IB}=1$ is the unique equilibrium. □

References

1. F. Allen and D. Gale, "Financial Contagion", *J. Polit. Economy*, **108** (2000), 1-33.

2. F. Allen and D. Gale, "Comparing Financial Systems", MIT University Press, Cambridge Massachusetts, 2000.

3. S. Bhattacharya and D. Gale. "Preference Shocks, Liquidity, and Central Bank Policy", in W.A. Barnett and J. Singleton (eds.), New Approaches to Monetary Theory, Cambridge University Press, New York, New York, 1987.

4. S. Bhattacharya and P. Fulghieri. "Uncertain Liquidity and Interbank Contracting", *Econ. Letters*, **44** (1994), 287-294.

5. J. Bryant. "A Model of Reserves, Bank Runs, and Deposit Insurance", *J. Banking Finance*, **4** (1980), 335-344.

6. C.W. Calomiris and C.M. Kahn. "The Role of Demandable Debt in structuring Optimal Banking Arrangements", *Amer. Econ. Rev.*, **81** (1991), 497-513.

7. V.V. Chari. "Banking Without Deposit Insurance or Bank Panics: Lessons From a Model of the U.S. National Banking System", *Fed. Reserve Bank Minneapolis Quart. Rev.*, **13** (1989).

8. D.W. Diamond and P.H. Dybvig. "Bank Runs, Deposit Insurance, and Liquidity", *J. Polit. Economy*, **91** (1983), 401-419.

9. D.W. Diamond and R.G. Rajan. "Bank Bailouts and Aggregate Liquidity Shortages", *Amer. Econ. Rev.*, **92** (2001), 38-42.

10. M. Ehrmann, L. Gambacorta, J. Martinez-Pages, P. Sevestre, and A. Worms. "Financial Systems and the Role of Banks in Monetary Policy Transmission in the Euro Area", *European Central Bank Working Paper No. 105* (2001).

11. X. Freixas, B. Parigi and J. Rochet. "Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank", *J. Money, Credit, Banking* **32** (2000), 611-638.

12. G. Gorton. “Clearinghouses and the Origin of Central Banking in the United States” *J. Econ. Hist.*, **45** (1985), 277-283.

13. N. Kocherlakota. “Risky Collateral and Deposit Insurance”, *Advances in Macroeconomics* **1** (2001), Article 2.

14. T.V. Koeppl and, J.C. MacGee. “Limited Enforcement and Efficient Interbank Arrangements”, Federal Reserve Bank of Minneapolis Working Paper 608, (2001).

15. J. Rochet and J. Tirole. “Interbank lending and Systemic Risk”, *J. Money, Credit, Banking*, **28** (1996), 733-762.

16. S.D. Williamson. “Bank Failures, Financial Restrictions, and Aggregate Fluctuations: Canada and the United States, 1870-1913”, *Fed. Reserve Bank Minneapolis Quart. Rev.*, **13** (1989).