PSY 9555A (Nov 6): Interactions (Moderation) in SEM

- Review of moderated multiple regression
- SEM
    - Multiple groups approach
    - Mplus approach
    - Plotting the interaction in Mplus
    - Curvilinear Effects

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$$y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + e$$

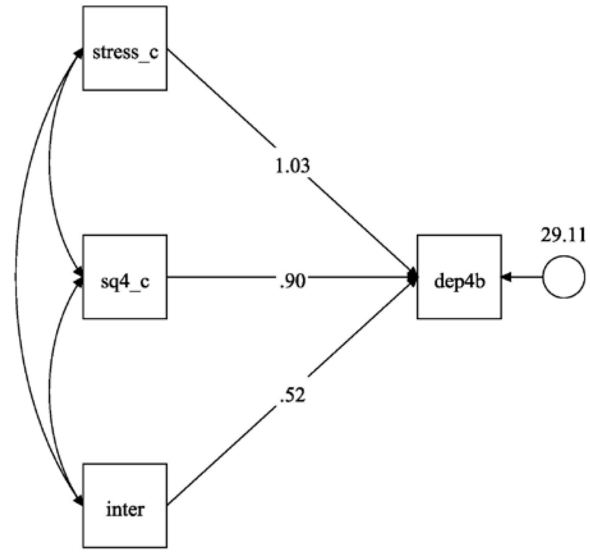
- β_3 is the amount of change in the effect of X on Y for a one unit increase in Z
- Z is the moderator, but we could instead specify X as the moderator of the Z on Y relation
- Rearrange equation so that you have only one predictor and assign values to the other one (e.g., - 1 SD + 1 SD):
- For example we will assign values to Z. Let's say that the SD of Z is 2.31. We can use values of -2.31 and 2.31.

$$y' = (\beta_0 + \beta_2 Z) + (\beta_1 + \beta_3 Z) X$$

simple intercept simple slope

Example 1. Using Multiple Regression with a Product Term

```
usevariables are dep4b stress_c sq1_c inter;
define:
stress_c = stress4b - 6.688; !centering first predictor variable
sq1_c = sq1 - 1.745; !centering second predictor variable
inter = stress_c * sq1_c; !creating product term (interaction)
model: dep4b on stress_c sq1_c inter;
output: sampstat stdyx cinterval;
```



MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
DEP4B ON				
STRESS_C	1.026	0.164	6.251	0.000
SQ1_C	0.895	0.286	3.135	0.002
INTER	0.521	0.166	3.135	0.002
Intercepts				
DEP4B	4.830	0.276	17.505	0.000
Residual Variances				
DEP4B	29.110	2.066	14.089	0.000

STDYX Standardization

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
DEP4B ON				
STRESS_C	0.294	0.045	6.504	0.000
SQ1_C	0.156	0.049	3.164	0.002
INTER	0.154	0.049	3.163	0.002
Intercepts				
DEP4B	0.819	0.056	14.635	0.000
Residual Variances				
DEP4B	0.837	0.034	24.685	0.000

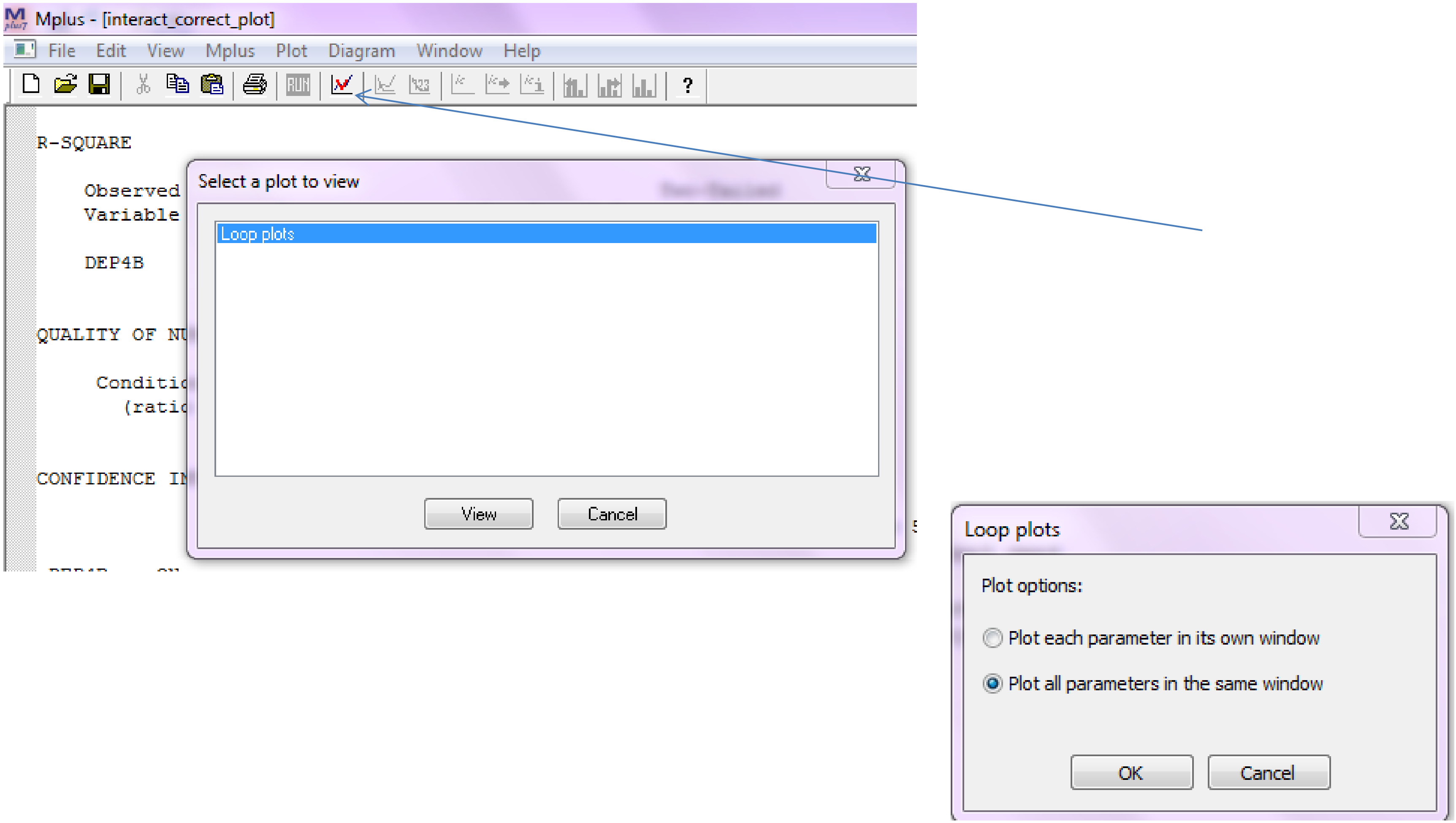
R-SQUARE

Observed Variable	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
DEP4B	0.163	0.034	4.802	0.000

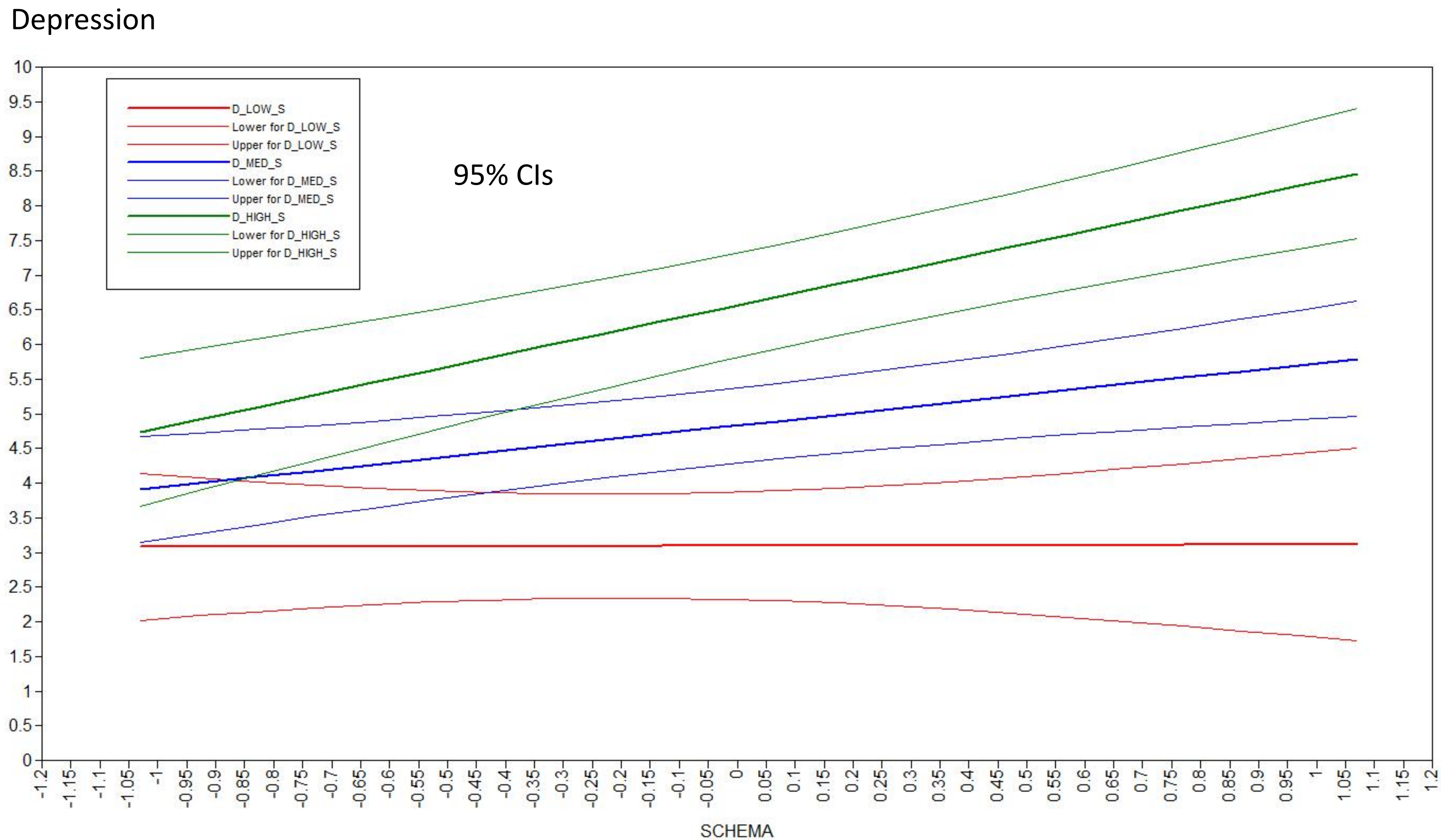
Example 1. Adding Code in Mplus to Plot Simple Slopes

```
define:
stress_c = stress4b - 6.688; !centering first predictor variable
sq1_c = sq1 - 1.745; !centering second predictor variable
inter = stress_c * sq1_c; !creating product term (interaction)
model: dep4b on stress_c (beta1); !added label for first regression coefficient
dep4b on sq1_c (beta2); !label for second regression coefficient
dep4b on inter (beta3); !label for third regression coefficient
[dep4b] (beta0); !label for intercept, [dep4b] asks for intercept; so now we have labels for all parts of the regression equation
model constraint: ! Use this line to make additional calculation that will be used to plot the simple slopes
plot (d_low_s d_med_s d_high_s); !here we specify that we want to plot three regression lines
!(simple slopes of dep4b regressed on schemas(sq1_c)for low, medium and high stress)
loop (schema, -1.03, 1.03, 0.1); !specify the range of
!schema values (-1 SD to +1 SD) look up the standard deviation of schema
d_low_s = beta0 + (beta1*(-1.69)) + (beta2*schema) + (beta3*schema*(-1.69)); !the three regression equations
!note we are using -1SD = 1.69 0 and 1SD = 1.69 (stress values)
d_med_s = beta0 + (beta2*schema);
d_high_s = beta0 + (beta1*1.69) + (beta2*schema) + (beta3*schema*1.69);
plot:
type = plot2;
output: sampstat stdyx cinterval;
```

Example 1. Using Multiple Regression with a Product Term



Example 1. Using Multiple Regression with a Product Term



Multiple Groups Approach to Interactions in SEM

- For categorical moderators use a multiple group SEM approach
- Each level of the moderator is specified as a group (e.g. male – female)
- Investigate differences in regression coefficients (slopes) across groups
- Tests of differences in slopes (i.e., moderation) involve nested models comparing constrained (to equality) vs. unconstrained regression coefficient parameters.
- Evidence of moderation when unconstrained model fits significantly better than the constrained model

Multiple-Groups Example: Gender

Test for metric invariance first

```
grouping is gender (1=male 2=female);
model:
dep by dep1b dep2b dep3b dep4b;
grade with dep;
```

In this model, the loadings for males and females are constrained to equality (Mplus default)

vs.

```
model:
dep by dep1b dep2b dep3b dep4b;
grade with dep;
model female:
dep by dep2b dep3b dep4b;
```

In this model, we specify to unconstrain the female loadings (i.e., not force them to equal the male loadings). Note. No need to do this for the first indicator set at 1.

$\chi^2_{(16)}$ 41.85 - $\chi^2_{(13)}$ 35.75 = $\chi^2_{(3)}$ 6.10, n.s.
(crit $\chi^2_{(3)}$ at p = .05 = 7.82)

Multiple-Groups Example: Gender

```
grouping is gender (1=male 2=female);
model:
dep by dep1b dep2b dep3b dep4b;
grade on dep (1);
```

In this model, the regression coefficient of grade regressed on the latent variable dep is constrained to equality across gender by adding the label (1).

vs.

```
model:
dep by dep1b dep2b dep3b dep4b;
grade on dep;
```

In this second model, the regression coefficient of grade regressed on the latent variable dep is left unconstrained across gender.

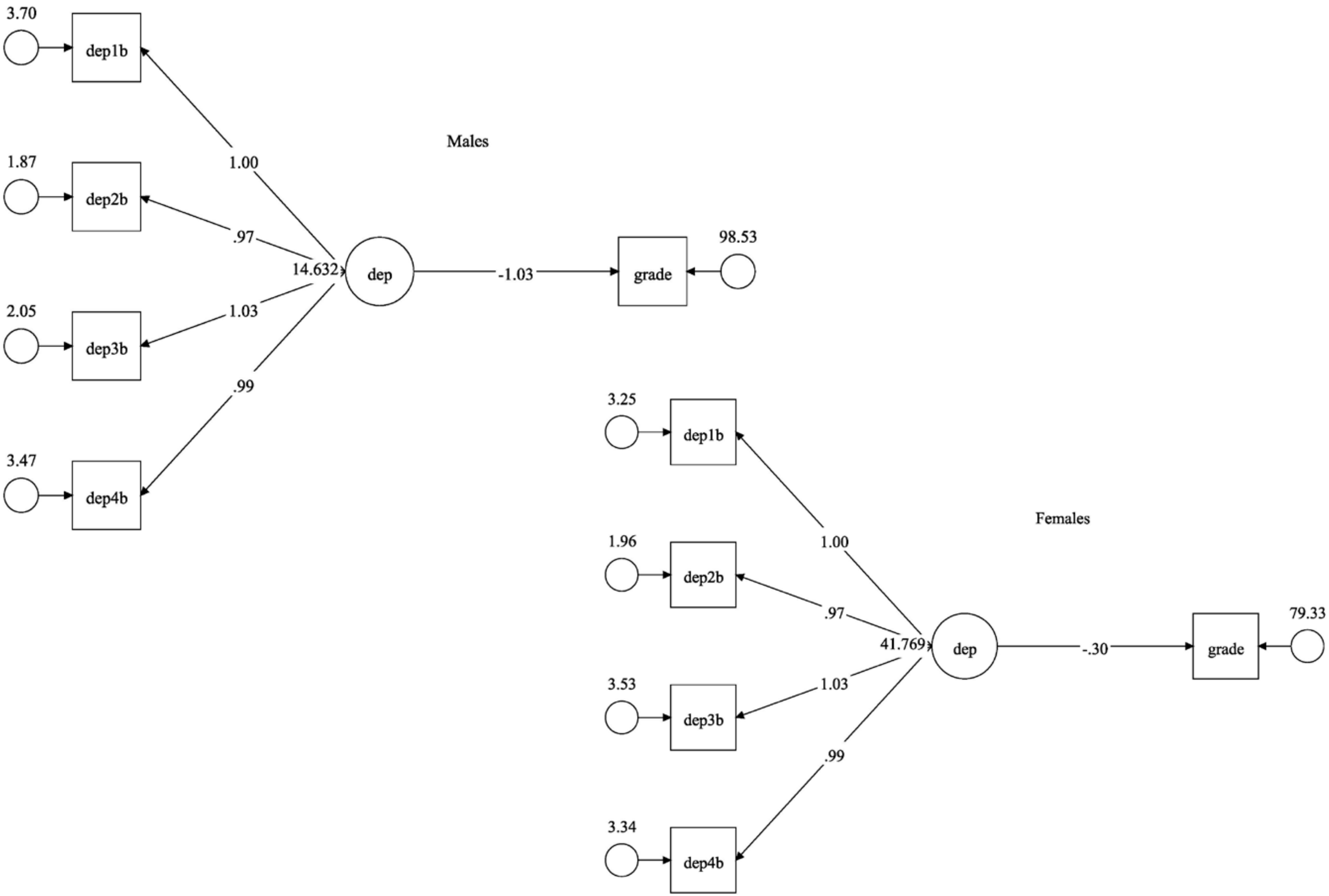
$\chi^2_{(17)}$ 50.41 - $\chi^2_{(16)}$ 41.85 = $\chi^2_{(1)}$ 8.56, $p < .001$
(crit $\chi^2_{(1)}$ at p = .05 = 3.84)

Multiple-Groups Example: Gender

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Group MALE				
DEP BY				
DEP1B	1.000	0.000	999.000	999.000
DEP2B	0.971	0.020	49.365	0.000
DEP3B	1.031	0.023	45.023	0.000
DEP4B	0.989	0.022	43.990	0.000
GRADE ON				
DEP	-1.032	0.231	-4.478	0.000
Means				
DEP	0.000	0.000	999.000	999.000
Intercepts				
DEP1B	4.420	0.333	13.280	0.000
DEP2B	3.994	0.317	12.580	0.000
DEP3B	3.938	0.338	11.655	0.000
DEP4B	3.737	0.329	11.349	0.000
GRADE	75.344	0.937	80.450	0.000
Variances				
DEP	14.632	1.836	7.970	0.000
Residual Variances				
DEP1B	3.696	0.544	6.799	0.000
DEP2B	1.874	0.348	5.384	0.000
DEP3B	2.049	0.382	5.369	0.000
DEP4B	3.474	0.510	6.807	0.000
GRADE	98.533	12.416	7.936	0.000

Group FEMALE				
DEP BY				
DEP1B	1.000	0.000	999.000	999.000
DEP2B	0.971	0.020	49.365	0.000
DEP3B	1.031	0.023	45.023	0.000
DEP4B	0.989	0.022	43.990	0.000
GRADE ON				
DEP	-0.301	0.089	-3.377	0.001
Means				
DEP	2.100	0.514	4.086	0.000
Intercepts				
DEP1B	4.420	0.333	13.280	0.000
DEP2B	3.994	0.317	12.580	0.000
DEP3B	3.938	0.338	11.655	0.000
DEP4B	3.737	0.329	11.349	0.000
GRADE	74.863	0.618	121.047	0.000
Variances				
DEP	41.769	3.864	10.810	0.000
Residual Variances				
DEP1B	3.253	0.369	8.805	0.000
DEP2B	1.965	0.272	7.213	0.000
DEP3B	3.531	0.400	8.819	0.000
DEP4B	3.337	0.381	8.751	0.000
GRADE	79.326	7.293	10.876	0.000

Multiple-Groups Example: Gender



Interactions with Latent Variables

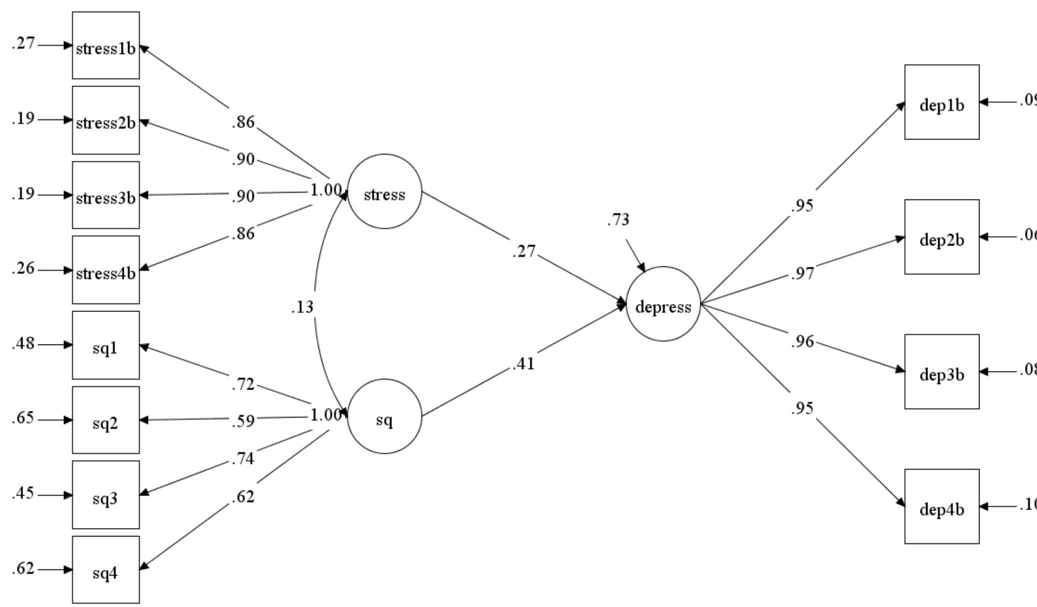
Mplus approach:

- Latent moderated structural equations (LMS) method
- QML – Quasi-maximum likelihood
- A. Klein and Muthen (2007)
- LMS/QML approach does not require separate indicators representing the interaction
- Works only with raw data
- See Kline p. 341-342 for description and Mplus manual (version 7) p. 76, 77, 687, 688

Interactions with Latent Variables
Model without interaction

```
model: stress by stress1b stress2b stress3b stress4b;
depress by dep1b dep2b dep3b dep4b;
sq by sq1 sq2 sq3 sq4;
depress on sq stress;
depress on stress;
output: sampstat residual stdyx modindices tech4;
```

```
Information Criteria

          Akaike (AIC)                   17572.333
          Bayesian (BIC)                 17729.342
          Sample-Size Adjusted BIC       17605.586
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             93.694
          Degrees of Freedom                    51
          P-Value                           0.0003

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.045
          90 Percent C.I.                    0.030  0.059
          Probability RMSEA <= .05           0.704

CFI/TLI

          CFI                                0.990
          TLI                                0.988
```



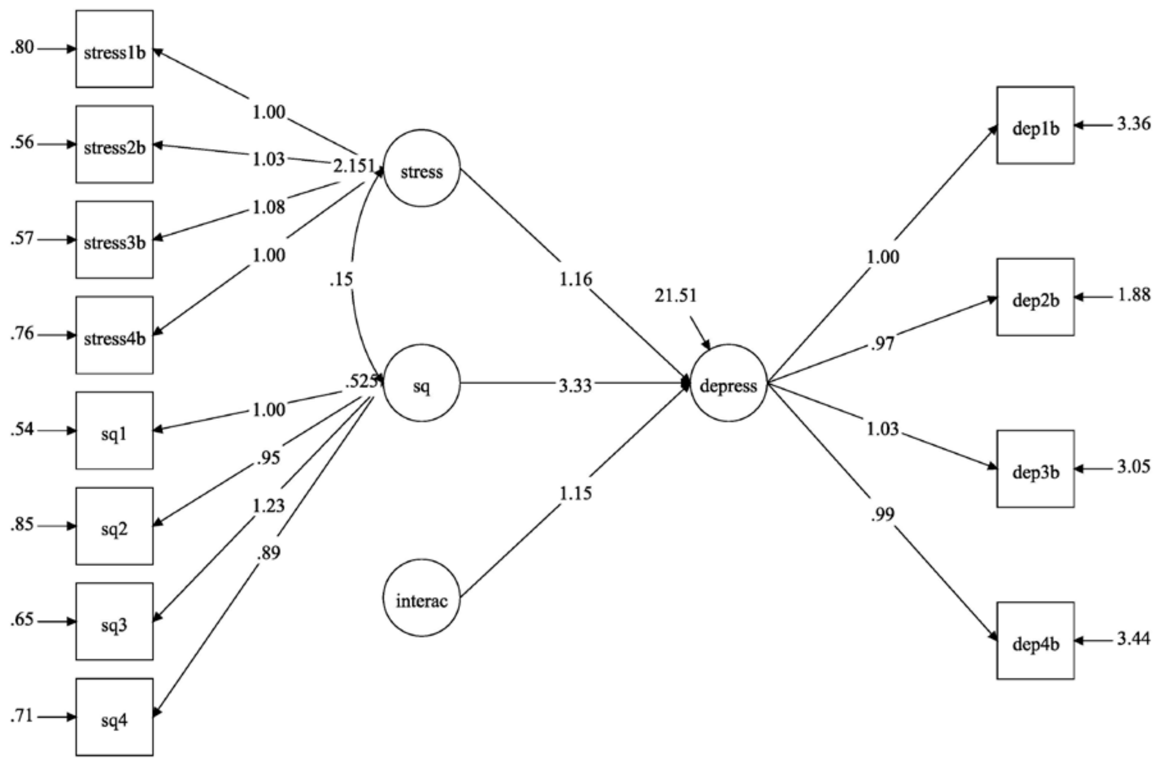
Model with Interaction

```
analysis:
type = random;
algorithm = integration;
model: stress by stress1b stress2b stress3b stress4b;
depress by dep1b dep2b dep3b dep4b;
sq by sq1 sq2 sq3 sq4;
interac | stress xwith sq;
depress on sq stress interac;
output: sampstat tech1 cinterval;
```

```
MODEL FIT INFORMATION

Number of Free Parameters                       40

Loglikelihood

          H0 Value                       -8737.694
          H0 Scaling Correction Factor      1.9191
            for MLR

Information Criteria

          Akaike (AIC)                   17555.389
          Bayesian (BIC)                 17716.424
          Sample-Size Adjusted BIC       17589.494
            (n* = (n + 2) / 24)
```

```
MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 STRESS   BY
    STRESS1B           1.000      0.000    999.000    999.000
    STRESS2B           1.034      0.048     21.343      0.000
    STRESS3B           1.083      0.053     20.491      0.000
    STRESS4B           0.996      0.049     20.276      0.000

 DEPRESS  BY
    DEP1B              1.000      0.000    999.000    999.000
    DEP2B              0.970      0.036     27.232      0.000
    DEP3B              1.031      0.033     31.129      0.000
    DEP4B              0.986      0.028     35.312      0.000

 SQ       BY
    SQ1                1.000      0.000    999.000    999.000
    SQ2                0.947      0.101      9.376      0.000
    SQ3                1.233      0.148      8.339      0.000
    SQ4                0.887      0.112      7.934      0.000

 DEPRESS    ON
    SQ                 3.331      0.669      4.979      0.000
    STRESS             1.160      0.210      5.515      0.000
    INTERAC            1.153      0.426      2.709      0.007

 SQ       WITH
    STRESS             0.154      0.070      2.189      0.029
```

Model with Interaction



How Much the Interaction Explains

Model without Interaction

```
Residual Variances
   STRESS1B           0.784      0.069     11.386      0.000
   STRESS2B           0.550      0.056      9.891      0.000
   STRESS3B           0.581      0.060      9.713      0.000
   STRESS4B           0.762      0.067     11.329      0.000
   DEP1B              3.360      0.301     11.161      0.000
   DEP2B              1.863      0.209      8.900      0.000
   DEP3B              3.044      0.289     10.548      0.000
   DEP4B              3.479      0.311     11.187      0.000
   SQ1                0.513      0.054      9.532      0.000
   SQ2                0.851      0.072     11.880      0.000
   SQ3                0.657      0.073      9.007      0.000
   SQ4                0.692      0.059     11.676      0.000
   DEPRESS           24.342      2.011     12.103      0.000
```

```
TECHNICAL 4 OUTPUT


     ESTIMATES DERIVED FROM THE MODEL


           ESTIMATED MEANS FOR THE LATENT VARIABLES
              STRESS        DEPRESS       SQ
              ________      ________      ________
      1         0.000         0.000         0.000


           S.E. FOR ESTIMATED MEANS FOR THE LATENT VARIABLES
              STRESS        DEPRESS       SQ
              ________      ________      ________
      1         0.000         0.000         0.000


           ESTIMATED COVARIANCE MATRIX FOR THE LATENT VARIABLES
              STRESS        DEPRESS       SQ
              ________      ________      ________
 STRESS         2.165
 DEPRESS        2.752        33.217
 SQ             0.143         1.883         0.548
```

Depression residual with no interaction
= 24.34/33.22 = 73.3%

Residual with interaction
= 21.51/33.22 = 64.8%

Model with Interaction

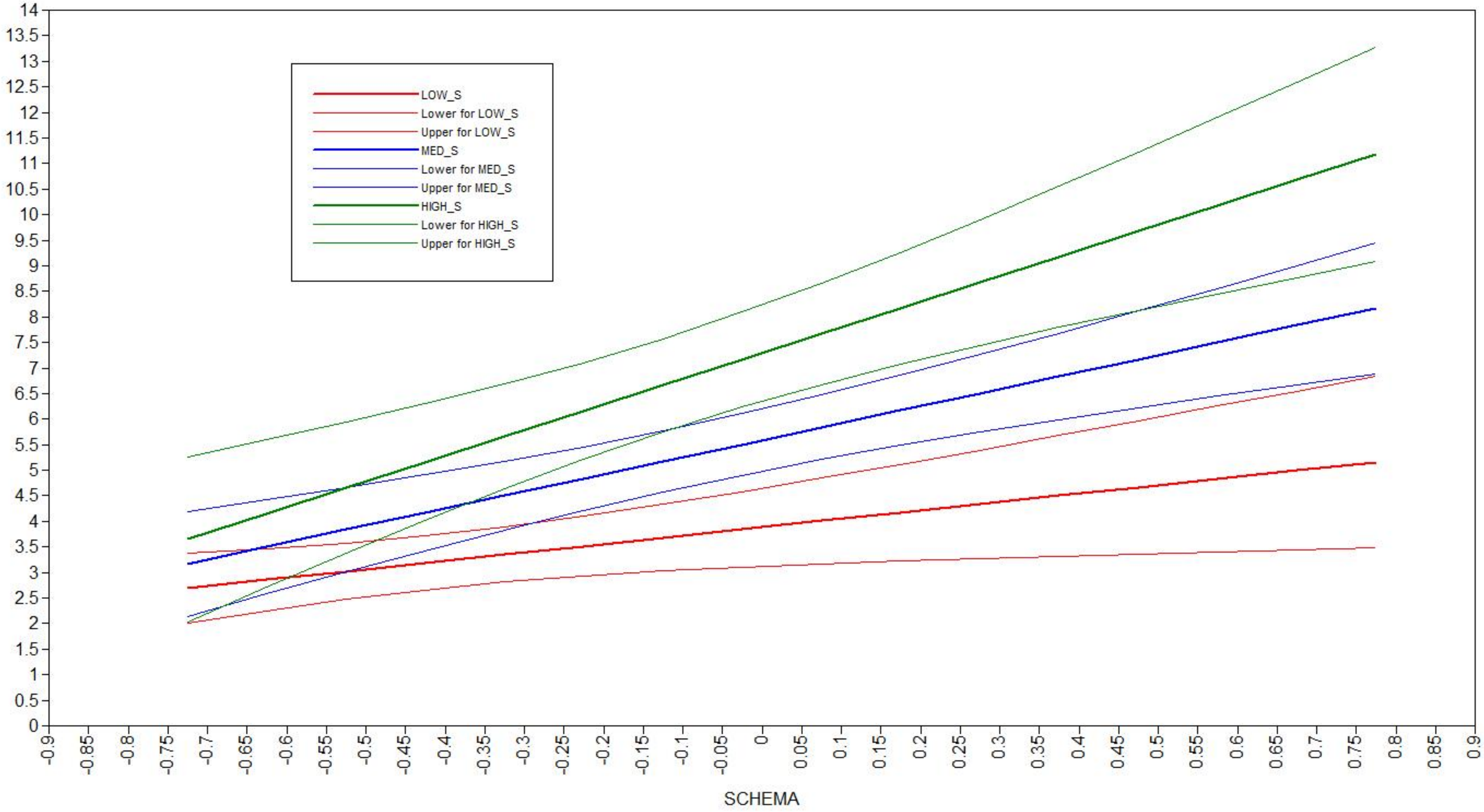
```
Residual Variances
   STRESS1B           0.798      0.154      5.176      0.000
   STRESS2B           0.556      0.077      7.218      0.000
   STRESS3B           0.573      0.073      7.830      0.000
   STRESS4B           0.763      0.111      6.901      0.000
   DEP1B              3.364      0.650      5.175      0.000
   DEP2B              1.885      0.355      5.314      0.000
   DEP3B              3.046      0.481      6.327      0.000
   DEP4B              3.443      0.595      5.785      0.000
   SQ1                0.538      0.077      6.961      0.000
   SQ2                0.848      0.078     10.893      0.000
   SQ3                0.652      0.090      7.224      0.000
   SQ4                0.706      0.078      9.047      0.000
   DEPRESS           21.508      4.531      4.746      0.000
```

Plotting the Interaction in Mplus

```
analysis:
type = random;
algorithm = integration;
model: stress by stress1b stress2b stress3b stress4b;
depress by dep1b dep2b dep3b dep4b;
[dep1b@0];!need to fix one intercept to allow latent intercept free
sq by sq1 sq2 sq3 sq4;
interac | stress xwith sq;
depress on sq (beta1);
depress on stress (beta2);
depress on interac (beta3);
[depress](beta0);
model constraint:
plot (low_s med_s high_s);
loop (schema, -0.725, 0.725, 0.1);
low_s = beta0 + (beta1*schema)+(beta2*(-1.467)) + (beta3*schema*(-1.467));
med_s = beta0 + (beta1*schema);
high_s = beta0 + (beta1*schema)+(beta2*1.467) + (beta3*schema*1.467);
plot:
type = plot2;
output: sampstat tech1;
```

Intercepts				
STRESS1B	6.832	0.084	80.920	0.000
STRESS2B	7.050	0.083	84.836	0.000
STRESS3B	6.808	0.087	78.621	0.000
STRESS4B	6.704	0.084	80.059	0.000
DEP1B	0.000	0.000	999.000	999.000
DEP2B	-0.296	0.183	-1.618	0.106
DEP3B	-0.606	0.183	-3.315	0.001
DEP4B	-0.622	0.160	-3.880	0.000
SQ1	1.748	0.052	33.745	0.000
SQ2	2.510	0.057	43.683	0.000
SQ3	2.089	0.060	34.635	0.000
SQ4	2.031	0.053	38.359	0.000
DEPRESS	5.586	0.308	18.109	0.000
Variances				
STRESS	2.151	0.257	8.373	0.000
SQ	0.525	0.109	4.825	0.000

Plotting the Interaction in Mplus



Model with a Quadratic Effect

```
analysis:
type = random;
algorithm = integration;
model: stress by stress1b stress2b stress3b stress4b;
depress by dep1b dep2b dep3b dep4b;
sq by sq1 sq2 sq3 sq4;
quad_s | stress xwith stress;
depress on sq stress quad_s;
output: sampstat tech1;
```

Residual Variances

STRESS1B	0.798	0.156	5.109	0.000
STRESS2B	0.556	0.077	7.199	0.000
STRESS3B	0.570	0.073	7.799	0.000
STRESS4B	0.764	0.111	6.910	0.000
DEP1B	3.366	0.650	5.179	0.000
DEP2B	1.871	0.349	5.359	0.000
DEP3B	3.038	0.478	6.360	0.000
DEP4B	3.467	0.598	5.798	0.000
SQ1	0.517	0.076	6.822	0.000
SQ2	0.846	0.078	10.883	0.000
SQ3	0.655	0.095	6.915	0.000
SQ4	0.693	0.078	8.843	0.000
DEPRESS	22.829	4.215	5.416	0.000

Information Criteria

Akaike (AIC)	17558.837
Bayesian (BIC)	17719.871
Sample-Size Adjusted BIC (n* = (n + 2) / 24)	17592.942

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
STRESS BY				
STRESS1B	1.000	0.000	999.000	999.000
STRESS2B	1.035	0.049	20.981	0.000
STRESS3B	1.084	0.054	20.063	0.000
STRESS4B	0.996	0.050	20.033	0.000
DEPRESS BY				
DEP1B	1.000	0.000	999.000	999.000
DEP2B	0.970	0.035	27.363	0.000
DEP3B	1.031	0.033	31.213	0.000
DEP4B	0.986	0.028	35.458	0.000
SQ BY				
SQ1	1.000	0.000	999.000	999.000
SQ2	0.930	0.100	9.296	0.000
SQ3	1.206	0.144	8.372	0.000
SQ4	0.883	0.113	7.816	0.000
DEPRESS ON				
SQ	3.164	0.606	5.220	0.000
STRESS	1.427	0.294	4.856	0.000
QUAD_S	0.328	0.119	2.764	0.006
SQ WITH				
STRESS	0.142	0.072	1.972	0.049

Model with a Quadratic Effect

```
analysis:
type = random;
algorithm = integration;
model: stress by stress1b stress2b stress3b stress4b;
depress by dep1b dep2b dep3b dep4b;
[dep1b@0];!need to fix one intercept to allow latent intercept free
sq by sq1 sq2 sq3 sq4;
quad_s | stress xwith stress;
depress on sq (beta1);
depress on stress (beta2);
depress on quad_s (beta3);
[depress] (beta0);
model constraint:
plot dep;
loop (stress, -1.467, 1.467, 0.1);
!note beta1 is multiplied by 0 which is the mean of sq
dep = beta0 + (beta2*stress) + (beta3*stress*stress);
plot:
type = plot2;
output: sampstat tech1;
```