

- JARL G. TAXERÅS FLATEN, *Internal injectivity of modules in higher toposes*.
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Any “internal” property in a (higher) topos ought to be stable by base change. While this is automatically the case for properties coming from an internal language, such properties are often unwieldy and one seeks simpler characterizations. Our present interest is in comparing the notion of injectivity of a module coming from HoTT, to the preexisting notion of internal injectivity. This is part of joint, ongoing work with Dan Christensen on the interpretation of homological algebra from HoTT into higher toposes.

In the 80s, Roswitha Harting proved that internal injectivity of abelian groups in an elementary topos \mathcal{E} is stable by base change. Central is her construction [1] of a left-exact left adjoint to pullback of abelian groups, called the *internal coproduct*:

$$\bigoplus_X : \text{Ab}(\mathcal{E}/X) \rightleftarrows \text{Ab}(\mathcal{E}) : X^*$$

In 2017, Ingo Blechschmidt remarked in his thesis that Harting’s construction works for modules as well, and further proves that internal injectivity of modules corresponds to injectivity in the *Stacks semantics* of an elementary topos.

We prove that for higher toposes satisfying the external axiom that “sets cover,” internal injectivity of modules coincides with the notion of injectivity coming from HoTT. Thus the notions coincide for ∞ -sheaves on a 1-site. In general, we show that internal injectivity is stable by pullback over 0-truncated objects as well as pointed, connected objects. When pulling back over untruncated objects, the analog of the internal coproduct is an internal colimit. We give a novel construction of this internal colimit in HoTT, and produce examples demonstrating that it may fail to be left-exact.

[1] ROSWITHA HARTING, *Internal coproduct of abelian groups in an elementary topos*, *Communications in Algebra*, vol. 10 (1982), no. 11, pp. 1173–1237.