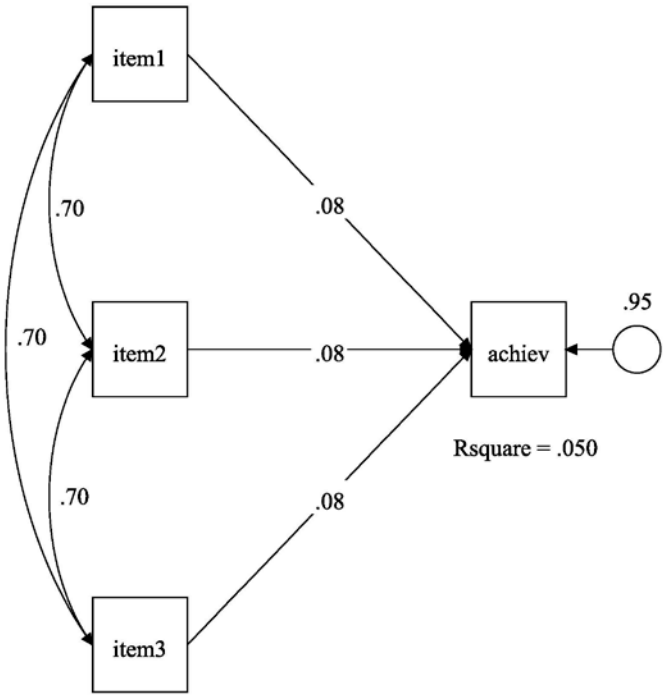
PSY 9555A (Oct 9) CFA Measurement and Test Construction
Example of the Principle of Aggregation

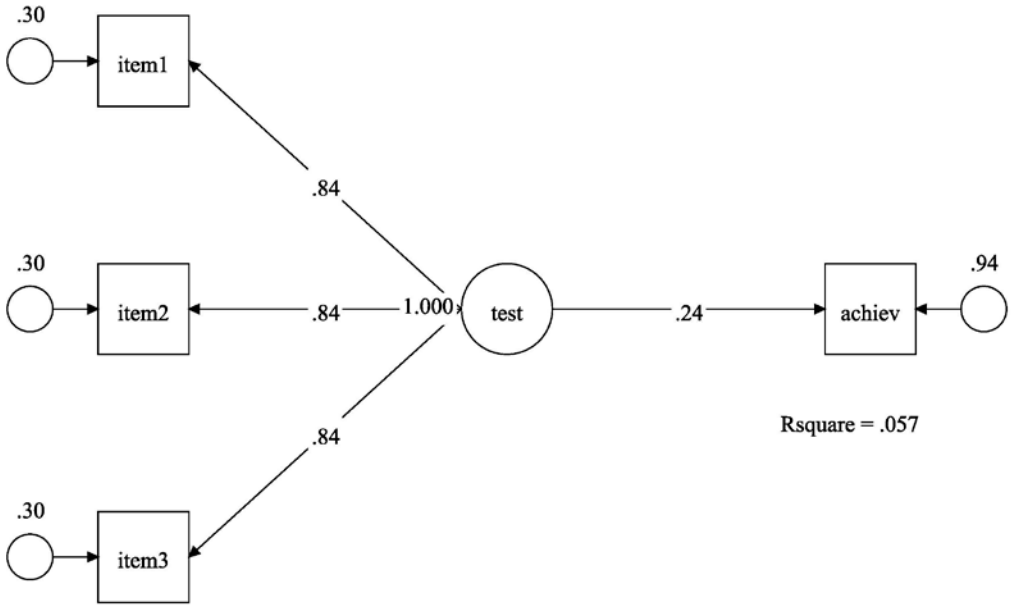
Consider a situation where we have three test items that all correlate .20 with an achievement score and .70 with each other. Let's see how well we can predict achievement in two different ways.

	ACHIEV	ITEM1	ITEM2	ITEM3
ACHIEV	1.000			
ITEM1	0.200	1.000		
ITEM2	0.200	0.700	1.000	
ITEM3	0.200	0.700	0.700	1.000

Example of the Principle of Aggregation

Explain the difference in R-square





A Perfectly Fitting Model

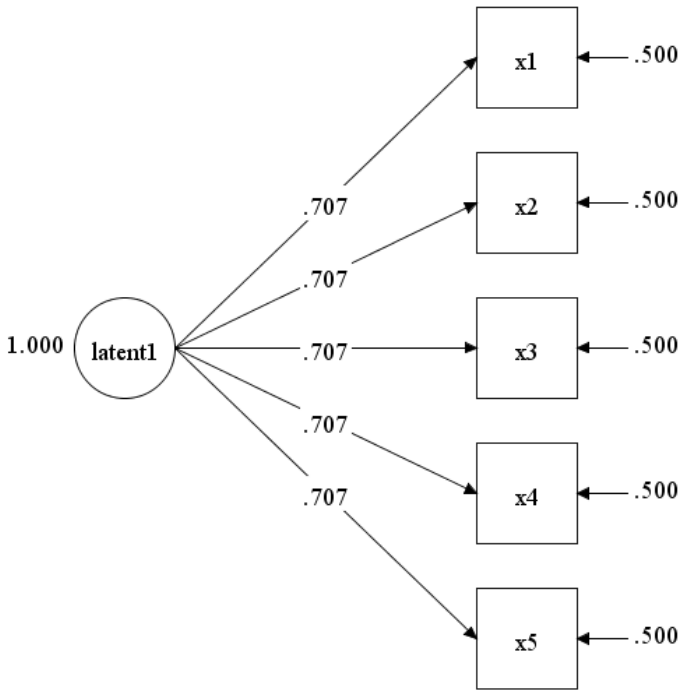
```
5.0 5.2 5.1 5.4 5.0
1.1 1.2 1.1 1.3 1.2
1.0
.50 1.0
.50 .50 1.0
.50 .50 .50 1.0
.50 .50 .50 .50 1.0
```

```
title: Example reading a correlation matrix !title of analysis
data:
file is example_corr.txt;
nobservations = 200; !the number of observations must be included
type=correlation means stdeviations; !type=correlation tells mplus that the data are correlations
variable:
names are x1 x2 x3 x4 x5; !variable names
usevariables are x1 x2 x3 x4 x5; !variables used in analysis
model:
latent1 by x1 x2 x3 x4 x5;
output:
stdyx residual;
```

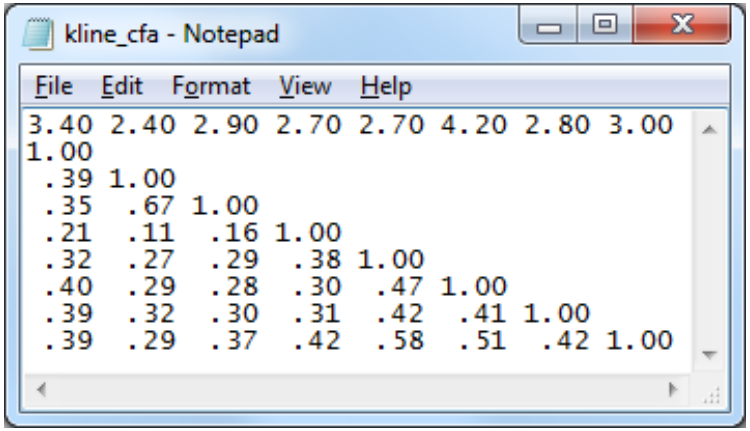
A Perfectly Fitting Model

```
MODEL FIT INFORMATION

Number of Free Parameters                       15

Loglikelihood

          H0 Value                       -1412.560
          H1 Value                       -1412.560

Information Criteria

          Akaike (AIC)                    2855.120
          Bayesian (BIC)                  2904.595
          Sample-Size Adjusted BIC        2857.074
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             0.000
          Degrees of Freedom                    5
          P-Value                           1.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                          0.000
          90 Percent C.I.                   0.000  0.000
          Probability RMSEA <= .05          1.000

CFI/TLI

          CFI                               1.000
          TLI                               1.031

Chi-Square Test of Model Fit for the Baseline Model

          Value                           334.795
          Degrees of Freedom                   10
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                             0.000
```

```
5.0 5.2 5.1 5.4 5.0
1.1 1.2 1.1 1.3 1.2
1.0
.50 1.0
.50 .50 1.0
.50 .50 .50 1.0
.50 .50 .50 .50 1.0
```



Example 1: Factor Structure of KABC-I (in Kline p. 117-118)

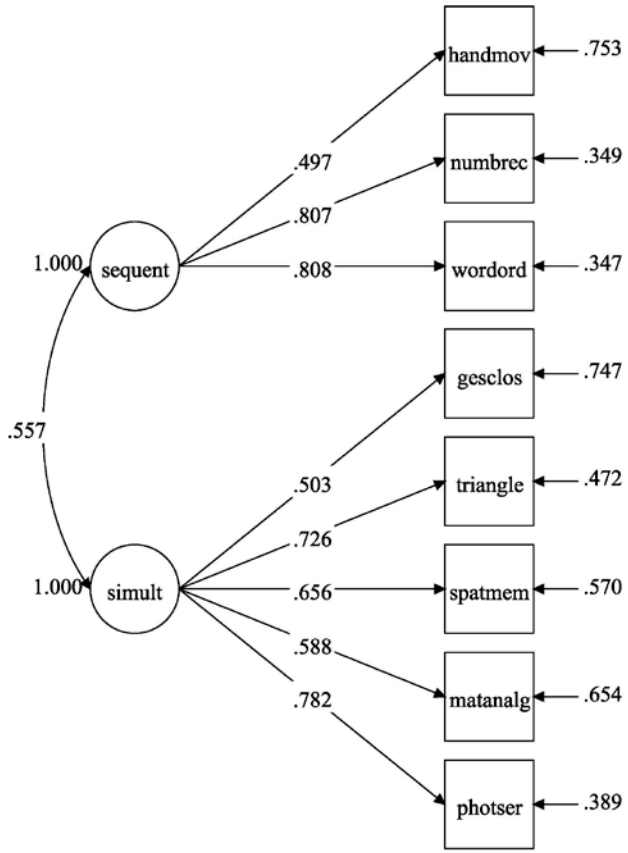



```
TITLE: Principles and Practice of SEM (3rd ed.), Rex Kline
       KABC-I 2-factor, Figure 9.1, Table 9.1
DATA:
  FILE IS kline_cfa.txt;
  TYPE IS STDEVIATIONS CORRELATION;
  NOBSERVATIONS = 200;
VARIABLE:
  NAMES ARE handmov numbrec wordord
  gesclos triangle spatmem matanalg photser;
MODEL:
  Sequent BY handmov numbrec wordord;
  Simult BY gesclos triangle spatmem matanalg photser;
OUTPUT:  SAMPSTAT RESIDUAL STDYX MODINDICES;
```

Example 1: Fit Indices

```
MODEL FIT INFORMATION

Number of Free Parameters                       17

Loglikelihood

          H0 Value                       -3779.041
          H1 Value                       -3759.878

Information Criteria

          Akaike (AIC)                    7592.082
          Bayesian (BIC)                  7648.153
          Sample-Size Adjusted BIC        7594.295
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             38.325
          Degrees of Freedom                    19
          P-Value                           0.0054

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.071
          90 Percent C.I.                    0.038  0.104
          Probability RMSEA <= .05           0.132

CFI/TLI

          CFI                                0.959
          TLI                                0.939
```

```
Chi-Square Test of Model Fit for the Baseline Model

          Value                            498.336
          Degrees of Freedom                    28
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.072
```



Example 1: Inspecting the Residuals

Standardized Residuals (z-scores) for Covariances/Correlations/Residual Corr

	HANDMOV	NUMBREC	WORDORD	GESCLOS	TRIANGLE
HANDMOV	999.000				
NUMBREC	-0.595	999.000			
WORDORD	-3.803	1.537	999.000		
GESCLOS	1.126	-2.329	-1.315	999.000	
TRIANGLE	2.046	-1.558	-1.001	0.427	999.000
SPATMEM	3.464	-0.112	-0.354	-0.785	-0.268
MATANALG	3.505	1.129	0.727	0.323	-0.246
PHOTSER	2.990	-2.001	0.524	0.909	0.676

Standardized Residuals (z-scores) for Covariances/Correlations/Residual Corr

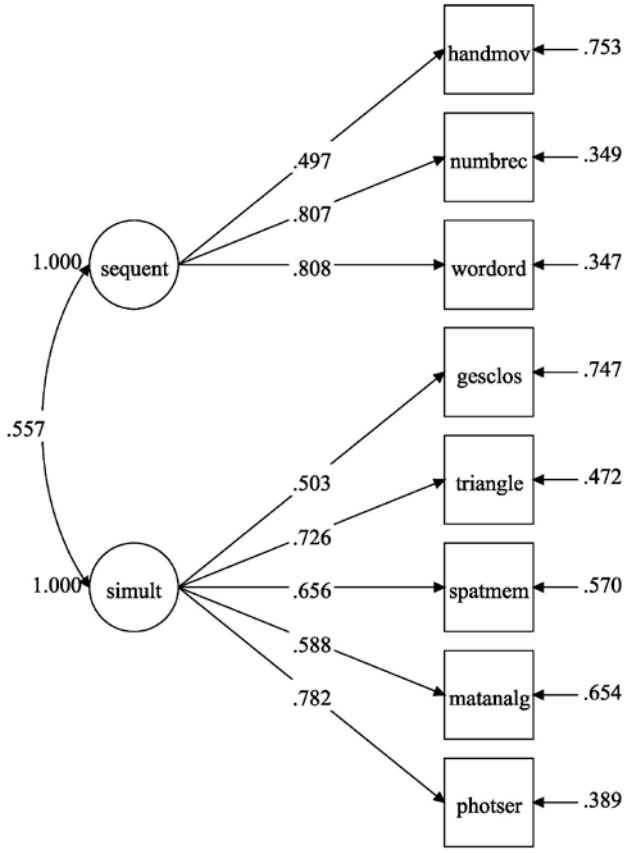
	SPATMEM	MATANALG	PHOTSER
SPATMEM	999.000		
MATANALG	0.664	0.042	
PHOTSER	-0.144	-1.978	999.000

Normalized Residuals for Covariances/Correlations/Residual Correlations

	HANDMOV	NUMBREC	WORDORD	GESCLOS	TRIANGLE
HANDMOV	-0.001				
NUMBREC	-0.144	0.000			
WORDORD	-0.688	0.208	0.000		
GESCLOS	0.981	-1.631	-0.927	0.000	
TRIANGLE	1.603	-0.772	-0.502	0.193	0.000
SPATMEM	2.869	-0.066	-0.208	-0.406	-0.085
MATANALG	2.996	0.751	0.479	0.194	-0.093
PHOTSER	2.289	-0.833	0.241	0.350	0.148

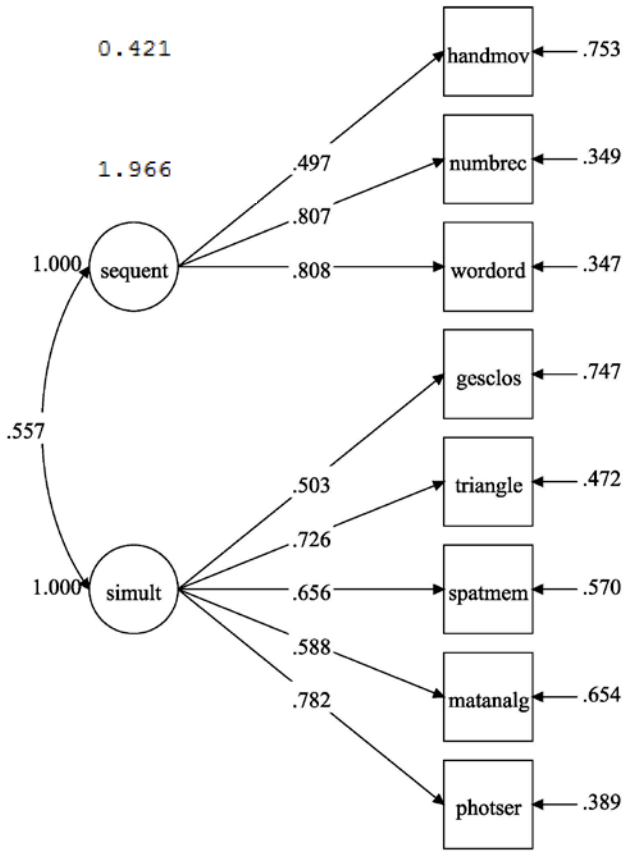
Normalized Residuals for Covariances/Correlations/Residual Correlations

	SPATMEM	MATANALG	PHOTSER
SPATMEM	-0.001		
MATANALG	0.318	0.000	
PHOTSER	-0.036	-0.516	0.000



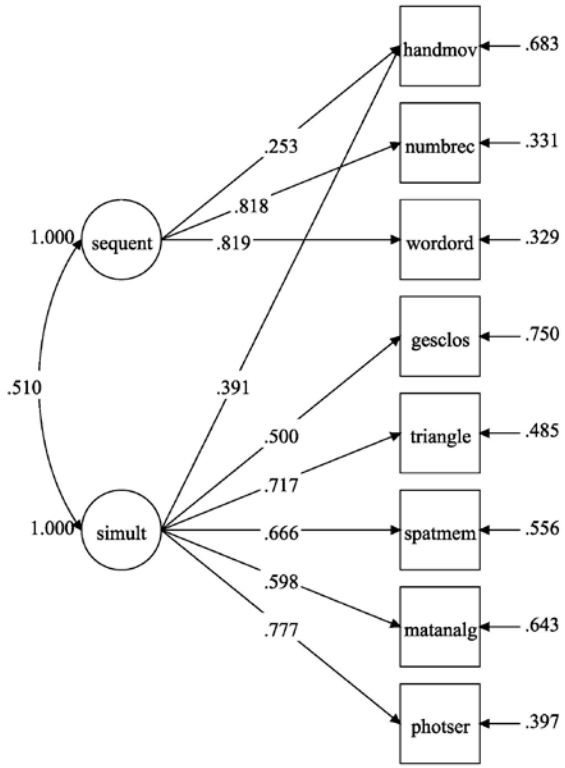
Example 1: Inspecting the Modification Indices

```
MODEL MODIFICATION INDICES

NOTE:  Modification indices for direct effects of observed dependent variables
regressed on covariates may not be included.  To include these, request
MODINDICES (ALL).

Minimum M.I. value for printing the modification index    10.000

                                  M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.

BY Statements

SIMULT   BY HANDMOV              20.091     1.054      1.427        0.421

WITH Statements

WORDORD  WITH NUMBREC            20.042     4.735      4.735        1.966
```



Example 1: Adding a Cross Loading

```
MODEL:
  Sequent BY handmov numbrec wordord;
  Simult BY gesclos triangle spatmem matanalg photser handmov;
OUTPUT:  SAMPSTAT RESIDUAL STDYX modindices;
```

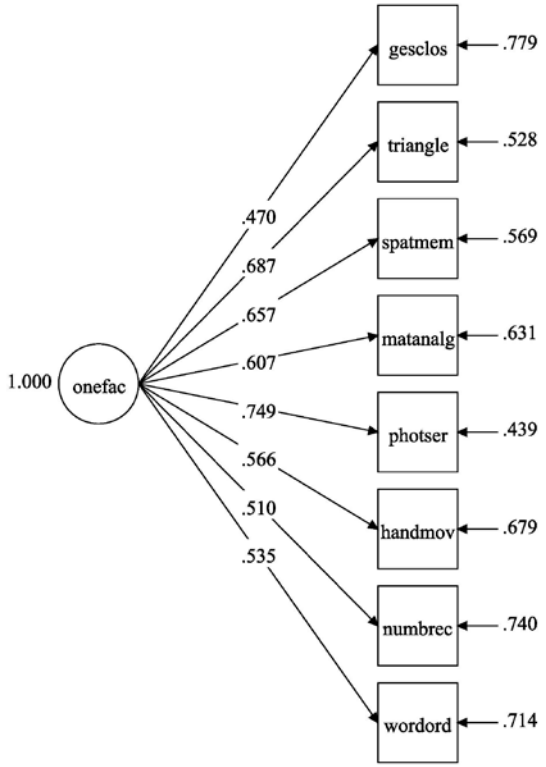



```
MODEL FIT INFORMATION

Number of Free Parameters                       18

Loglikelihood

          H0 Value                       -3768.932
          H1 Value                       -3759.878

Information Criteria

          Akaike (AIC)                    7573.864
          Bayesian (BIC)                  7633.234
          Sample-Size Adjusted BIC        7576.208
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             18.108
          Degrees of Freedom                    18
          P-Value                           0.4486

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.005
          90 Percent C.I.                    0.000  0.063
          Probability RMSEA <= .05           0.859

CFI/TLI

          CFI                                1.000
          TLI                                1.000

Chi-Square Test of Model Fit for the Baseline Model

          Value                            498.336
          Degrees of Freedom                    28
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.035
```

Example 1: Another Approach – Testing a One-Factor Solution

```
MODEL:
  onefac BY gesclos triangle spatmem matanalg photser handmov numbrec wordord;
OUTPUT:  SAMPSTAT RESIDUAL STDYX modindices;
```

```
MODEL FIT INFORMATION

Number of Free Parameters                       16

Loglikelihood

          H0 Value                       -3812.592
          H1 Value                       -3759.878

Information Criteria

          Akaike (AIC)                    7657.183
          Bayesian (BIC)                  7709.956
          Sample-Size Adjusted BIC        7659.267
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                            105.427
          Degrees of Freedom                    20
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.146
          90 Percent C.I.                    0.119  0.174
          Probability RMSEA <= .05           0.000

CFI/TLI

          CFI                                0.818
          TLI                                0.746
```

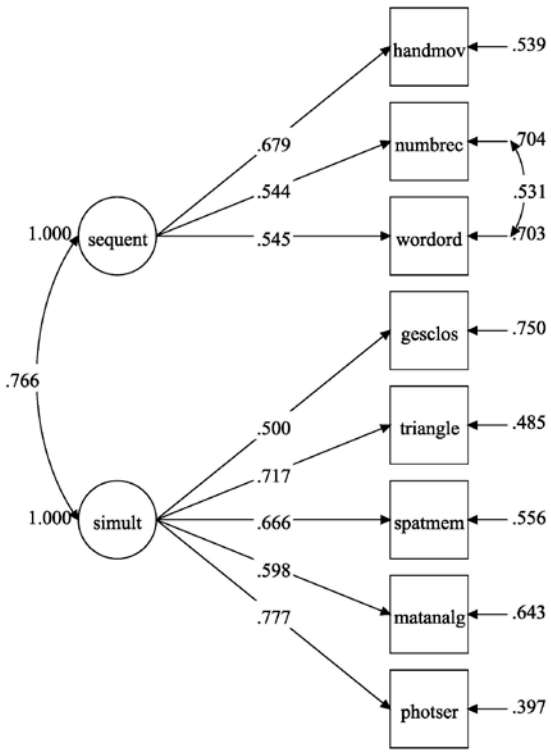



```
Chi-Square Test of Model Fit for the Baseline Model

          Value                            498.336
          Degrees of Freedom                    28
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.084
```

Example 1: Third Approach to Modification: Correlated Errors

```
MODEL:
  Sequent BY handmov numbrec wordord;
  numbrec with wordord;
  Simult BY gesclos triangle spatmem matanalg photser;
```

```
MODEL FIT INFORMATION

Number of Free Parameters                       18

Loglikelihood

          H0 Value                       -3768.932
          H1 Value                       -3759.878

Information Criteria

          Akaike (AIC)                    7573.864
          Bayesian (BIC)                  7633.234
          Sample-Size Adjusted BIC        7576.208
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             18.108
          Degrees of Freedom                    18
          P-Value                           0.4486

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.005
          90 Percent C.I.                    0.000  0.063
          Probability RMSEA <= .05           0.859

CFI/TLI

          CFI                                1.000
          TLI                                1.000
```

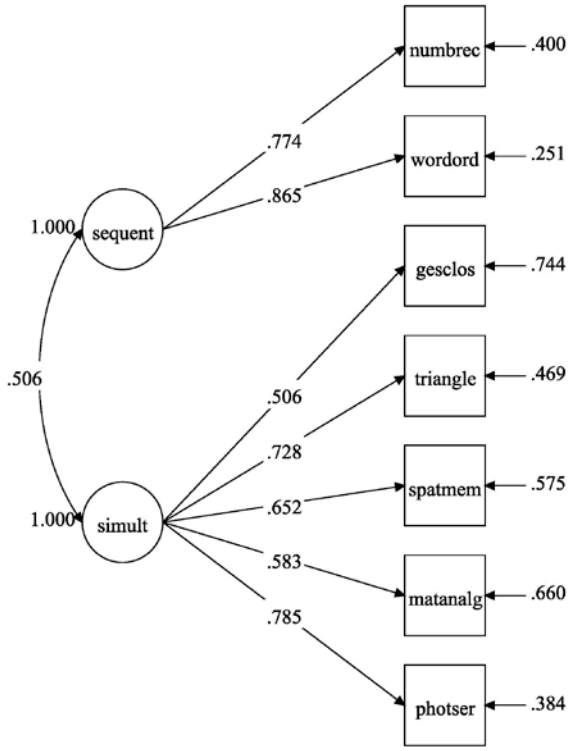



```
Chi-Square Test of Model Fit for the Baseline Model

          Value                            498.336
          Degrees of Freedom                    28
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.035
```

Example 1: Fourth Approach to Modification: Removal of Poor Indicator

```
MODEL:
  Sequent BY numbrec wordord;
  Simult BY gesclos triangle spatmem matanalg photser;
```

```
MODEL FIT INFORMATION

Number of Free Parameters                       15

Loglikelihood

          H0 Value                       -3272.389
          H1 Value                       -3266.849

Information Criteria

          Akaike (AIC)                    6574.778
          Bayesian (BIC)                  6624.253
          Sample-Size Adjusted BIC        6576.731
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             11.080
          Degrees of Freedom                    13
          P-Value                           0.6041

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.000
          90 Percent C.I.                    0.000  0.061
          Probability RMSEA <= .05           0.895

CFI/TLI

          CFI                                1.000
          TLI                                1.008
```

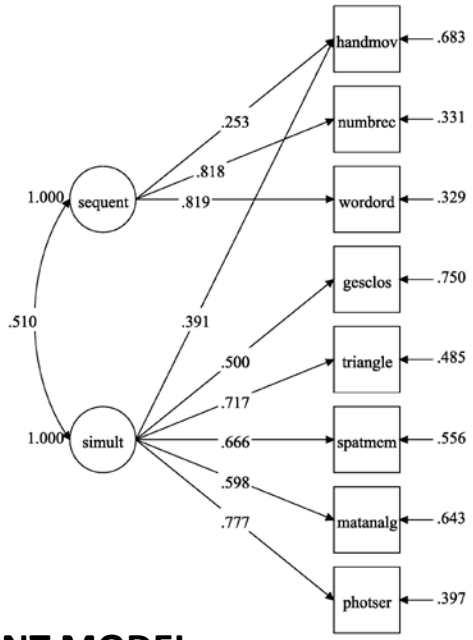



```
Chi-Square Test of Model Fit for the Baseline Model

          Value                            428.313
          Degrees of Freedom                    21
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.032
```

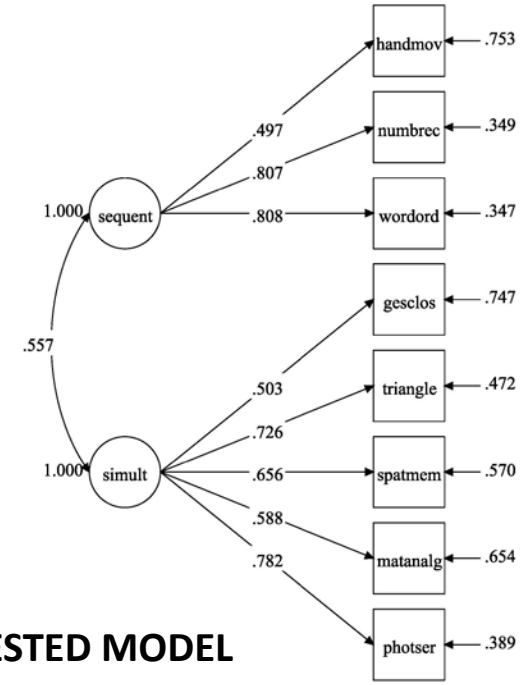
Example 1: Summary of Four Modified Models

Model	$\chi^2_{(df)}$	RMSEA	CFI	SRMR	AIC
1. Original	38.325 (19) p = .005	.071	.959	.072	7592.082
2. Cross-loading	18.108 (18) p = .449	.005	1.000	.035	7573.864
3. One-factor	105.427 (20) p = .000	.146	.818	.084	7657.183
4. Correlated errors	18.108 (18) p = .449	.005	1.000	.035	7573.864
5. Remove indicator	11.080 (13) p = .604	.000	1.000	.032	6574.778

Understanding Nested Models: Cross Loading Model vs. Original Model



PARENT MODEL



NESTED MODEL

NESTED MODEL: $\chi^2_{(19)} = 38.325$, $p < .001$

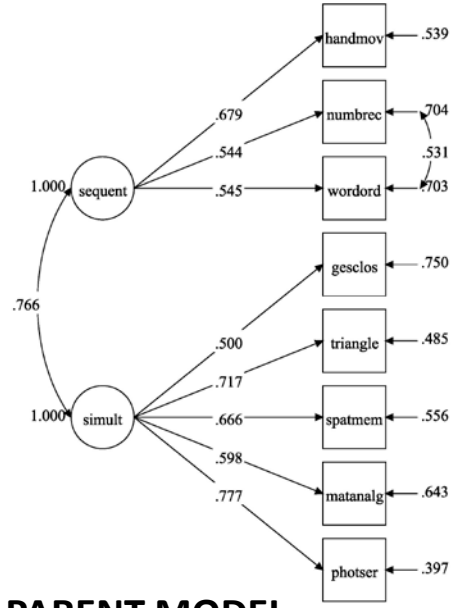
PARENT MODEL: $\chi^2_{(18)} = 18.108$, $p = .449$

Chi-square Difference test:

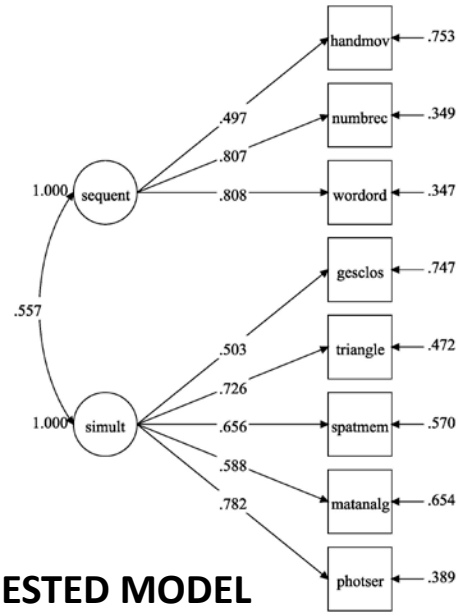
$\chi^2_{(19)}$ 38.325 - $\chi^2_{(18)}$ 18.108 = $\chi^2_{(1)}$ 20.217, $p < .01$

(crit $\chi^2_{(1)} = 3.84$ at $p = .05$ or 6.64 at $p = .01$)

Nested Models: Correlated Errors Model vs. Original Model



PARENT MODEL



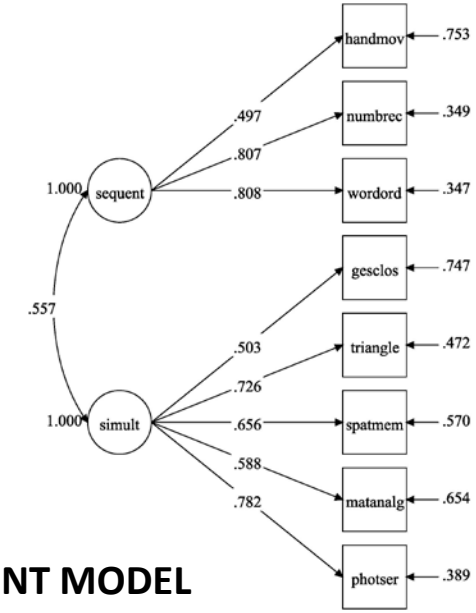
NESTED MODEL

NESTED MODEL: $\chi^2_{(19)} = 38.325$, $p < .001$

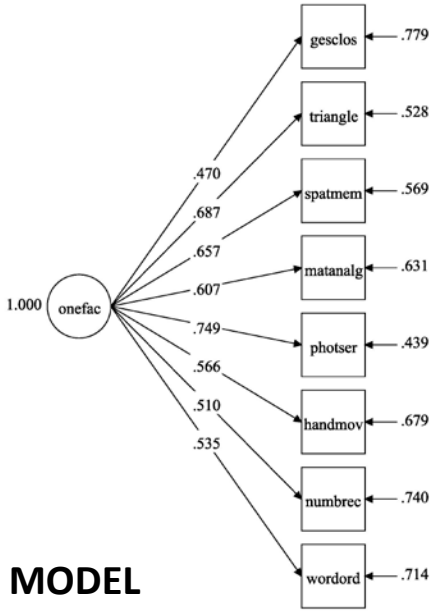
PARENT MODEL: $\chi^2_{(18)} = 18.108$, $p = .449$ (same values as cross-loading model)

Chi-square Difference test:
$\chi^2_{(19)}$ 38.325 - $\chi^2_{(18)}$ 18.108 = $\chi^2_{(1)}$ 20.217, $p < .01$
(crit $\chi^2_{(1)} = 3.84$ at $p = .05$ or 6.64 at $p = .01$)

Nested Models: One-Factor Model vs. Original (Two-Factor) Model



PARENT MODEL



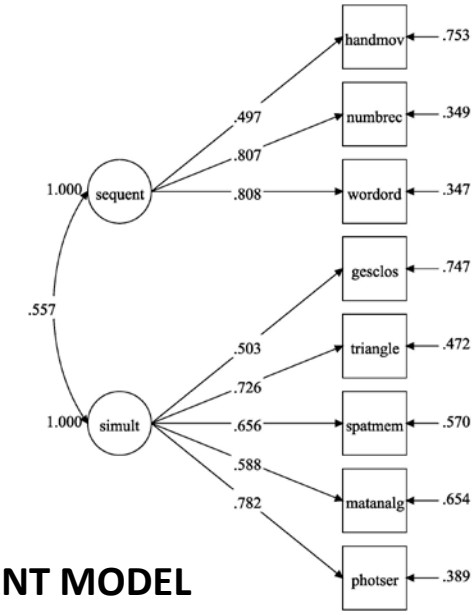
NESTED MODEL

NESTED MODEL: $\chi^2_{(20)} = 105.427$, $p < .001$ (nested because a one factor model is conceptually like two factors with a correlation fixed at one)

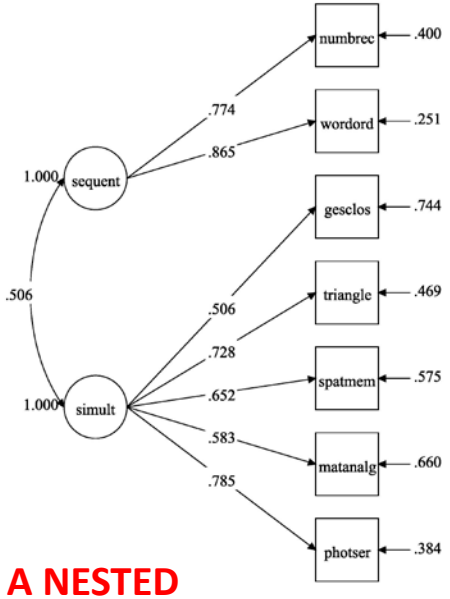
PARENT MODEL: $\chi^2_{(19)} = 38.325$, $p < .005$

Chi-square Difference test:
$\chi^2_{(20)}$ 105.427 - $\chi^2_{(19)}$ 38.325 = $\chi^2_{(1)}$ 67.102, $p < .01$
(crit $\chi^2_{(1)} = 3.84$ at $p = .05$ or 6.64 at $p = .01$)

Nested Models: Model with Removed Indicator vs. Original Model



PARENT MODEL



NOT A NESTED MODEL

Example with Test/Questionnaire Items Using Mplus CATEGORICAL

Analytic Methods

Confirmatory factor analysis of the items was conducted in Mplus Version 6.12 (Muthén & Muthén, 1998-2012) with a weighted least squares estimator with mean and variance adjusted chi-square test statistics (WLSMV). The “categorical” outcome variable option was specified to indicate that items were measured on an ordered discrete category “scale” rather than on a continuous scale. In Mplus, this specification refers to Samejima’s graded response models (Baker and Kim, 2004; Samejima, 1969).

Test Items: Confirmatory Factor Analysis of WHO-DAS- II Items

Item Analyses

A series of nested models presented in Table 1 was estimated: a one-factor model, followed by a one-factor model with one, two, and three correlated residuals. These correlated residuals are theoretically valid because they refer to items from the same subdomains (i.e., items 1 and 7 “getting around”, items 8 and 9 “self-care”, and items 10 and 11 “getting along with people). These models are summarized in Table 1. and show that the final model has a good fit. The items loadings and response proportions are presented in Table 2. It can be seen that all items have substantial loadings and variability in the responses. The coefficient alpha for the scale was .90.

Test Items: Confirmatory Factor Analysis of WHO-DAS- II Items

Table 2. Confirmatory Factor Analysis of WHO-DAS- II Items

Items	CFA Loadings	Means (SD)	Response proportions				
			1	2	3	4	5
1. Standing for long periods such as 30 minutes?	.68	2.05 (1.25)	.51	.13	.19	.13	.04
2. Taking care of your household responsibilities?	.80	2.05 (1.07)	.42	.25	.22	.10	.01
3. Learning a new task, for example, learning to get to a new place?	.71	1.70 (0.97)	.60	.18	.17	.05	.01
4. How much of a problem did you have joining in community activities (for example, festivities, religious or other activities) in the same way as anyone else can?	.76	2.12 (1.25)	.45	.19	.18	.13	.05
5. How much have you been emotionally affected by your health problems?	.80	2.65 (1.28)	.26	.21	.22	.25	.07

Test Items: Confirmatory Factor Analysis of WHO-DAS- II Items

Items	CFA Loadings	Means (SD)	Response proportions				
			1	2	3	4	5
6. Concentrating on doing something for ten minutes?	.72	1.86 (1.09)	.53	.19	.18	.08	.02
7. Walking a long distance such as a kilometre?	.71	2.07 (1.34)	.52	.16	.14	.11	.08
8. Washing your whole body?	.73	1.34 (0.76)	.80	.10	.07	.03	.00
9. Getting dressed?	.75	1.35 (0.76)	.78	.13	.06	.03	.00
10. Dealing with people you do not know?	.74	1.91 (1.14)	.52	.21	.15	.09	.03
11. Maintaining a friendship?	.73	1.72 (1.05)	.61	.18	.13	.06	.02
12. Your day-to-day work?	.85	2.08 (1.21)	.47	.21	.19	.08	.05

Note. Response categories: 1 = none; 2 = mild; 3 = moderate; 4 = severe; 5 = extreme or cannot do

Test Items: Confirmatory Factor Analysis of WHO-DAS- II Items

Table 1. CFA WHO-DAS II item analyses.

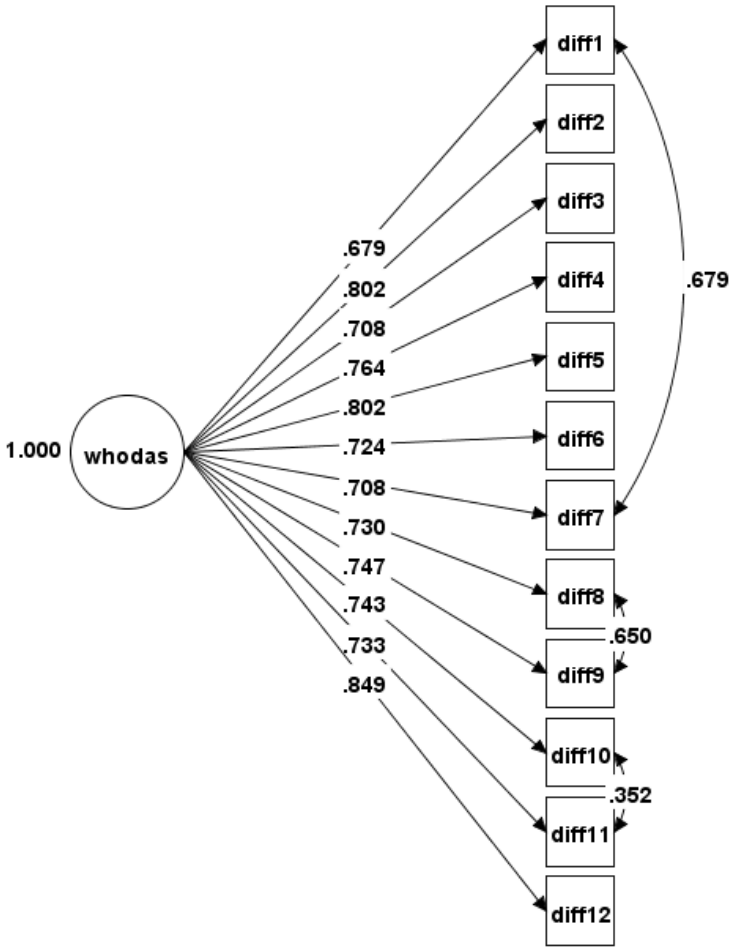
Model	χ^2	df	$\Delta\chi^2$	CFI	TLI	RMSEA
one-factor	525.14	54		.94	.92	.14
one-factor, cov e1-e7	310.79	53	93.97*	.97	.96	.10
one-factor, cov e1-e7, e8-e9	214.34	52	39.91*	.98	.97	.08
one-factor, cov e1-e7, e8-e9, e10-e11	188.91	51	23.68*	.98	.98	.08

Note *p < .001; $\Delta\chi^2$ are based on Mplus Difference Tests for WLSMV estimation

Test Items: Confirmatory Factor Analysis of WHO-DAS- II Items

```
categorical are Diff1 Diff2 Diff3 Diff4 Diff5 Diff6 Diff7 Diff8 Diff9
Diff10 Diff11 Diff12;
usevariables are Diff1 Diff2 Diff3 Diff4 Diff5 Diff6 Diff7 Diff8 Diff9
Diff10 Diff11 Diff12;
MODEL:
whodas by Diff1 Diff2 Diff3 Diff4 Diff5 Diff6 Diff7 Diff8 Diff9
Diff10 Diff11 Diff12;
diff1 with diff7;
diff9 with diff8;
diff10 with diff11;
```

```
UNIVARIATE PROPORTIONS AND COUNTS FOR CATEGORICAL VARIABLES

    DIFF1
      Category 1    0.511      227.000
      Category 2    0.128       57.000
      Category 3    0.191       85.000
      Category 4    0.133       59.000
      Category 5    0.036       16.000
    DIFF2
      Category 1    0.422      183.000
      Category 2    0.249      108.000
      Category 3    0.221       96.000
      Category 4    0.097       42.000
      Category 5    0.012        5.000
    DIFF3
      Category 1    0.596      263.000
      Category 2    0.177       78.000
      Category 3    0.170       75.000
      Category 4    0.050       22.000
      Category 5    0.007        3.000
    DIFF4
      Category 1    0.451      200.000
      Category 2    0.194       86.000
      Category 3    0.176       78.000
      Category 4    0.131       58.000
      Category 5    0.047       21.000
```



Could we Constrain the Item Loadings to Equality?

- Equal factor loadings would satisfy property of tau-equivalence
- We will run such a model; it will be the nested model
- We will compare it to our previous parent model
- If the chi-square difference test is significant then it will indicate that there is significant misfit with the introduction of this restriction
- One problem is that with the CATEGORICAL approach, chi-square tests need to be calculated in a different way
- Mplus provides an appropriate chisquare difference test but some additional syntax is required

Could we Constrain the Item Loadings to Equality?

Parent Model (no constraints)

```
MODEL:
whodas by Diff1 Diff2 Diff3 Diff4 Diff5 Diff6 Diff7 Diff8 Diff9
Diff10 Diff11 Diff12;
diff1 with diff7;
diff9 with diff8;
diff10 with diff11;
savedata:
difftest is mydiff.dat
```

Nested Model (equality constraints)

```
analysis:
difftest = mydiff.dat;
MODEL:
whodas by Diff1* Diff2 Diff3 Diff4 Diff5 Diff6 Diff7 Diff8 Diff9 (1)
Diff10 Diff11 Diff12 (1);
diff1 with diff7;
diff9 with diff8;
diff10 with diff11;
whodas@1;
!savedata:
!difftest is mydiff.dat
```

Could we Constrain the Item Loadings to Equality?

MODEL FIT INFORMATION

Number of Free Parameters 52

Chi-Square Test of Model Fit

Value	189.984*
Degrees of Freedom	62
P-Value	0.0000

Chi-Square Test for Difference Testing

Value	37.791
Degrees of Freedom	11
P-Value	0.0001

* The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.068	
90 Percent C.I.	0.057	0.079
Probability RMSEA <= .05	0.004	

CFI/TLI

CFI	0.983
TLI	0.982

Chi-Square Test of Model Fit for the Baseline Model

Value	7463.430
Degrees of Freedom	66
P-Value	0.0000

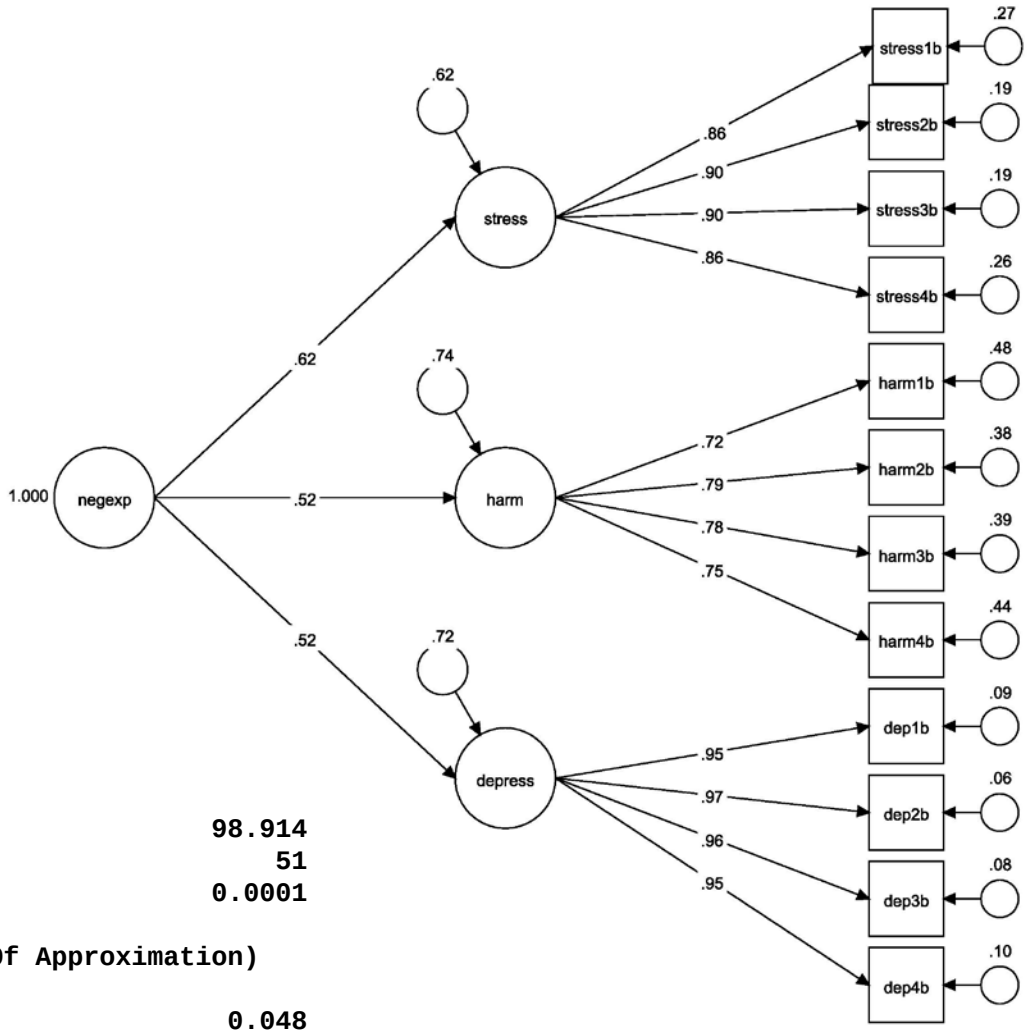
WRMR (Weighted Root Mean Square Residual)

Value	1.348

STDYX Standardization

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
WHODAS BY				
DIFF1	0.756	0.013	58.972	0.000
DIFF2	0.756	0.013	58.972	0.000
DIFF3	0.756	0.013	58.972	0.000
DIFF4	0.756	0.013	58.972	0.000
DIFF5	0.756	0.013	58.972	0.000
DIFF6	0.756	0.013	58.972	0.000
DIFF7	0.756	0.013	58.972	0.000
DIFF8	0.756	0.013	58.972	0.000
DIFF9	0.756	0.013	58.972	0.000
DIFF10	0.756	0.013	58.972	0.000
DIFF11	0.756	0.013	58.972	0.000
DIFF12	0.756	0.013	58.972	0.000
DIFF1 WITH				
DIFF7	0.610	0.042	14.464	0.000
DIFF9 WITH				
DIFF8	0.628	0.052	12.115	0.000
DIFF10 WITH				
DIFF11	0.312	0.068	4.586	0.000

An Example of Hierarchical CFA



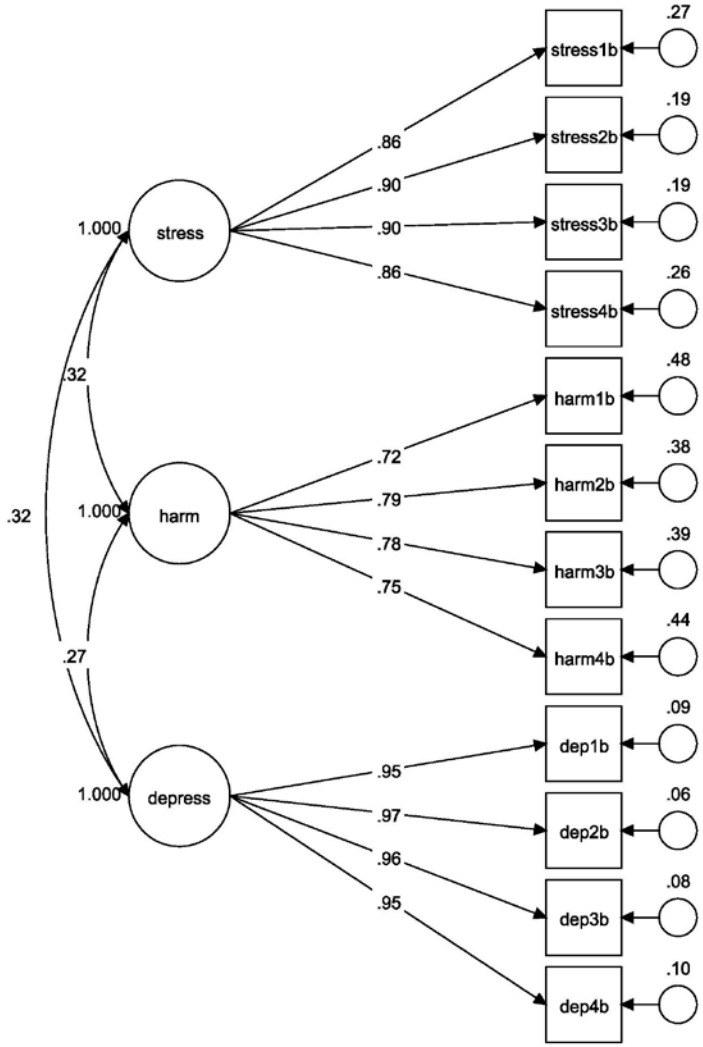
Chi-Square Test of Model Fit

Value	98.914	
Degrees of Freedom	51	
P-Value	0.0001	

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.048	
90 Percent C.I.	0.033	0.062
Probability RMSEA <= .05	0.590	

An Equivalent Model



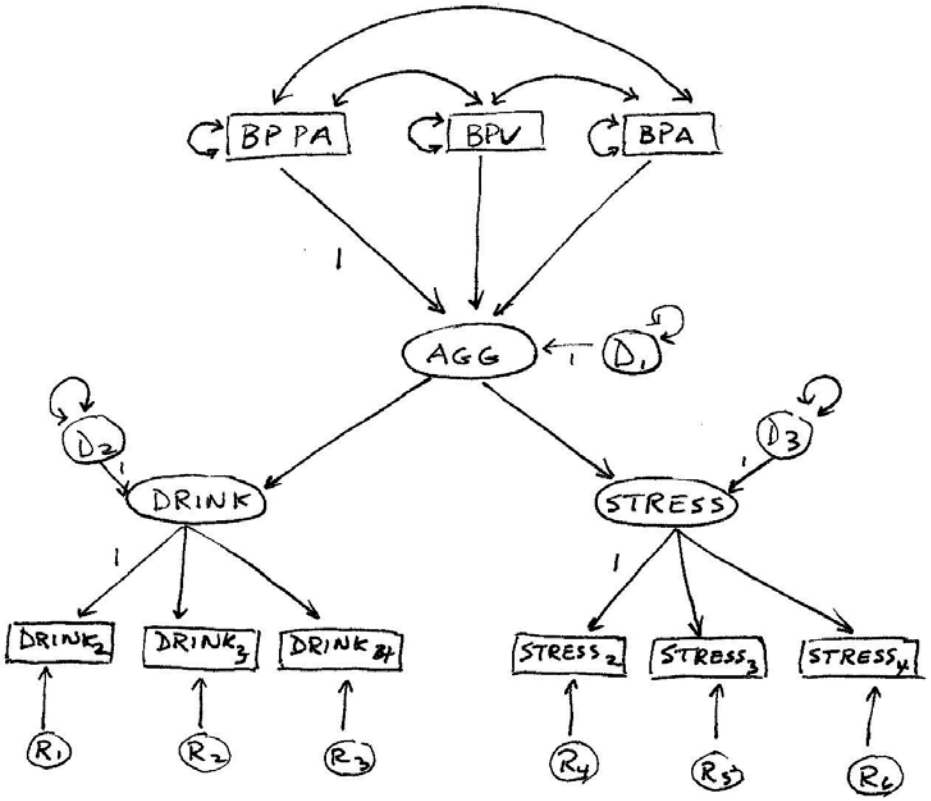
Chi-Square Test of Model Fit

Value	98.914	
Degrees of Freedom	51	
P-Value	0.0001	

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.048	
90 Percent C.I.	0.033	0.062
Probability RMSEA <= .05	0.590	

Causal Indicators



Causal Indicators