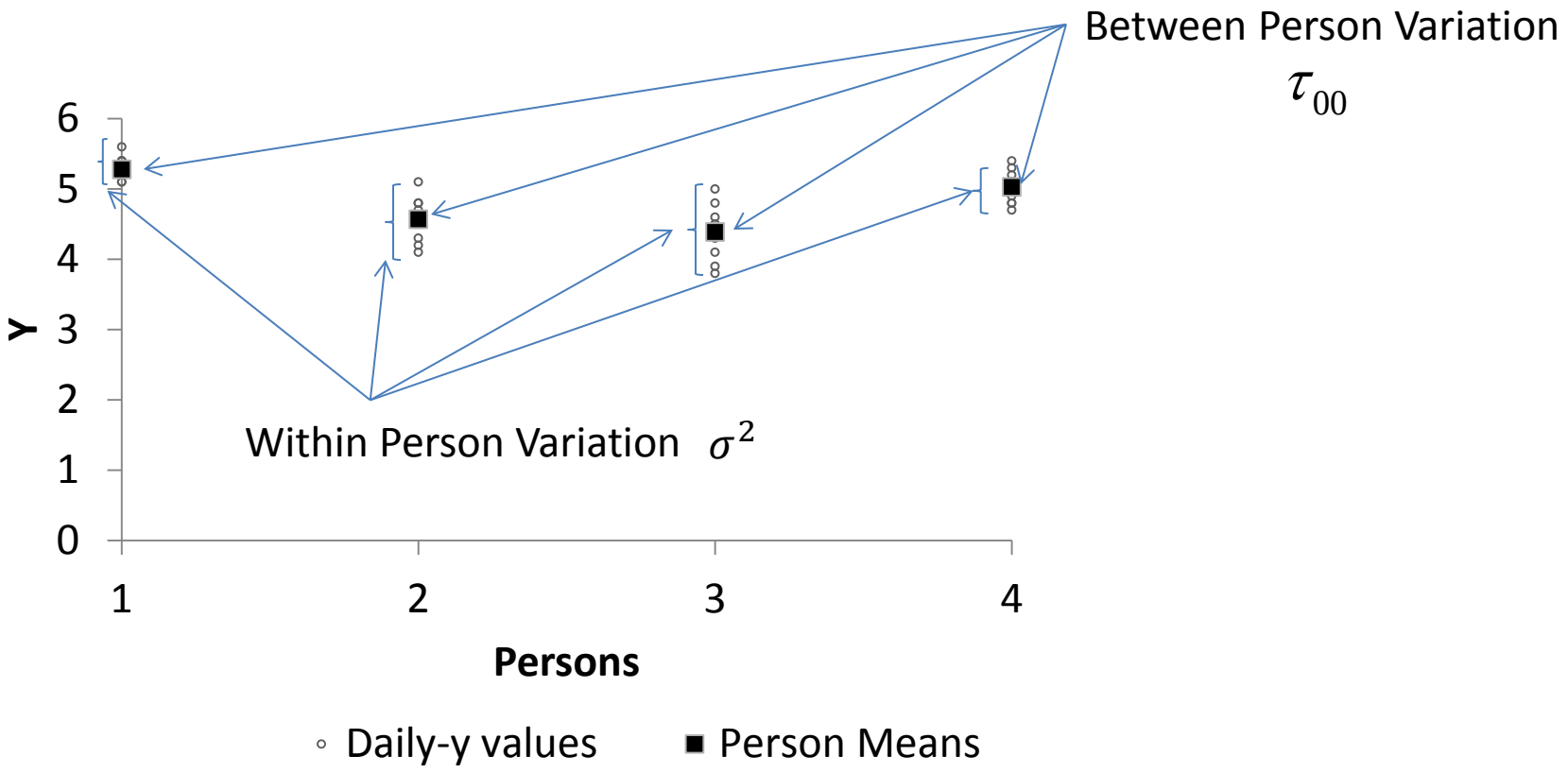
PSY 9556B (March 26) MLM for Diary Data

Recall Distinction between “fixed” and “random” effects in MLM and LGM:

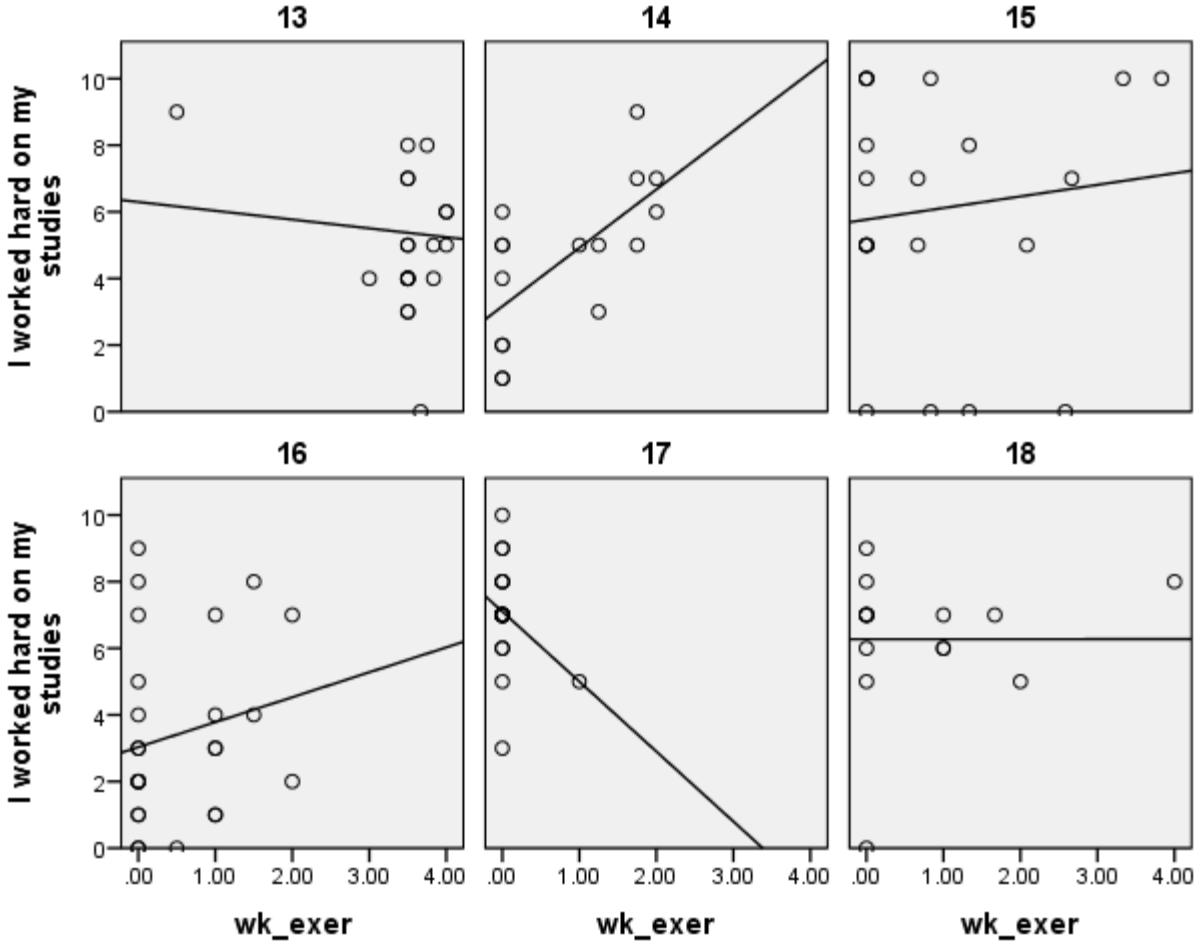
- Fixed effect: single values that estimate of population parameters
  - (e.g., a regression coefficient, a mean intercept or mean slope)
- Random effect: provide information about the variation in the individual regression coefficient or intercept parameters across the clustering units (i.e., persons)
  - (e.g., variance of intercepts and slopes in LGM or MLM)

Between and Within Person Variation

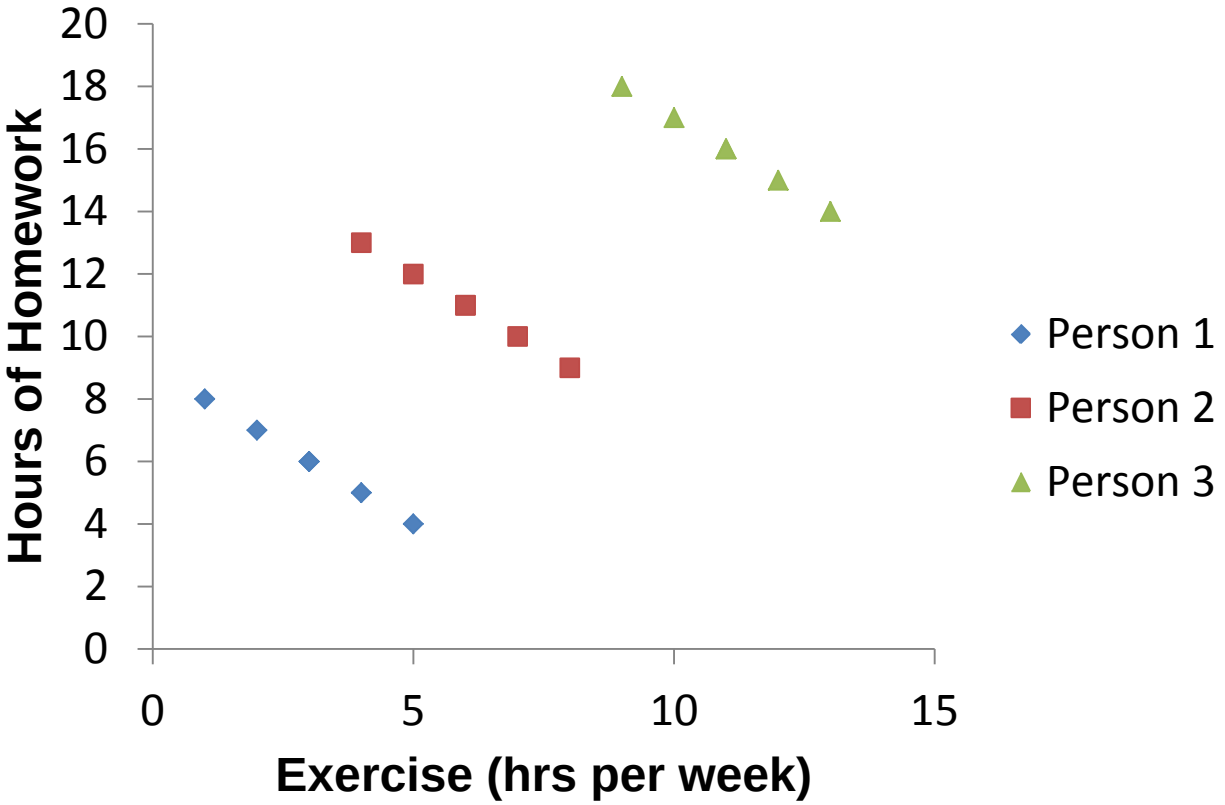


Individual Scatterplots for Six Subjects

Notice the variation in the intercepts and the slopes



Between and Within Group Correlations



Unconditional Means or Intercept-only MLM model

Level 1: $$y_{ti} = \beta_{0i} + r_{ti}$$

y_{ti} is observation *y* at time *t* for individual *i*
β_{0i} is the mean *y* for individual *i* based on the observations aggregated across time
r_{ti} is the deviation of observation at time *t* for individual *i* from that individual's mean

Level 2:

$$\beta_{0i} = \gamma_{00} + \mu_{0i}$$

β_{0i} same as above
γ_{00} is the mean across individuals and μ_{0i} is the deviation of an individual's mean from the total sample mean.

With substitution:

$$y_{ti} = \gamma_{00} + \mu_{0i} + r_{ti}$$

Unconditional Means or Intercept-only MLM model

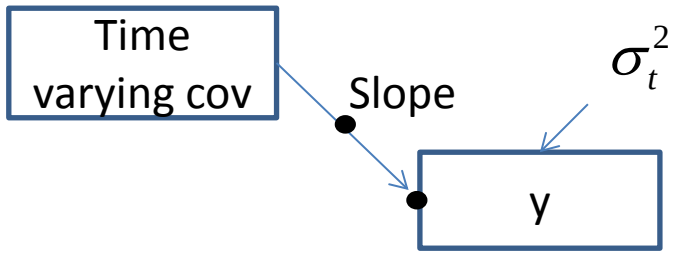
$$y_{ti} = \gamma_{00} + \mu_{0i} + r_{ti}$$

$$\rho = \frac{var(\mu_{0i})}{var(\mu_{0i}) + var(r_{ti})}$$

$$\rho = \frac{\tau_{00}}{(\tau_{00} + \sigma_t{}^2)}$$

Adding a Time-Varying Covariate

$$y_{ti} = \beta_{0i} + \beta_{1i} x_{ti} + r_{ti}$$



$$\beta_{0i} = \gamma_{00} + \mu_{0i}$$
$$\beta_{1i} = \gamma_{10} + \mu_{1i}$$

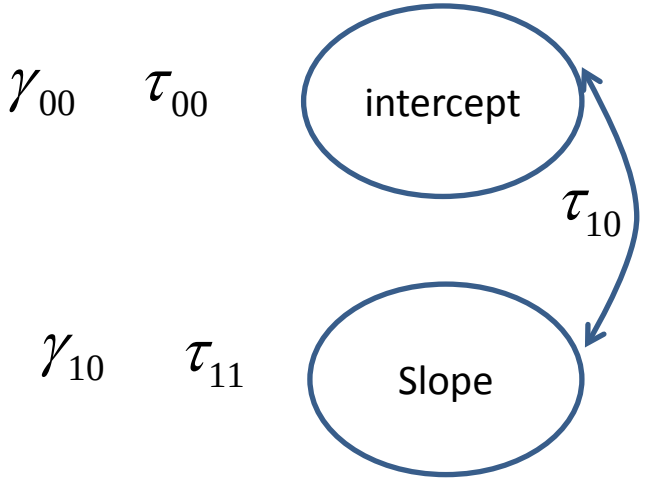
$$y_{ti} = \gamma_{00} + \mu_{0i} + \left(\gamma_{10} + \mu_{1i}\right) x_{ti} + r_{ti}$$
$$y_{ti} = \gamma_{00} + \gamma_{10} x_{ti} + \mu_{0i} + \mu_{1i} x_{ti} + r_{ti}$$

WITHIN

BETWEEN

var of:

$$\mu_{0i} = \tau_{00}$$
$$\mu_{1i} = \tau_{11}$$
$$r_{ti} = \sigma_t^2$$
$$\text{COV}_{\mu_{0i}\,\mu_{1i}} = \tau_{10}$$

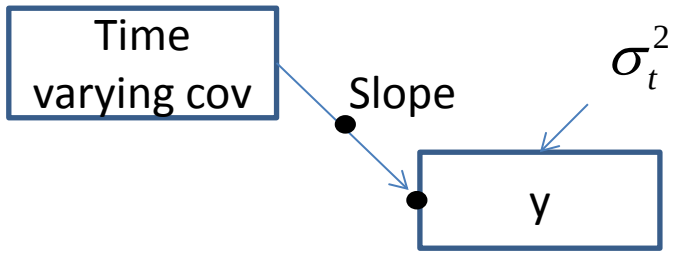


Adding a Contextual Predictor (Level-2 Time-Invariant Covariate

$$y_{ti} = \beta_{0i} + \beta_{1i} x_{ti} + r_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_i + \mu_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_i + \mu_{1i}$$



$$y_{ti} = \gamma_{00} + \gamma_{01} w_i + \mu_{0i} + \left(\gamma_{10} + \gamma_{11} w_i + \mu_{1i}\right) x_{ti} + r_{ti}$$

$$y_{ti} = \gamma_{00} + \gamma_{01} w_i + \gamma_{10} x_{ti} + \gamma_{11} w_i x_{ti} + \mu_{0i} + \mu_{1i} x_{ti} + r_{ti}$$

WITHIN

BETWEEN

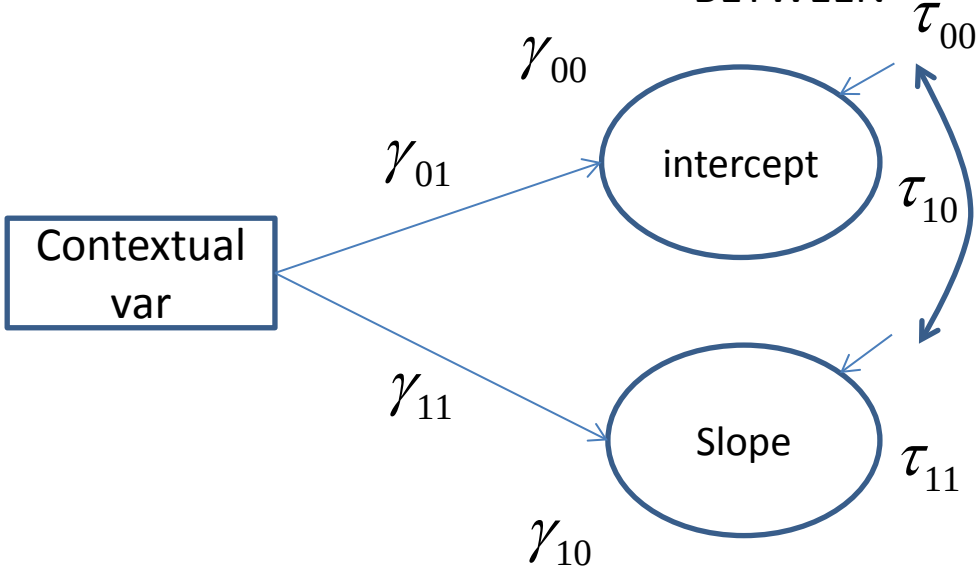
var of:

$$\mu_{0i} = \tau_{00}$$

$$\mu_{1i} = \tau_{11}$$

$$r_{ti} = \sigma_t^2$$

$$\text{COV}_{\mu_{0i}\,\mu_{1i}} = \tau_{10}$$



Example: Depression regressed on Drinking (26 time points)

```
Title: LGM Depression;
data: File is c:\paul\mplus\dec09\mplus_conti.dat;
    Format is 1F4, 2F1, 26F6.2, 52F2, 1F8.2, 26F6.2, 1F6, 26F2, 79F6.2;
data widetolong:
    wide = w1dep-w26dep | w1dr-w26dr | ac1-ac26 | ha1-ha26;
    long = dep | drink | acad | harm;
    idvariable = person;
    repetition = time;
variable: names are studid gender genderc
w1dep w2dep w3dep w4dep w5dep w6dep w7dep w8dep w9dep w10dep
w11dep w12dep w13dep w14dep w15dep w16dep w17dep w18dep w19dep w20dep
w21dep w22dep w23dep w24dep w25dep w26dep
w1depo w2depo w3depo w4depo w5depo w6depo w7depo w8depo w9depo w10depo
w11depo w12depo w13depo w14depo w15depo w16depo w17depo w18depo w19depo w20depo
w21depo w22depo w23depo w24depo w25depo w26depo
w1depd w2depd w3depd w4depd w5depd w6depd w7depd w8depd w9depd w10depd
w11depd w12depd w13depd w14depd w15depd w16depd w17depd w18depd w19depd w20depd
w21depd w22depd w23depd w24depd w25depd w26depd
w0dr w1dr w2dr w3dr w4dr w5dr w6dr w7dr w8dr w9dr w10dr w11dr w12dr w13dr
w14dr w15dr w16dr w17dr w18dr w19dr w20dr w21dr w22dr w23dr w24dr w25dr w26dr
g0dr g1dr g2dr g3dr g4dr g5dr g6dr g7dr g8dr g9dr g10dr g11dr g12dr g13dr
g14dr g15dr g16dr g17dr g18dr g19dr g20dr g21dr g22dr g23dr g24dr g25dr g26dr
ac1 ac2 ac3 ac4 ac5 ac6 ac7 ac8 ac9 ac10 ac11 ac12 ac13 ac14 ac15 ac16
ac17 ac18 ac19 ac20 ac21 ac22 ac23 ac24 ac25 ac26
ha1 ha2 ha3 ha4 ha5 ha6 ha7 ha8 ha9 ha10 ha11 ha12 ha13 ha14 ha15 ha16
ha17 ha18 ha19 ha20 ha21 ha22 ha23 ha24 ha25 ha26
failure iscl aban mist edep ssac enme sali einh usta depe defe enti subj vuln
mqsoc mqcop mqenh mqcon bppa bpv bpa bph bptot esea espa esfi;
missing = blank;
USEVARIABLES ARE dep person;
CLUSTER IS person;
ANALYSIS:
TYPE IS TWOLEVEL RANDOM;
ESTIMATOR IS ML;
model:
%WITHIN%
dep;
%BETWEEN%
dep;
output: sampstat;
```

When your data file is structured in the conventional one line per subject with repeated measures on the same line

Example: Depression regressed on Drinking (26 time points)
Intercept Only Model

```
SUMMARY OF DATA

     Number of missing data patterns             1
     Number of clusters                        414

     Average cluster size       21.225

     Estimated Intraclass Correlations for the Y Variables

                Intraclass
     Variable   Correlation

     DEP            0.643
```

```
MODEL FIT INFORMATION

Number of Free Parameters                       3

Loglikelihood

          H0 Value                       -25899.075
          H1 Value                       -25899.075
```

```
MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Within Level

 Variances
    DEP               17.926      0.277     64.683      0.000

Between Level

 Means
    DEP                5.372      0.284     18.932      0.000

 Variances
    DEP               32.352      2.331     13.881      0.000
```

σ_t^2

γ_{00}

τ_{00}

Example: Depression regressed on Drinking (26 time points)
Level-1 Predictor (Fixed)

```
USEVARIABLES ARE dep drink person;
CLUSTER IS person;
WITHIN IS drink;
DEFINE:
center drink(grandmean);
ANALYSIS:
TYPE IS TWOLEVEL RANDOM;
ESTIMATOR IS ML;
model:
%WITHIN%
dep on drink;
%BETWEEN%
dep;
output: sampstat;
```

```
     ESTIMATED SAMPLE STATISTICS FOR WITHIN

                Means
                   DEP           DRINK
                _______       _______
1                 0.000         0.000

                Covariances
                   DEP           DRINK
                _______       _______
DEP              17.954
DRINK             0.938        74.149

                Correlations
                   DEP           DRINK
                _______       _______
DEP               1.000
DRINK             0.026         1.000
```

```
MODEL FIT INFORMATION

Number of Free Parameters                        4

Loglikelihood

          H0 Value                       -25836.717
          H1 Value                       -25836.717
```

Example: Depression regressed on Drinking (26 time points)
Level-1 Predictor (Fixed)

γ_{10}

σ_t^2

γ_{00}

τ_{00}

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Within Level				
DEP ON				
DRINK	0.013	0.007	1.797	0.072
Residual Variances				
DEP	17.942	0.278	64.595	0.000
Between Level				
Means				
DEP	5.374	0.284	18.930	0.000
Variances				
DEP	32.382	2.333	13.880	0.000

Example: Depression regressed on Drinking (26 time points)
Level-1 Predictor (Random slopes)

```
USEVARIABLES ARE dep drink person;
CLUSTER IS person;
WITHIN IS drink;
DEFINE:
center drink(grandmean);
ANALYSIS:
TYPE IS TWOLEVEL RANDOM;
ESTIMATOR IS ML;
model:
%WITHIN%
s1|dep on drink;
%BETWEEN%
dep s1;
dep with s1;
output: sampstat;
```

```
MODEL FIT INFORMATION

Number of Free Parameters                        6

Loglikelihood

          H0 Value                       -25824.615
```

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Within Level				
σ_t^2 Residual Variances				
DEP	17.722	0.280	63.304	0.000
Between Level				
DEP WITH				
τ_{10} S1	0.190	0.064	2.954	0.003
Means				
γ_{00} DEP	5.386	0.286	18.837	0.000
γ_{10} S1	0.023	0.010	2.274	0.023
Variances				
τ_{00} DEP	32.734	2.368	13.823	0.000
τ_{11} S1	0.007	0.003	2.599	0.009

Example: Depression regressed on Drinking (26 time points)
Level-2 Contextual Variable: Method 1 (conventional)

```
USEVARIABLES ARE dep drink person tdrink;
CLUSTER IS person;
WITHIN are drink;
BETWEEN are tdrink;
DEFINE:
center drink(grandmean);
tdrink = cluster_mean(drink);
center tdrink(grandmean);
ANALYSIS:
TYPE IS TWOLEVEL RANDOM;
ESTIMATOR IS ML;
model:
%WITHIN%
s1|dep on drink;
%BETWEEN%
dep on tdrink;
dep with s1;
output: sampstat;
```

```
MODEL FIT INFORMATION

Number of Free Parameters                        7

Loglikelihood

          H0 Value                       -25824.066
```

ESTIMATED SAMPLE STATISTICS FOR WITHIN

Covariances

	DEP	DRINK	TDRINK
DEP	17.956		
DRINK	1.014	74.149	
TDRINK	0.000	0.000	0.000

Correlations

	DEP	DRINK	TDRINK
DEP	1.000		
DRINK	0.028	1.000	
TDRINK	0.000	0.000	0.000

ESTIMATED SAMPLE STATISTICS FOR BETWEEN

Covariances

	DEP	DRINK	TDRINK
DEP	32.383		
DRINK	0.000	0.000	
TDRINK	-1.644	0.000	35.580

Correlations

	DEP	DRINK	TDRINK
DEP	1.000		
DRINK	0.000	0.000	
TDRINK	-0.048	0.000	1.000

Example: Depression regressed on Drinking (26 time points)
Level-2 Contextual Variable: Method 1 (conventional)

MODEL RESULTS

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
	Within Level				
σ_t^2	Residual Variances				
	DEP	17.720	0.280	63.314	0.000
	Between Level				
γ_{01}	DEP ON				
	TDRINK	-0.050	0.048	-1.049	0.294
τ_{10}	DEP WITH				
	S1	0.189	0.064	2.936	0.003
γ_{10}	Means				
	S1	0.022	0.010	2.240	0.025
γ_{00}	Intercepts				
	DEP	5.386	0.285	18.870	0.000
τ_{11}	Variances				
	S1	0.007	0.003	2.626	0.009
τ_{00}	Residual Variances				
	DEP	32.625	2.361	13.820	0.000

Example: Depression regressed on Drinking (26 time points)
Level-2 Contextual Variable: Method 2 (Ludke et al. 2008)

Psychological Methods
2008, Vol. 13, No. 3, 203–229

1082-989X/08/$12.00 DOI: 10.1037/a0

The Multilevel Latent Covariate Model: A New, More Reliable Approach to Group-Level Effects in Contextual Studies

Oliver Lüdtke
Max Planck Institute for Human Development

Herbert W. Marsh
Oxford University

Alexander Robitzsch
Institute for Educational Progress

Ulrich Trautwein
Max Planck Institute for Human Development

Tihomir Asparouhov
Muthén & Muthén

Bengt Muthén
University of California, Los Angeles

Example: Depression regressed on Drinking (26 time points)
Level-2 Contextual Variable: Method 2 (Ludke et al. 2008)

```
USEVARIABLES ARE dep drink person;
CLUSTER IS person;
DEFINE:
center drink(grandmean);
ANALYSIS:
TYPE IS TWOLEVEL RANDOM;
ESTIMATOR IS ML;
model:
%WITHIN%
s1|dep on drink;
%BETWEEN%
dep on drink;
dep with s1;
output: sampstat;
```

```
MODEL FIT INFORMATION

Number of Free Parameters                        7

Loglikelihood

          H0 Value                       -55262.266
```

ESTIMATED SAMPLE STATISTICS FOR WITHIN

Covariances

	DEP	DRINK
DEP	17.950	
DRINK	0.582	42.387

Correlations

	DEP	DRINK
DEP	1.000	
DRINK	0.021	1.000

ESTIMATED SAMPLE STATISTICS FOR BETWEEN

Covariances

	DEP	DRINK
DEP	32.343	
DRINK	-1.247	33.113

Correlations

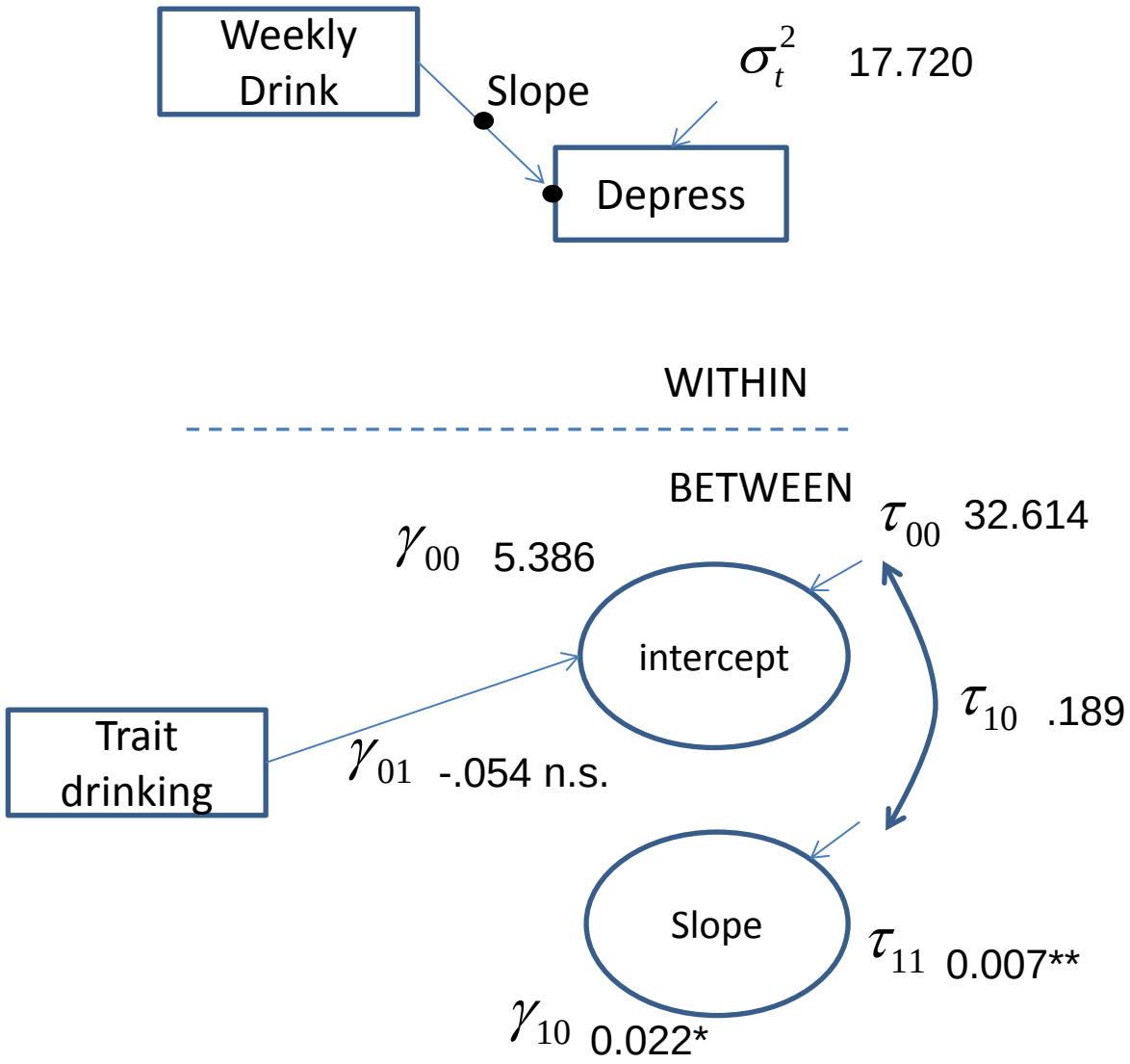
	DEP	DRINK
DEP	1.000	
DRINK	-0.038	1.000

Example: Depression regressed on Drinking (26 time points)
Level-2 Contextual Variable: Method 2 (Ludke et al. 2008)

MODEL RESULTS

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
	Within Level				
σ_t^2	Residual Variances				
	DEP	17.720	0.280	63.314	0.000
	Between Level				
γ_{01}	DEP ON				
	DRINK	-0.054	0.051	-1.054	0.292
τ_{10}	DEP WITH				
	S1	0.189	0.064	2.931	0.003
γ_{10}	Means				
	S1	0.022	0.010	2.238	0.025
γ_{00}	Intercepts				
	DEP	5.386	0.285	18.870	0.000
τ_{11}	Variances				
	S1	0.007	0.003	2.626	0.009
τ_{00}	Residual Variances				
	DEP	32.614	2.361	13.815	0.000

Example: Depression regressed on Drinking (26 time points)
Level-2 Contextual Variable: Method 2 (Ludke et al. 2008)



Standardizing
(Example p. 22 Table 2.4 Hox 2010)

$$coeff_{std} = \frac{coeff_{unstd} \times sd_{predictor}}{sd_{outcome}}$$

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Within Level				
Residual Variances				
DEP	17.720	0.280	63.314	0.000
Between Level				
DEP ON				
DRINK	-0.054	0.051	-1.054	0.292
DEP WITH				
S1	0.189	0.064	2.931	0.003
Means				
S1	0.022	0.010	2.238	0.025
Intercepts				
DEP	5.386	0.285	18.870	0.000
Variances				
S1	0.007	0.003	2.626	0.009
Residual Variances				
DEP	32.614	2.361	13.815	0.000

$$\frac{-.054 \times 5.754}{5.687} = -.055$$

$$\frac{0.022 \times 6.511}{4.237} = .034$$

Gender Differences

```
USEVARIABLES ARE dep drink person;
CLUSTER IS person;
grouping is gender (1=male 2=female)
DEFINE:
center drink(grandmean);
ANALYSIS:
TYPE IS TWOLEVEL RANDOM;
ESTIMATOR IS ML;
model:
%WITHIN%
s1|dep on drink;
%BETWEEN%
dep on drink;
dep with s1;
output: sampstat;
```

MODEL FIT INFORMATION

Number of Free Parameters 14

Loglikelihood

H0 Value -54767.256

ESTIMATED SAMPLE STATISTICS FOR MALE WITHIN

Covariances

	DEP	DRINK
DEP	15.479	
DRINK	0.505	69.103

Correlations

	DEP	DRINK
DEP	1.000	
DRINK	0.015	1.000

ESTIMATED SAMPLE STATISTICS FOR FEMALE WITHIN

Covariances

	DEP	DRINK
DEP	19.251	
DRINK	0.619	28.395

Correlations

	DEP	DRINK
DEP	1.000	
DRINK	0.026	1.000

ESTIMATED SAMPLE STATISTICS FOR MALE BETWEEN

Covariances

	DEP	DRINK
DEP	14.105	
DRINK	2.075	52.992

Correlations

	DEP	DRINK
DEP	1.000	
DRINK	0.076	1.000

ESTIMATED SAMPLE STATISTICS FOR FEMALE BETWEEN

Covariances

	DEP	DRINK
DEP	40.679	
DRINK	-0.508	18.232

Correlations

	DEP	DRINK
DEP	1.000	
DRINK	-0.019	1.000

Gender Differences

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Group MALE				
Within Level				
Residual Variances				
DEP	15.174	0.410	36.974	0.000
Between Level				
DEP ON				
DRINK	0.024	0.047	0.503	0.615
DEP WITH				
S1	-0.013	0.051	-0.254	0.800
Means				
S1	0.007	0.013	0.541	0.588
Intercepts				
DEP	3.961	0.318	12.457	0.000
Variances				
S1	0.006	0.003	2.058	0.040
Residual Variances				
DEP	13.800	1.767	7.811	0.000
Group FEMALE				
Within Level				
Residual Variances				
DEP	19.010	0.370	51.351	0.000
Between Level				
DEP ON				
DRINK	-0.022	0.096	-0.231	0.818
DEP WITH				
S1	0.354	0.104	3.401	0.001
Means				
S1	0.035	0.015	2.289	0.022
Intercepts				
DEP	6.146	0.397	15.473	0.000
Variances				
S1	0.010	0.005	2.121	0.034
Residual Variances				
DEP	40.902	3.661	11.171	0.000

Measures within Persons: How to code Time in Mplus (MLM)

```
Title: PSY9555 Regression examples;
!note that two outliers of 0 were removed in average;
data: File is sem_mplus2.dat;
    Format is 1F4, 1F1, 1F2, 23F8.2, 1F11.3, 72F8.2;
data widetolong:
wide = drink1 drink2 drink3 drink4;
long = drink;
idvariable = person;
repetition = time;
variable: names are studid gender age
bppa bpv bpa bph bptot
sq1 sq2 sq3 sq4 sq5 sq6 sq7 sq8 sq9 sq10 sq11 sq12 sq13 sq14 sq15
es es_pt es_fin grade
drink1 drink2 drink3 drink4 epis1 epis2 epis3 epis4
stress1 stress2 stress3 stress4 pleased1 pleased2 pleased3 pleased4
enjoyc1 enjoyc2 enjoyc3 enjoyc4 enjoyu1 enjoyu2 enjoyu3 enjoyu4
effort1 effort2 effort3 effort4 harm1 harm2 harm3 harm4 dep1 dep2 dep3 dep4
drink1b drink2b drink3b drink4b epis1b epis2b epis3b epis4b
stress1b stress2b stress3b stress4b please1b please2b please3b please4b
enjoyc1b enjoyc2b enjoyc3b enjoyc4b enjoyu1b enjoyu2b enjoyu3b enjoyu4b
effort1b effort2b effort3b effort4b harm1b harm2b harm3b harm4b
dep1b dep2b dep3b dep4b;
missing = blank;
usevariables are drink person time;
cluster = person;
within = time;
analysis:
type = twolevel random;
model:
%within%
s | drink on time;
%between%
s with drink;
output: sampstat tech1;
```

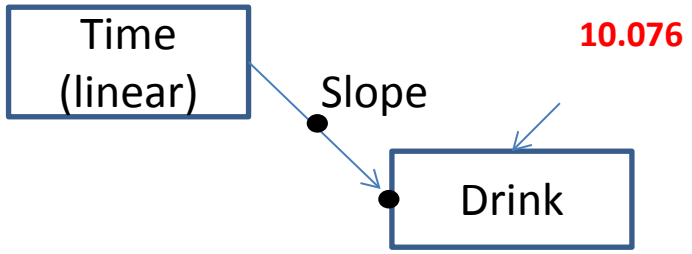
LGM as a MLM

```
Loglikelihood

          H0 Value                          -4756.126
          H0 Scaling Correction Factor          3.5791
            for MLR

Information Criteria

          Akaike (AIC)                       9524.253
          Bayesian (BIC)                     9556.568
          Sample-Size Adjusted BIC           9537.507
            (n* = (n + 2) / 24)
```



WITHIN

BETWEEN

```
MODEL RESULTS

                                                    Two-Tailed
                  Estimate       S.E.  Est./S.E.    P-Value

Within Level

 Residual Variances
    DRINK            10.076      1.428      7.056      0.000

Between Level

 S        WITH
    DRINK            -0.139      1.168     -0.119      0.905

 Means
    DRINK             5.236      0.306     17.090      0.000
    S                -0.118      0.085     -1.396      0.163

 Variances
    DRINK            31.765      4.088      7.769      0.000
    S                 0.782      0.478      1.636      0.102
```

Mean = 5.236
Var = 31.765

Drink
(intercept)

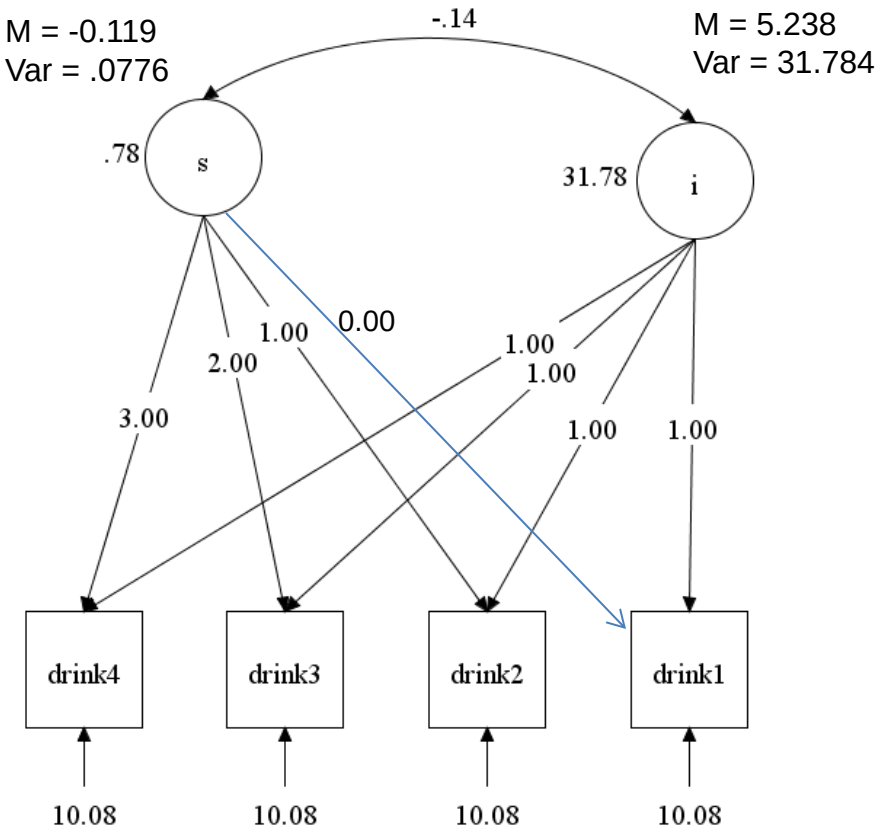
-0.139

Mean = -.118
Var = .782

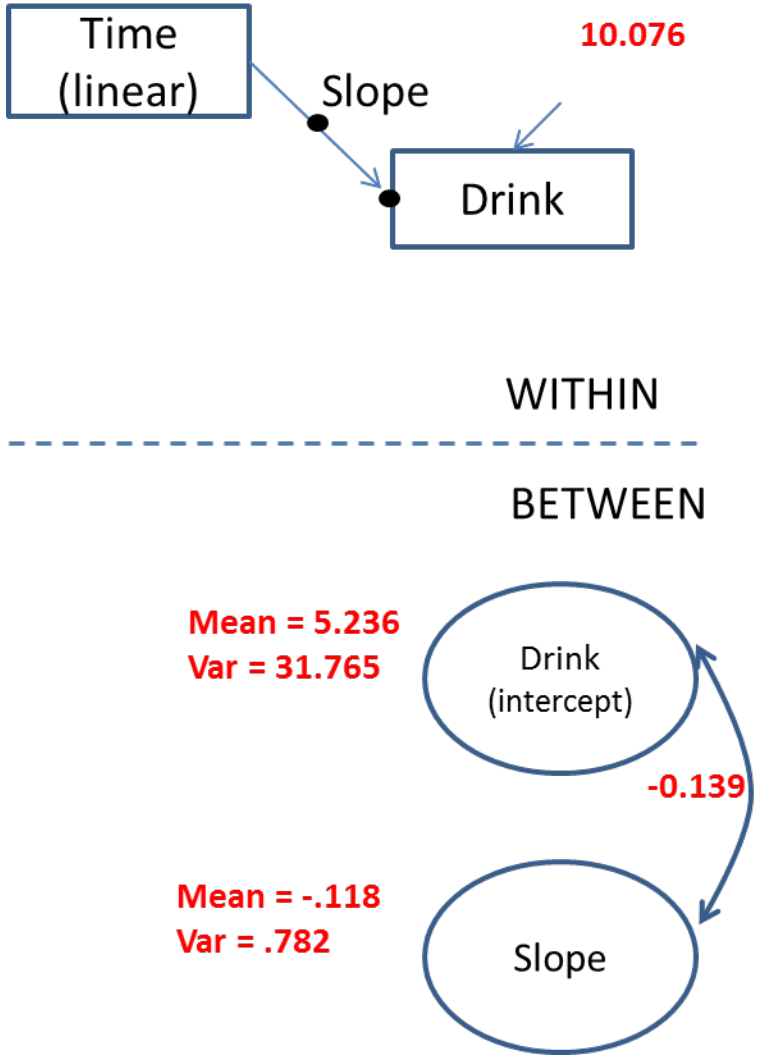
Slope

LGM as a MLM

LGM

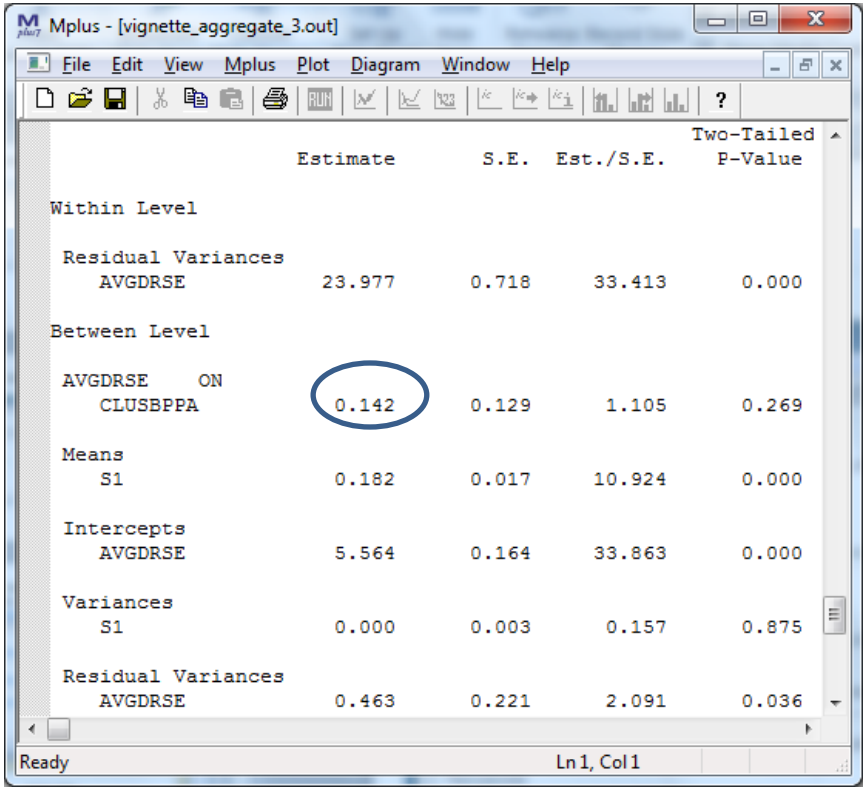


MLM

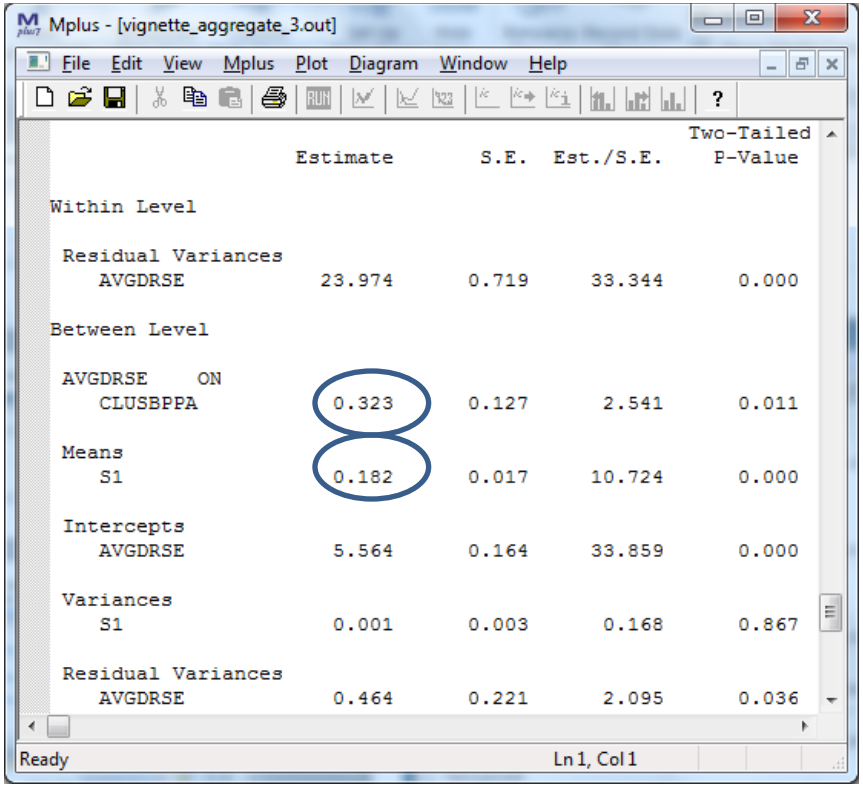


Contextual Variable : Impact of Different Types of Centering

L2 variable grand-mean c.
L1 variable grand-mean c.



L2 variable grand-mean c.
L1 variable group-mean c.



see interpretation in Enders and Tofighi (2007) p. 131