PSY 9555A (Nov 13): SEM Approach to Multilevel Modeling with Mplus

- MLM in Mplus Data structure and syntax
- Intercept only model
- Level 1 predictors
- Intercepts and slopes as outcomes
- Measures within persons
- LGM and MLM
- Multilevel CFA

Additional resources

Mplus manual (especially Chapter 9)

Hox, J. J. (2010). Multilevel analysis. Techniques and applications. 2nd ed.

Example (from Joop Hox book on Multilevel Analysis, 2010)

Data

```
Level 1: pupils (2000 in total)
```

Level 2: Classes (100 classes of approx 20 pupils each)

Outcome variable:

```
pupil popularity (scale 0-10)
```

Predictor variables at pupil level (level 1)

pupil gender (1=girl 0=boy)

pupil extraversion (scale 1-10)

Predictor variable(s) at class level (level 2)

teacher experience (scale 2-25)

Data Structure

```
Mplus - [popular2modified]

File Edit View Mplus Plot Diagram Window Help

TITLE: Random Effects ANOVA using Mplus;

DATA:

FILE IS "popular2.dat";

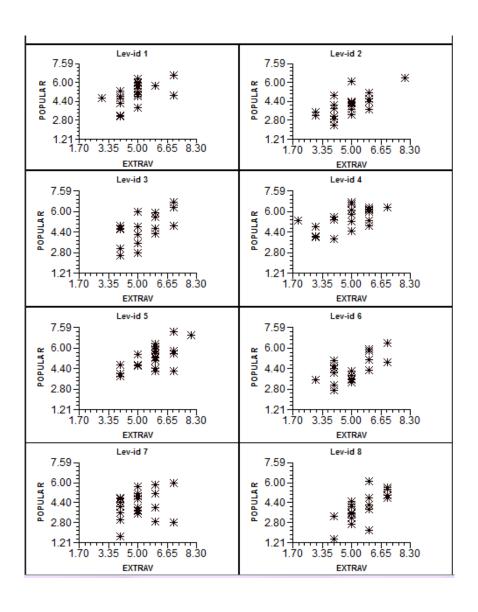
VARIABLE:

NAMES ARE class pupil cons extrav sex texp popular popteach zextrav zsex ztexp zpopular zpoptch; usevariables are popular extrav;

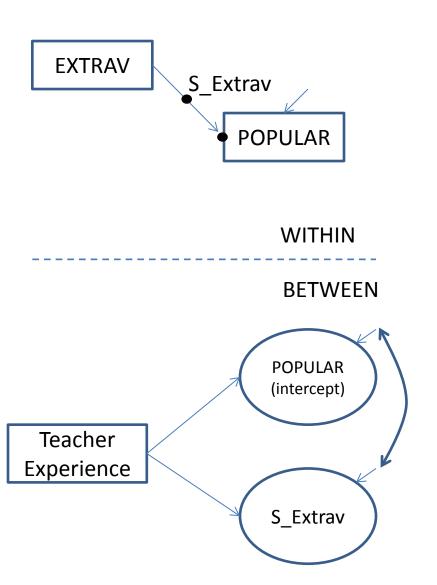
CLUSTER IS class;
```

popular2 - Notepad File Edit Format View Help 1 1 1 5 1 24 6.3 6 -.17031 .98881 1.48615 .88501 .66906 1 2 1 7 0 24 4.9 5 1.41401 -1.01081 1.48615 -.12763 -.04308 1 3 1 4 1 24 5.3 6 -.96248 .98881 1.48615 .1617 .66906 1 4 1 3 1 24 4.7 5 -1.75464 .98881 1.48615 -.27229 -.04308 1 5 1 5 1 24 6 6 -.17031 .98881 1.48615 .66802 .66906 1 6 1 4 0 24 4.7 5 -.96248 -1.01081 1.48615 -.27229 -.04308 1 7 1 5 0 24 5.9 5 -.17031 -1.01081 1.48615 .59569 -.04308 1 8 1 4 0 24 4.2 5 -.96248 -1.01081 1.48615 -.63395 -.04308 1 9 1 5 0 24 5.2 5 -.17031 -1.01081 1.48615 .08937 -.04308 1 10 1 5 0 24 3.9 3 -.17031 -1.01081 1.48615 -.85095 -1.46737 1 11 1 5 1 24 5.7 5 -.17031 .98881 1.48615 .45102 -.04308 1 12 1 5 1 24 4.8 5 -.17031 .98881 1.48615 -.19996 -.04308 1 13 1 5 0 24 5 5 -.17031 -1.01081 1.48615 -.0553 -.04308 1 14 1 5 1 24 5.5 6 -.17031 .98881 1.48615 .30636 .66906 1 15 1 5 1 24 6 5 -.17031 .98881 1.48615 .66802 -.04308 1 16 1 6 1 24 5.7 5 .62185 .98881 1.48615 .45102 -.04308 1 17 1 4 0 24 3.2 2 -.96248 -1.01081 1.48615 -1.35727 -2.17951 1 18 1 4 0 24 3.1 3 -.96248 -1.01081 1.48615 -1.4296 -1.46737 1 19 1 7 1 24 6.6 7 1.41401 .98881 1.48615 1.10201 1.3812 1 20 1 4 0 24 4.8 4 -.96248 -1.01081 1.48615 -.19996 -.75523 2 21 1 8 1 14 6.4 6 2.20617 .98881 -.04014 .95734 .66906 2 22 1 4 0 14 2.4 3 -.96248 -1.01081 -.04014 -1.93592 -1.46737 2 23 1 6 0 14 3.7 4 .62185 -1.01081 -.04014 -.99561 -.75523 2 24 1 5 1 14 4.4 4 -.17031 .98881 -.04014 -.48929 -.75523 2 25 1 5 1 14 4.3 4 -.17031 .98881 -.04014 -.56162 -.75523 2 26 1 5 0 14 4 4 -.17031 -1.01081 -.04014 -.77861 -.75523 2 27 1 4 1 14 3.8 5 -.96248 .98881 -.04014 -.92328 -.04308 2 28 1 5 0 14 4.2 5 -.17031 -1.01081 -.04014 -.63395 -.04308

Scatterplot of Extraversion by Popularity for Each Class



Intercepts and Slopes as Outcome Variables Model Mplus Diagram



Example: Intercept Only Model

```
TITLE: Random Effects ANOVA using Mplus;
DATA:
 FILE IS "popular2.dat";
VARIABLE:
NAMES ARE class pupil cons extrav sex texp popular
popteach zextrav zsex ztexp zpopular zpoptch;
usevariables are popular;
CLUSTER IS class;
ANALYSIS:
TYPE IS TWOLEVEL;
ESTIMATOR IS ML; !default is MLR
MODEL:
  %within%
 popular;
 %between%
 popular;
OUTPUT: SAMPSTAT;
```

Intercept Only Model

SUMMARY OF DATA

Number of clusters 100

Average cluster size 20.000

Estimated Intraclass Correlations for the Y Variables

Intraclass

Variable Correlation

POPULAR 0.362

0.695/(0.695 + 1.222) = .362

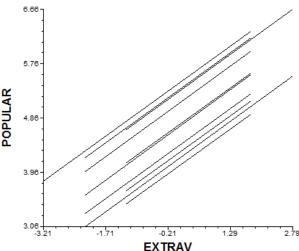
MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Within Level				
Variances POPULAR	1.222	0.040	30.822	0.000
Between Level				
Means POPULAR	5.078	0.087	58.390	0.000
Variances POPULAR	0.695	0.107	6.489	0.000

Level-1 Predictor Fixed

Note: in this model, we are estimating random variation in the intercepts but not in the slopes

```
TITLE: Random Effects ANOVA using Mplus;
DATA:
  FILE IS "popular2.dat";
VARIABLE:
NAMES ARE class pupil cons extrav sex texp popular
popteach zextrav zsex ztexp zpopular zpoptch;
usevariables are popular extrav;
                                                                       5.76
CLUSTER IS class:
                                                                    POPULAR
within are extrav:
define:
center extrav (groupmean);
!centering = groupmean (extrav); use this statement instead of
previous two lines in Mplus version 6 or earlier.
ANALYSIS:
TYPE IS TWOLEVEL:
ESTIMATOR IS ML; !default is MLR
MODEL:
  %within%
  popular on extrav;
  %between%
  popular;
OUTPUT: SAMPSTAT;
```



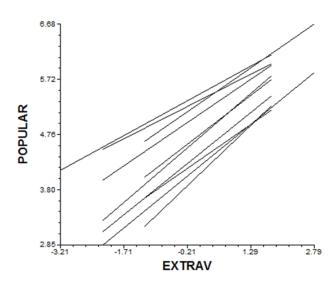
Level-1 Predictor Fixed

	Number of Free Para	meters		4		
	Loglikelihood					
	HO Value H1 Value			-2904.238 -2904.238		C Extrav
	Information Criteri	.a				EXTRAV S_Extrav .930
	_			5816.476 5838.880 5826.172		POPULAR
1	MODEL RESULTS					
		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value	WITHIN
1	Within Level					BETWEEN
	POPULAR ON EXTRAV	0.498	0.020	24.430	0.000	
	Residual Variances POPULAR	0.930	0.030	30.822	0.000	Mean = 5.078 Var = .710 POPULAR (intercept)
	Between Level					
	Means POPULAR	5.078	0.087	58.378	0.000	
	Variances POPULAR	0.710	0.107	6.627	0.000	

Level-1 Predictor Random

Note: in this model, we are estimating random variation in the intercepts and in the slopes (popularity regressed on extraversion)

```
TITLE: Random Effects ANOVA using Mplus;
DATA:
  FILE IS "popular2.dat";
VARIABLE:
NAMES ARE class pupil cons extrav sex texp popular
popteach zextrav zsex ztexp zpopular zpoptch;
usevariables are popular extrav;
CLUSTER IS class:
within are extrav:
define:
center extrav (groupmean);
!centering = groupmean (extrav); use this statement instead of
!previous two lines in Mplus version 6 or earlier.
ANALYSIS:
TYPE IS TWOLEVEL RANDOM:
ESTIMATOR IS ML: !default is MLR
MODEL:
  %within%
  s extrav|popular on extrav;
  %between%
  popular;
OUTPUT: SAMPSTAT;
```



Level-1 Predictor Random

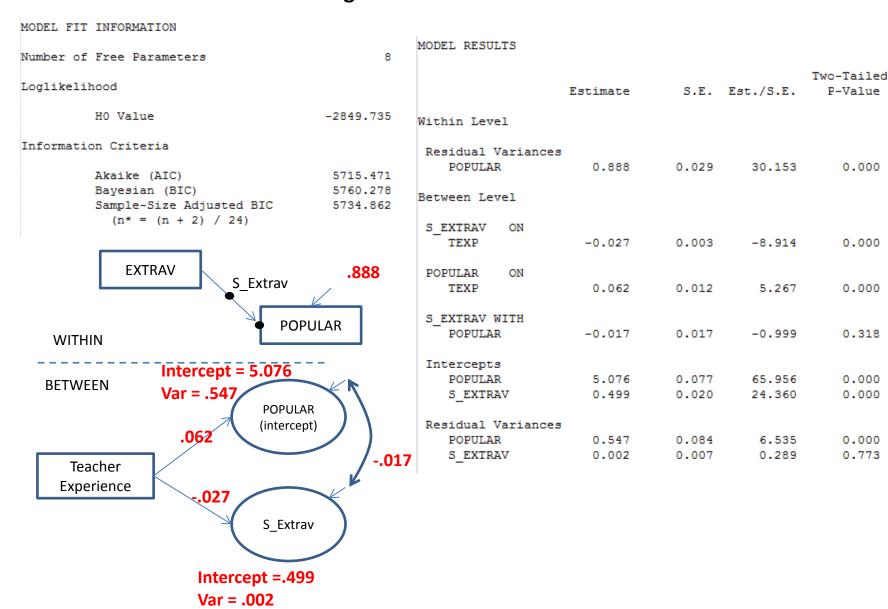
Number of Loglikeli	Free Para	ameters			5 E	EXTRAV S EX	ktrav . <mark>892</mark>
000000000000000000000000000000000000000	H0 Value			-2893.75	57	•	
Informati	on Criteri	la					POPULAR
	Akaike (A	AIC)		5797.51	.5		
	Bayesian			5825.51			
	_	ze Adjusted		5809.63	34		
0000000000	(n* = ((n + 2) / 24	1)				WITHIN
MODEL RESU	JLTS						BETWEEN
					Two-Tailed		DETVICEIV
		Estimate	S.E.	Est./S.E.			
Within Lev	7el					Mean =5.078	POPULAR
Residual POPULA	Variances AR	0.892	0.030	30.018	0.000	Var = 0.711	(intercept)
Between Le	evel						
Means							
POPULA	AR	5.078	0.087	58.389	0.000	Mean = 0.497	
S_EXT	RAV	0.497	0.027	18.236	0.000	Var = .033	S_Extrav
Variance	3						
POPUL	AR.	0.711	0.107	6.648	0.000		
S_EXT	RAV	0.033	0.010	3.103	0.002		

Adding a Level-2 Predictor

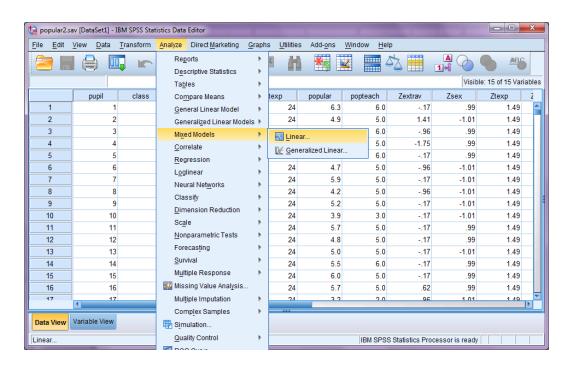
Note: in this model, we add a Level-2 predictor. We regress both the intercepts and the slopes on this predictor. (I also included a correlation between the intercept and slope residuals)

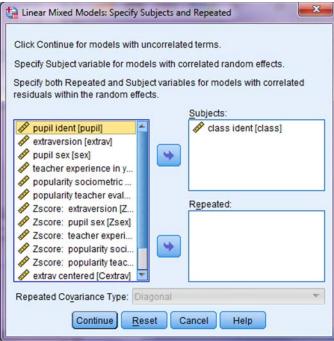
```
TITLE: Random Effects ANOVA using Mplus;
DATA:
  FILE IS "popular2.dat";
VARIABLE:
NAMES ARE class pupil cons extrav sex texp popular
popteach zextrav zsex ztexp zpopular zpoptch;
usevariables are popular extrav texp;
CLUSTER IS class:
within are extrav:
between are texp;
define:
center extrav (groupmean);
!centering = groupmean (extrav); use this statement instead of
!previous two lines in Mplus version 6 or earlier.
center texp (grandmean);
ANALYSIS:
TYPE IS TWOLEVEL RANDOM;
ESTIMATOR IS ML; !default is MLR
MODEL:
 %within%
  s extrav|popular on extrav;
  %between%
  popular on texp;
  s extrav on texp;
  s extrav with popular;
OUTPUT: SAMPSTAT:
```

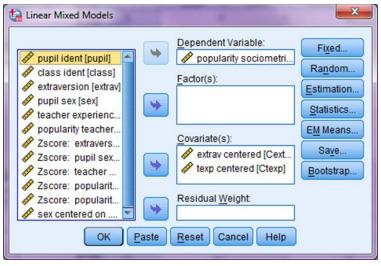
Adding a Level-2 Predictor



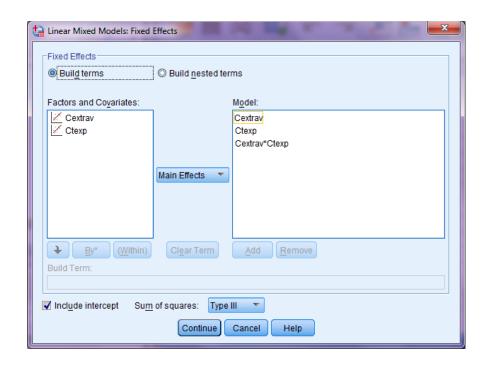
Replicating in SPSS Mixed Models

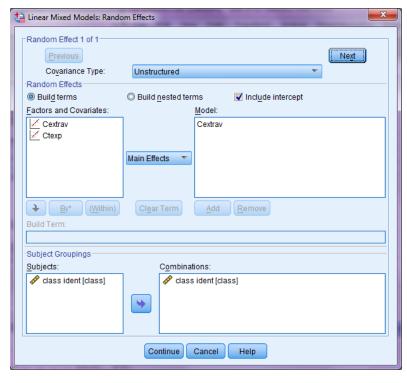






Previous Model in SPSS Mixed Models





Previous Model in SPSS Mixed Models

Note: model not exactly the same (level-1 predictor grand-mean centered instead of group-mean centered)

```
MIXED popular WITH Cextrav Ctexp

/CRITERIA=CIN(95) MXITER(100) MXSTEP(10) SCORING(1) SINGULAR(0.00000000001) HCONVERGE(0, ABSOLUTE

LCONVERGE(0, ABSOLUTE) PCONVERGE(0.000001, ABSOLUTE)

/FIXED=Cextrav Ctexp Cextrav*Ctexp | SSTYPE(3)

/METHOD=ML

/PRINT=SOLUTION TESTCOV

/RANDOM=INTERCEPT Cextrav | SUBJECT(class) COVTYPE(UN).
```

Model Dimension^a Number of Covariance Number of Subject Levels Structure Parameters Variables Fixed Effects Intercept Cextrav Ctexp Cextrav * Ctexp Intercept + Cextravb Random Effects Unstructured class Residual Total

a. Dependent Variable: popular popularity sociometric score.

b. As of version 11.5, the syntax rules for the RANDOM subcommand have changed. Your command
syntax may yield results that differ from those produced by prior versions. If you are using version 11
syntax, please consult the current syntax reference guide for more information.

Information Criteria ^a								
-2 Log Likelihood	5667.925							
Akaike's Information Criterion (AIC)	5683.925							
Hurvich and Tsai's Criterion (AICC)	5683.997							
Bozdogan's Criterion (CAIC)	5736.732							
Schwarz's Bayesian Criterion (BIC)	5728.732							

The information criteria are displayed in smaller-is-better forms.

 a. Dependent Variable: popular popularity sociometric score.

Previous Model in SPSS Mixed Models

Fixed Effects

Type III Tests of Fixed Effects^a

Source	Numerator df	Denominator df	F	Sig.
Intercept	1	103.056	5611.552	.000
Cextrav	1	1981.068	637.815	.000
Ctexp	1	105.672	113.751	.000
Cextrav * Ctexp	1	1787.297	91.701	.000

a. Dependent Variable: popular popularity sociometric score.

Estimates of Fixed Effects^a

						95% Confidence Interval	
Parameter	Estimate	Std. Error	df	t	Sig.	Lower Bound	Upper Bound
Intercept	4.986636	.066568	103.056	74.910	.000	4.854614	5.118657
Cextrav	.499467	.019777	1981.068	25.255	.000	.460681	.538253
Ctexp	.108705	.010192	105.672	10.665	.000	.088497	.128913
Cextrav * Ctexp	027452	.002867	1787.297	-9.576	.000	033074	021829

a. Dependent Variable: popular popularity sociometric score.

Covariance Parameters

Estimates of Covariance Parameters^a

						95% Confidence Interval	
Parameter		Estimate	Std. Error	Wald Z	Sig.	Lower Bound	Upper Bound
Residual		.887014	.028723	30.882	.000	.832467	.945134
Intercept + Cextrav [subject = class]	UN (1,1)	.389512	.061633	6.320	.000	.285650	.531139
	UN (2,1)	012524	.013061	959	.338	038123	.013075
	UN (2,2)	.000403 ^b	.000000				

a. Dependent Variable: popular popularity sociometric score.

b. This covariance parameter is redundant. The test statistic and confidence interval cannot be computed.

Exact Same Model in Mplus (all predictors grand-mean centered)

MODEL FIT INFORMATION						
Number of Free Parameters	8					
Loglikelihood						
HO Value	-2834.152					
Information Criteria						
Akaike (AIC)	5684.304					
Bayesian (BIC)	5729.112					
Sample-Size Adjusted BIC $(n* = (n + 2) / 24)$	5703.695					

MODEL RESULTS				
	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Within Level				
Residual Variances POPULAR	0.888	0.029	30.210	0.000
Between Level				
S_EXTRAV ON TEXP	-0.028	0.003	-9.348	0.000
POPULAR ON TEXP	0.109	0.010	10.662	0.000
POPULAR WITH S_EXTRAV	-0.011	0.014	-0.814	0.416
Intercepts POPULAR S_EXTRAV			74.919 24.714	
Residual Variances POPULAR S_EXTRAV	0.389	0.062 0.007	6.281 0.262	

Measures within Persons

```
Title: PSY9555 Regression examples;
!note that two outliers of 0 were removed in average;
data: File is sem mplus2.dat;
    Format is 1F4, 1F1, 1F2, 23F8.2, 1F11.3, 72F8.2;
data widetolong: <
wide = drink1 drink2 drink3 drink4:
long = drink:
idvariable = person;
repetition = time;
variable: names are studid gender age
bppa bpv bpa bph bptot
sq1 sq2 sq3 sq4 sq5 sq6 sq7 sq8 sq9 sq10 sq11 sq12 sq13 sq14 sq15
es es pt es fin grade
drink1 drink2 drink3 drink4 epis1 epis2 epis3 epis4
stress1 stress2 stress3 stress4 pleased1 pleased2 pleased3 pleased4
enjoyc1 enjoyc2 enjoyc3 enjoyc4 enjoyu1 enjoyu2 enjoyu3 enjoyu4
effort1 effort2 effort3 effort4 harm1 harm2 harm3 harm4 dep1 dep2 dep3 dep4
drink1b drink2b drink3b drink4b epis1b epis2b epis3b epis4b
stress1b stress2b stress3b stress4b please1b please2b please3b please4b
enjoyc1b enjoyc2b enjoyc3b enjoyc4b enjoyu1b enjoyu2b enjoyu3b enjoyu4b
effort1b effort2b effort3b effort4b harm1b harm2b harm3b harm4b
dep1b dep2b dep3b dep4b;
missing = blank;
usevariables are drink person time;
cluster = person;
within = time:
analysis:
type = twolevel random;
model:
%within%
s | drink on time;
%between%
s with drink:
output: sampstat tech1;
```

When your data file is structured in the conventional one line per subject with repeated measures on the same line

Measures within Persons

Loglikelihood						
H0 Value H0 Scaling for MLR	g Correction		-4756.126 3.5791			0.076
Information Criteri	a				(linear) Slope	
			9524.253 9556.568 9537.507		Drink	
MODEL RESULTS	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value	WITH	
Within Level					BEIV	VEEN
Residual Variances DRINK Between Level		1.428	7.056	0.000	Mean = 5.236 Var = 31.765 Drink (intercept	11
S WITH DRINK	-0.139	1.168	-0.119	0.905	(interes)	-0.139
Means DRINK S		0.306 0.085	17.090 -1.396	0.000 0.163	Mean =118	
Variances DRINK S	31.765 0.782	4.088 0.478	7.769 1.636	0.000 0.102	Var = .782 Slope	

Measures within Persons (Previous Model specified as LGM)

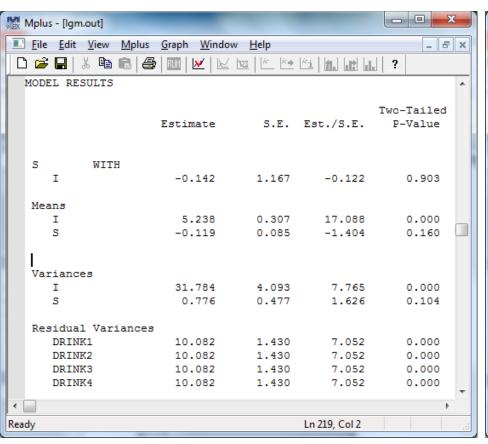
```
Title: PSY9555 Regression examples;
!note that two outliers of 0 were removed in average;
data: File is sem mplus2.dat;
    Format is 1F4, 1F1, 1F2, 23F8.2, 1F11.3, 72F8.2;
!LISTWISE = ON:
variable: names are studid gender age
bppa bpv bpa bph bptot
sq1 sq2 sq3 sq4 sq5 sq6 sq7 sq8 sq9 sq10 sq11 sq12 sq13 sq14 <del>sq1</del>5
es es pt es fin grade
drink1 drink2 drink3 drink4 epis1 epis2 epis3 epis4
stress1 stress2 stress3 stress4 pleased1 pleased2 pleased3 pleased4
enjoyc1 enjoyc2 enjoyc3 enjoyc4 enjoyu1 enjoyu2 enjoyu3 enjoyu4
effort1 effort2 effort3 effort4 harm1 harm2 harm3 harm4 dep1 dep2 dep3 dep4
drink1b drink2b drink3b drink4b epis1b epis2b epis3b epis4b
stress1b stress2b stress3b stress4b please1b please2b please3b please4b
enjoyc1b enjoyc2b enjoyc3b enjoyc4b enjoyu1b enjoyu2b enjoyu3b enjoyu4b
effort1b effort2b effort3b effort4b harm1b harm2b harm3b harm4b
dep1b dep2b dep3b dep4b;
missing = blank;
usevariables are drink1 drink2 drink3 drink4;
analysis:
type = general;
estimator = mlr;
I S | drink1@0 drink2@1 drink3@2 drink4@3;
drink1-drink4 (1); \leftarrow
plot:
type is plot3;
series = drink1 (0) drink2 (1) drink3 (2) drink4 (3);
output: sampstat residual stdyx tech4 modindices;
```

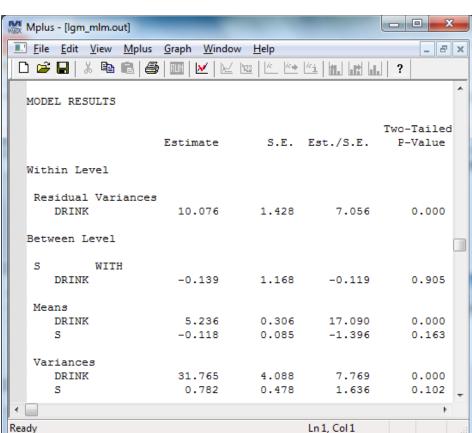
Back to the conventional data structure specification

IN LGM these residuals are usually not constrained to equality but they are in MLM. I constrained them here.

Example: Measures within Persons (LGM and MLM)

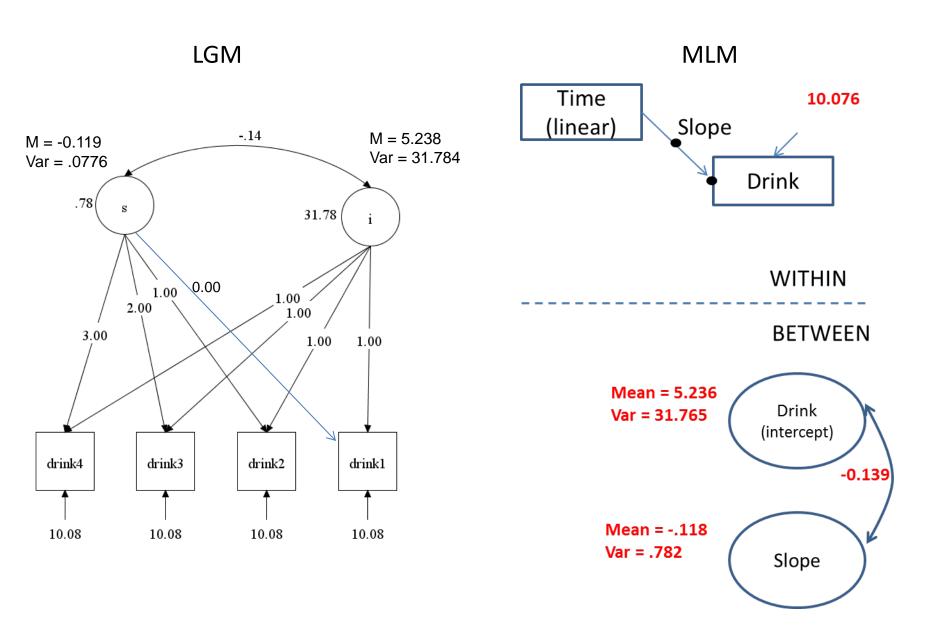
LGM MLM





Same as in slide 13

Example: Measures within Persons (LGM and MLM)



Example: Measures within Persons (LGM and MLM)

LGM MLM

WODEL EIT	THEODYPETON		WODEL BIT INFORMATION		
MODEL FII	INFORMATION		MODEL FIT INFORMATION		
Number of	Free Parameters	6	Number of Free Parameters	6	
Loglikeli	hood		Loglikelihood		
	HO Value	-4756.126	HO Value	-4756.126	
	HO Scaling Correction Factor for MLR	3.5794	HO Scaling Correction Factor for MLR	3.5791	
	H1 Value	-4731.180			
	H1 Scaling Correction Factor for MLR	3.6974	Information Criteria		
			Akaike (AIC)	9524.253	
Informati	on Criteria		Bayesian (BIC)	9556.568	
			Sample-Size Adjusted BIC	9537.507	
	Akaike (AIC)	9524.252	(n* = (n + 2) / 24)		
	Bayesian (BIC)	9548.407			
	Sample-Size Adjusted BIC	9529.368			
	(n* = (n + 2) / 24)				
Chi-Squar	e Test of Model Fit				

13.178*

0.1059

3.7860

8

Value

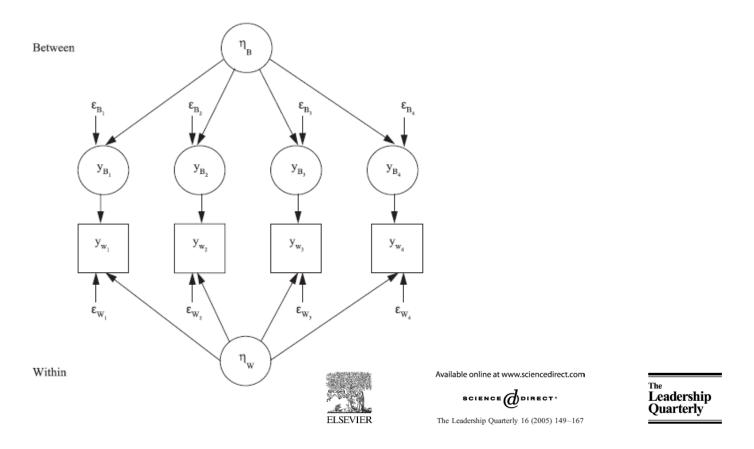
P-Value

for MLR

Degrees of Freedom

Scaling Correction Factor

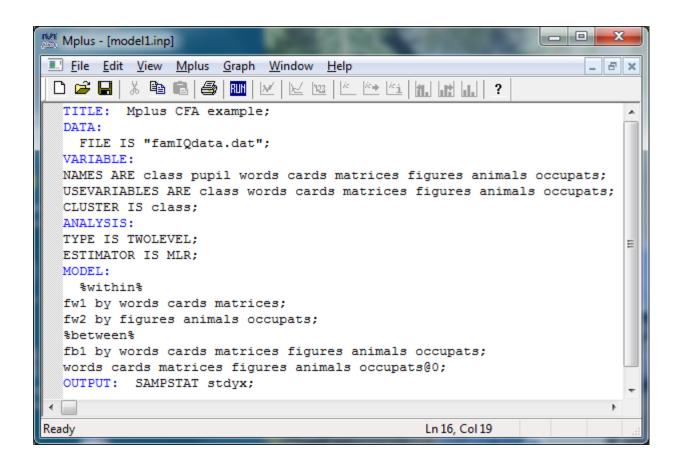
Multilevel CFA



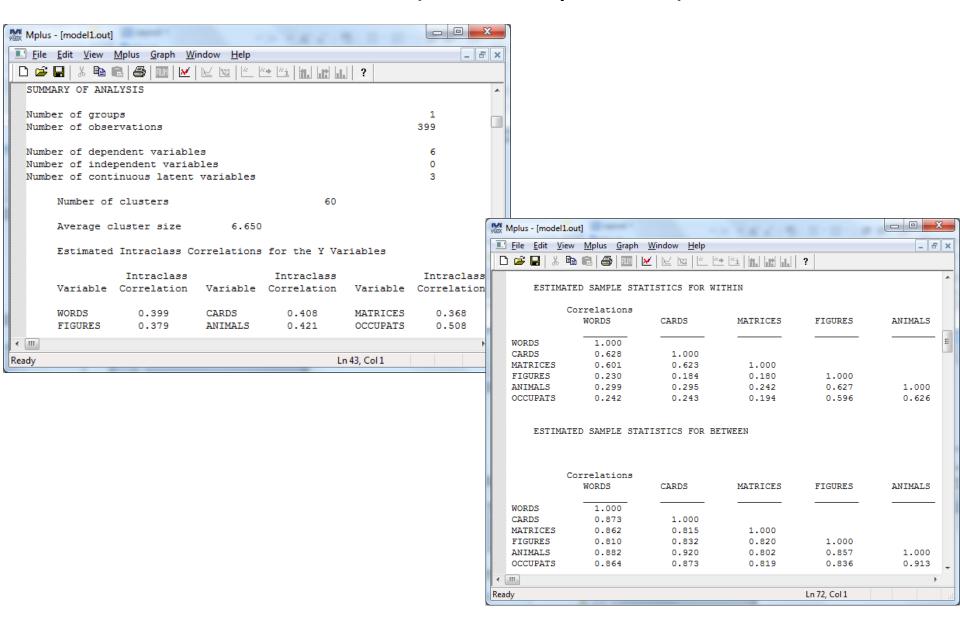
Applying multilevel confirmatory factor analysis techniques to the study of leadership

Naomi G. Dyer^{a,*}, Paul J. Hanges^a, Rosalie J. Hall^b

Multilevel CFA (data from Joop Hox, 2010)



Multilevel CFA (data from Joop Hox, 2010)



Multilevel CFA (data from Joop Hox, 2010)

Mplus - [model1.out]				_	
<u> File Edit V</u> iew <u>M</u> plus	<u>G</u> raph <u>W</u> indow	<u>H</u> elp		_ &	×
	RUH 🔀 🖂 🛚	23 /c /c+] ?	
		- 1		Two-Tailed	Ĭ
	Estimate	S.E.	Est./S.E.	P-Value	
Within Level					
FW1 BY					
WORDS	1.000	0.000	999.000	999.000	
CARDS	0.991	0.075	13.236	0.000	
MATRICES	0.959	0.081	11.773	0.000	
FW2 BY					
FIGURES	1.000	0.000	999.000	999.000	
ANIMALS	1.035	0.066	15.708	0.000	
OCCUPATS	0.880	0.054	16.403	0.000	
FW2 WITH					Ξ
FW1	3.806	0.877	4.338	0.000	
	0.000	0.077	1.000	3.333	
Variances					
FW1	10.357	1.943	5.330	0.000	
FW2	9.583	1.344	7.128	0.000	
Residual Variances					
WORDS	6.181	0.782	7.909	0.000	
CARDS	5.382	0.647	8.323	0.000	
MATRICES	6.415	0.786	8.159	0.000	
FIGURES	6.816	0.785	8.684	0.000	
ANIMALS	4.704	0.631	7.450	0.000	
OCCUPATS	5.908	0.759	7.788	0.000	Ŧ
∢ 📗				+	
Ready			Ln 72, Col 1		at
,					

Mplus - [model1.out]				_	
<u>File Edit View Mplus</u>	<u>G</u> raph <u>W</u> indow	/ <u>H</u> elp		_ 5	×
	RUN 💹 💆	<u>923</u> 1/c 1/c → 1/		?	
					_
Between Level					
FB1 BY					
WORDS	1.000	0.000	999.000	999.000	
CARDS	0.994	0.095	10.427	0.000	
MATRICES	0.864	0.093	9.312	0.000	
FIGURES	0.924	0.114	8.095	0.000	
ANIMALS	1.049	0.112	9.336	0.000	
OCCUPATS	1.211	0.140	8.661	0.000	
Intercepts					
WORDS	29.874	0.472	63.227	0.000	
CARDS	29.876	0.466	64.071	0.000	
MATRICES	29.735	0.441	67.478	0.000	
FIGURES	30.037	0.460	65.315	0.000	
ANIMALS	30.138	0.472	63.879	0.000	
OCCUPATS	29.980	0.508	58.968	0.000	
Variances					
FB1	8.928	2.333	3.827	0.000	
Residual Variances					
WORDS	1.334	0.514	2.597	0.009	
CARDS	1.467	0.575	2.554	0.011	
MATRICES	1.956	0.609	3.212	0.001	
FIGURES	2.453	0.619	3.962	0.000	
ANIMALS	1.273	0.636	2.000	0.045	
OCCUPATS	0.000	0.000	999.000	999.000	Ŧ
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