# PSY9556b (April 2) Dichotomous (Binary), Categorical-Ordinal, and Count Outcomes

**Continuous**
- continuous ordinal
- interval
- ratio

**Discrete**
- dichotomous, binary
- nominal (can have more than two unordered categories)
- ordinal

In Mplus after "variable: names are…" include
**categorical are** …..
- For dichotomous variables
  - 1 and 2 will be automatically recoded as 0 and 1
- For ordinal variables
  - no more than 10 categories
  - integer values only
  - categories automatically recoded as 0, 1, 2, …

**nominal are** …..
- categories automatically recoded as 0, 1, 2, …
- last category is reference

## Dichotomous (Binary), Categorical-Ordinal, and Count Outcomes

**Count Variables**

- e.g., number of accidents at a particular highway interchange in a 24 hour period (without knowing total number of cars that went through
- Poisson distribution
  - To model count data that varies randomly over time
  - Often used when probability is small
  - Discrete values (positive integers)

In Mplus after "variable: names are…" include
**count =** var or
**count =** var **(p)** for Poisson
**count =** var **(i)** or
**count =** var **(pi)** for zero-inflated poisson (ZIP)
**count =** var **(nb)** for negative binomial model (dispersion parameter is calculated; variance exceeds the mean)
See other models in Mplus manual

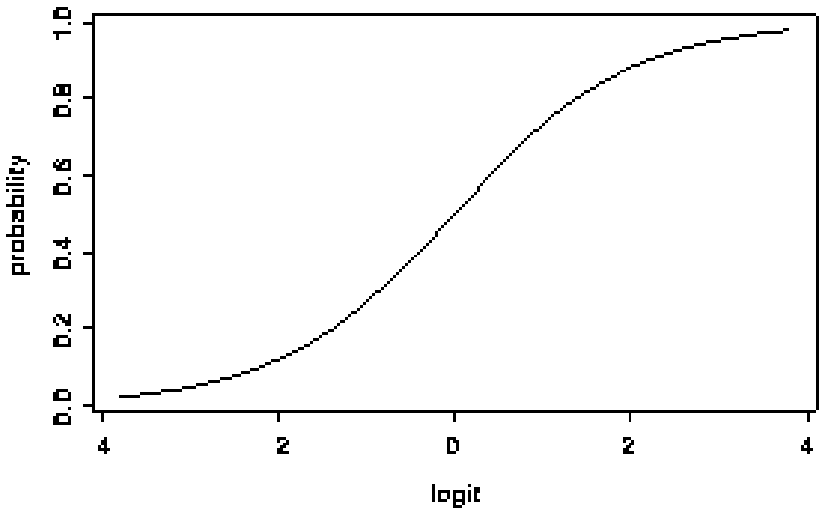
## Binary Outcome

- 1 vs. 0, success vs. failure
- Expected mean = proportion of cases who have 1
  - $mean\ y = p_y$
  - $\pi_y$in population
  - $variance\ y = p_y(1 - p_y)$

$$\pi_i = x_1\beta$$

$$odds_i = \frac{\pi_i}{1-\pi_i}$$

$$logit(\pi_i) = \log\frac{\pi_i}{1-\pi_i}$$



## Generalized Linear Models

- General linear model is a special case of generalized linear models
- These models are available in SPSS
- Components include:
  - Random component referring to distribution of outcome variable
  - Systematic component refers to predictor variables (Xs)
  - Link function: the way in which the outcome is transformed so that a linear relationship can be assessed

- Generalized linear model for logistic regression
  - Random component: Outcome is binary, probability $\pi_i$
  - Systematic component:

    $$\beta_0 + \beta_1 X_1 + B_2 X_2 + \cdots + B_p X_p$$

  - Link function:

    $ln\left(\frac{\pi}{1-\pi}\right)$ = *logit of* $\pi$

  - Full equation

    $y = logistic\left(\beta_0 + \beta_1 X_1 + B_2 X_2 + \cdots + B_p X_p\right)$

    (y expressed in logits)

## Model for Polytomous Items

$$P_{ik}^{*}\left(\theta\right)=\frac{e^{1.7a_i\left(\theta-b_{ik}\right)}}{1+e^{1.7a_i\left(\theta-b_{ik}\right)}}$$

For an item *i*, $P_{ik}^{*}\left(\theta\right)$ is the probability of a response in or above a particular category *k* (except in the first category) given the value of $\theta$ and the parameters $a_i$ (slope or discrimination) and $b_{ik}$ (thresholds). The * refers to probability of a response or *higher*.

## Understanding Thresholds

- Let us consider the example of subjects' responses to a 5-point Likert scale item. This is an example of a polytomous item (an item with more than two ordered categories).

- IRT (in this case the Graded-Response Model) uses item characteristics curves (ICCs) that depict the probability of each response as a function of a person's trait level.

- The parameter estimates in our example include one slope (discrimination parameter) and four thresholds.

- The number of thresholds equals the number of response categories minus one.

- An item characteristic curve (ICC) can be produced for each response category as shown below in Figure 1. In this case, each curve is a category response curve.

# Understanding Thresholds

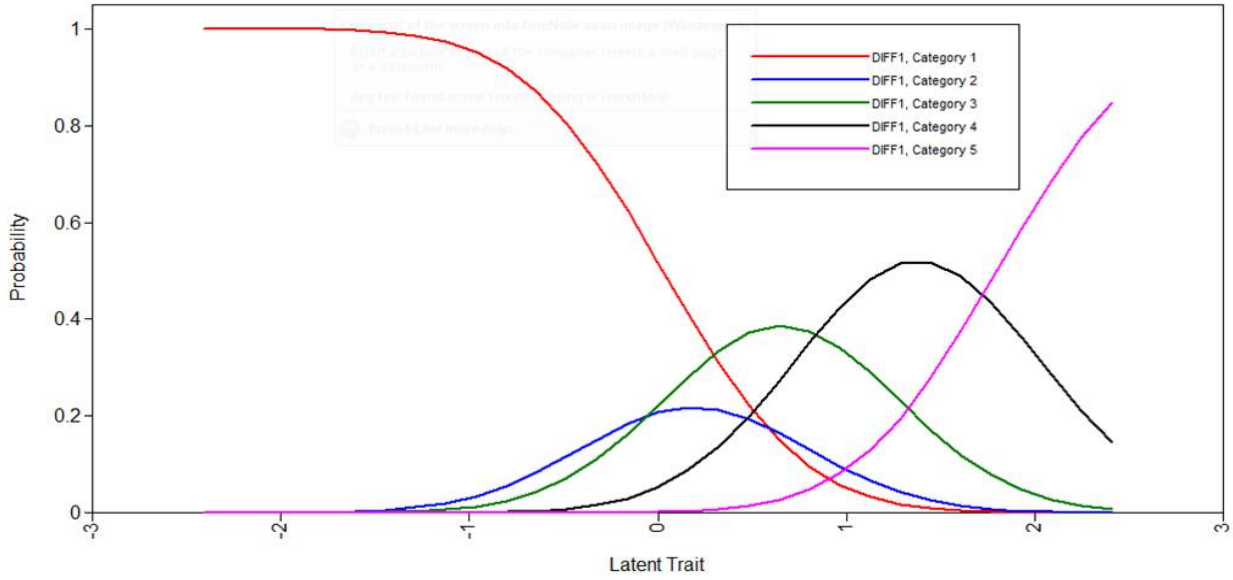



**Figure 1**. Category response curves for a five-point Likert scale item.

## Understanding Thresholds

- A threshold is the value of the trait level $\theta$ on the "threshold" (.50 probability) of crossing over into the next highest response category. More specifically the four thresholds in our example distinguish:

- Categories 1 (very unlikely) **vs**. 2 (unlikely), 3 (neutral), 4 (likely), 5 (very likely)
- Categories 1 (very unlikely), 2 (unlikely) **vs**. 3 (neutral), 4 (likely), 5 (very likely)
- Categories 1 (very unlikely), 2 (unlikely), 3 (neutral) **vs**. 4 (likely), 5 (very likely)
- Categories 1 (very unlikely), 2 (unlikely), 3 (neutral), 4 (likely) **vs**. 5 very likely)

- Thus the first threshold which in our example has a value of 0.028 is the trait level at which there is a .50 probability of endorsing "unlikely" or higher. This is shown in Figure 2.

# Understanding Thresholds

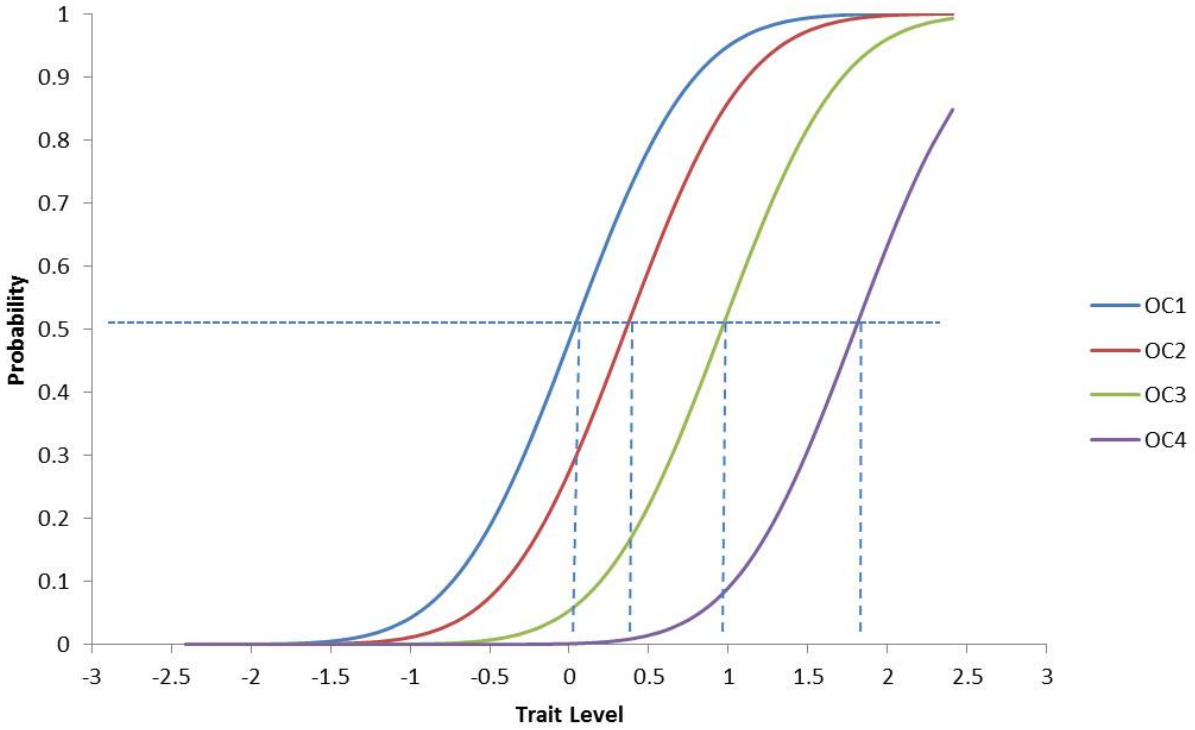



**Figure 2**. Operating characteristic curves for a five-point Likert scale item. Note that the threshold values (0.028, 0.357, 0.958, and 1.799) correspond to the intersection points between the vertical dotted lines and the X-axis referring to the latent trait value.

## Understanding Thresholds

- Note that IRT methodologists have differed in the way they label item characteristic curves (or category characteristic curves for polytomous items) and operating characteristic curves. I have used the terminology by Embreston and Reise (2000); others have used opposite labels to define these two types of curves (e.g., DeMars, 2010).

DeMars, C. (2010). *Item response theory*. New York: Oxford University Press.

Embreston, S. E., & Reise, S. P. (2000). *Item response theory for psychologists*. Mahwah, NJ: Erlbaum.

## Thresholds in Mplus

- Mplus reports thresholds (instead of means) for outcome variables specified as CATEGORICAL
- CATEGORICAL in Mplus specifies that the outcome variables are ordered-categorical or dichotomous
- This approach is ideal when your outcome variables are test or questionnaire items such as Likert-scale responses or any other types of responses with 10 or less ordered categories

# Example of Longitudinal Invariance Analysis: BDI-II

```
UNIVARIATE PROPORTIONS AND COUNTS FOR CATEGORICAL VARIABLES

    W1BDI1
      Category 1    0.702      271.000
      Category 2    0.267      103.000
      Category 3    0.021        8.000
      Category 4    0.010        4.000
    W1BDI2
      Category 1    0.578      222.000
      Category 2    0.362      139.000
      Category 3    0.052       20.000
      Category 4    0.008        3.000
    W1BDI3
      Category 1    0.657      253.000
      Category 2    0.234       90.000
      Category 3    0.088       34.000
      Category 4    0.021        8.000
    W1BDI4
      Category 1    0.699      270.000
      Category 2    0.249       96.000
      Category 3    0.044       17.000
      Category 4    0.008        3.000
    W1BDI5
      Category 1    0.620      238.000
      Category 2    0.320      123.000
      Category 3    0.047       18.000
      Category 4    0.013        5.000
    W1BDI6
      Category 1    0.858      332.000
      Category 2    0.114       44.000
      Category 3    0.021        8.000
      Category 4    0.008        3.000
    W1BDI7
      Category 1    0.627      242.000
      Category 2    0.251       97.000
      Category 3    0.073       28.000
      Category 4    0.049       19.000
    W1BDI8
      Category 1    0.584      226.000
      Category 2    0.318      123.000
      Category 3    0.070       27.000
      Category 4    0.028       11.000
```

4.

| | |
|---|---|
| 0 | I get as much satisfaction out of things as I used to. |
| 1 | I don't enjoy things the way I used to. |
| 2 | I don't get real satisfaction out of anything anymore. |
| 3 | I am dissatisfied or bored with everything. |

5.

| | |
|---|---|
| 0 | I don't feel particularly guilty |
| 1 | I feel guilty a good part of the time. |
| 2 | I feel quite guilty most of the time. |
| 3 | I feel guilty all of the time. |

6.

| | |
|---|---|
| 0 | I don't feel I am being punished. |
| 1 | I feel I may be punished. |
| 2 | I expect to be punished. |
| 3 | I feel I am being punished. |

# BDI-II: Items Specified as Continuous (ML) vs. Categorical (WLSMV)

## ML

```
MODEL FIT INFORMATION

Number of Free Parameters                       63

Loglikelihood

          H0 Value                       -6704.908
          H1 Value                       -6382.964

Information Criteria

          Akaike (AIC)                   13535.817
          Bayesian (BIC)                 13785.198
          Sample-Size Adjusted BIC       13585.305
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                            643.889
          Degrees of Freedom                   189
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.079
          90 Percent C.I.                    0.072  0.086
          Probability RMSEA <= .05           0.000

CFI/TLI

          CFI                                0.841
          TLI                                0.823
```

## WLSMV

```
MODEL FIT INFORMATION

Number of Free Parameters                       84

Chi-Square Test of Model Fit

          Value                            530.654*
          Degrees of Freedom                   189
          P-Value                           0.0000

*   The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM an
    for chi-square difference testing in the regular way.
    chi-square difference testing is described on the Mplus
    and ULSMV difference testing is done using the DIFFTEST

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.068
          90 Percent C.I.                    0.062  0.075
          Probability RMSEA <= .05           0.000

CFI/TLI

          CFI                                0.934
          TLI                                0.926

Chi-Square Test of Model Fit for the Baseline Model

          Value                           5359.205
          Degrees of Freedom                   210
          P-Value                           0.0000

WRMR (Weighted Root Mean Square Residual)

          Value                              1.269
```

# BDI-II: Items Specified as Continuous (ML) vs. Categorical (WLSMV)

## ML

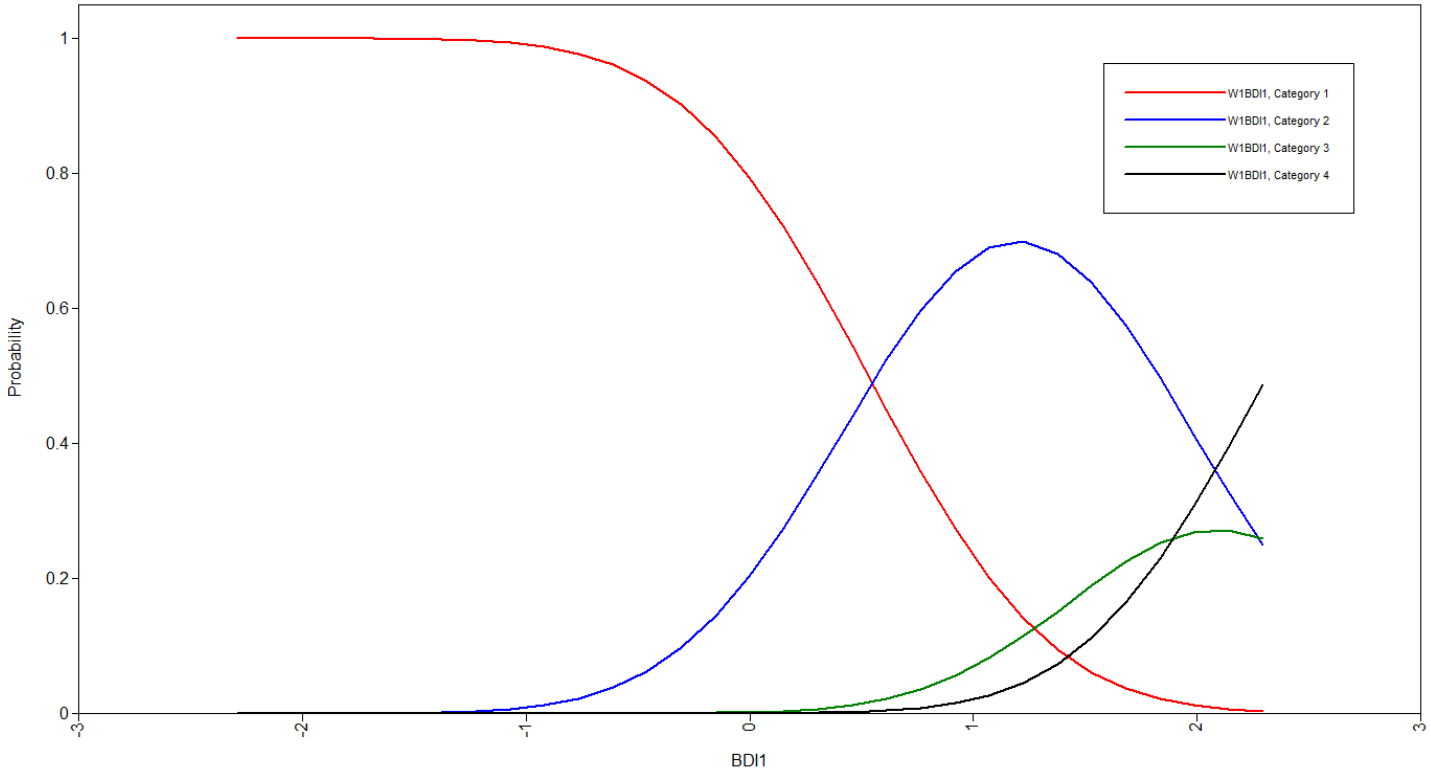
STDYX Standardization

| | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|---|---|---|---|---|
| BDI1 BY | | | | |
| W1BDI1 | 0.653 | 0.032 | 20.565 | 0.000 |
| W1BDI2 | 0.573 | 0.037 | 15.687 | 0.000 |
| W1BDI3 | 0.697 | 0.029 | 24.257 | 0.000 |
| W1BDI4 | 0.576 | 0.036 | 15.833 | 0.000 |
| W1BDI5 | 0.598 | 0.035 | 17.083 | 0.000 |
| W1BDI6 | 0.479 | 0.042 | 11.531 | 0.000 |
| W1BDI7 | 0.732 | 0.026 | 27.919 | 0.000 |
| W1BDI8 | 0.616 | 0.034 | 18.175 | 0.000 |
| W1BDI9 | 0.453 | 0.043 | 10.613 | 0.000 |
| W1BDI10 | 0.434 | 0.044 | 9.936 | 0.000 |
| W1BDI11 | 0.406 | 0.045 | 9.037 | 0.000 |
| W1BDI12 | 0.533 | 0.039 | 13.765 | 0.000 |
| W1BDI13 | 0.597 | 0.035 | 17.000 | 0.000 |
| W1BDI14 | 0.754 | 0.025 | 30.636 | 0.000 |
| W1BDI15 | 0.666 | 0.031 | 21.497 | 0.000 |
| W1BD16R | 0.414 | 0.044 | 9.298 | 0.000 |
| W1BDI17 | 0.602 | 0.035 | 17.321 | 0.000 |
| W1BD18R | 0.520 | 0.039 | 13.187 | 0.000 |
| W1BDI19 | 0.608 | 0.034 | 17.663 | 0.000 |
| W1BDI20 | 0.558 | 0.038 | 14.818 | 0.000 |
| W1BDI21 | 0.458 | 0.043 | 10.747 | 0.000 |

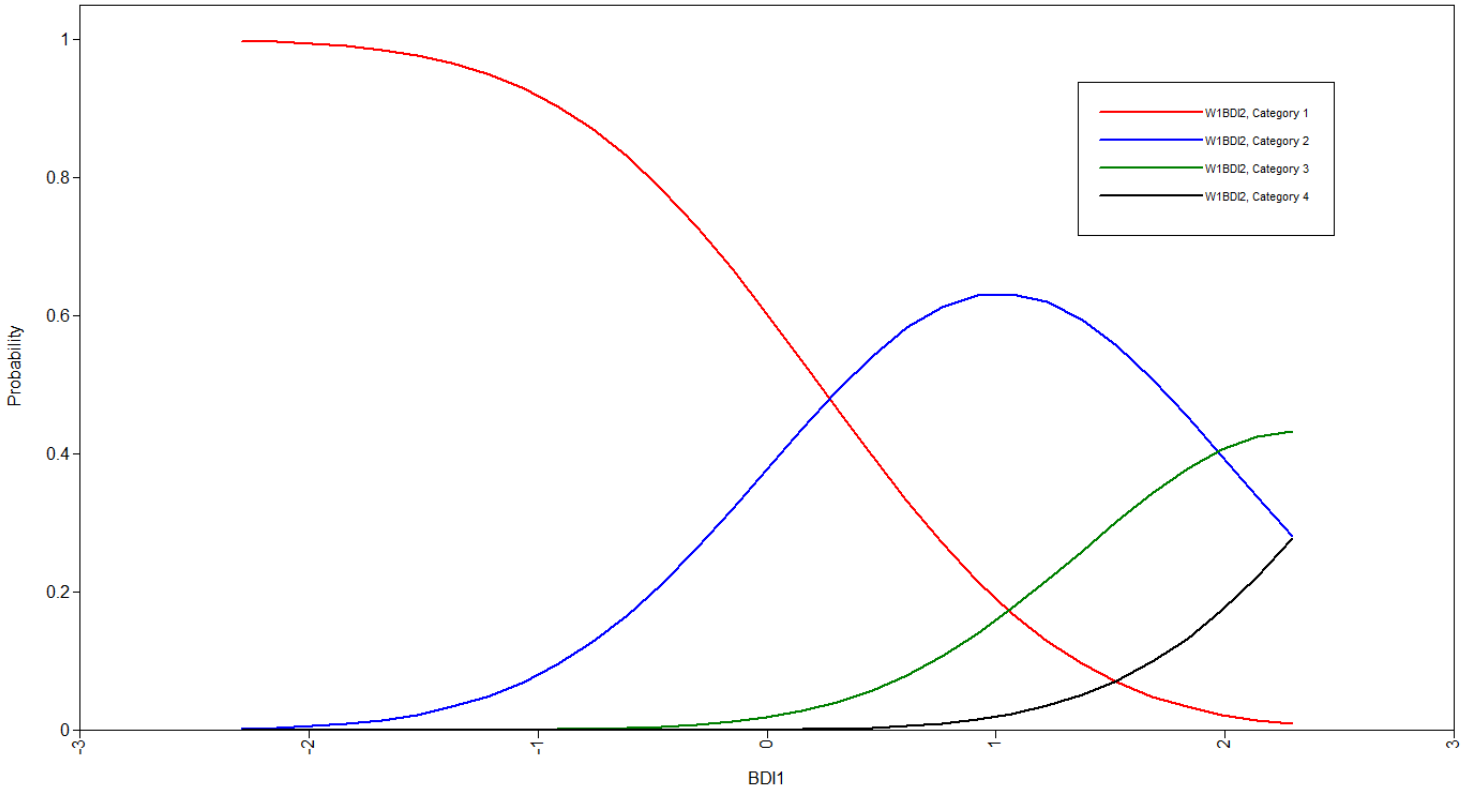
## WLSMV

| | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|---|---|---|---|---|
| BDI1 BY | | | | |
| W1BDI1 | 0.764 | 0.033 | 23.018 | 0.000 |
| W1BDI2 | 0.657 | 0.037 | 17.866 | 0.000 |
| W1BDI3 | 0.765 | 0.030 | 25.400 | 0.000 |
| W1BDI4 | 0.658 | 0.037 | 17.745 | 0.000 |
| W1BDI5 | 0.662 | 0.040 | 16.413 | 0.000 |
| W1BDI6 | 0.643 | 0.055 | 11.693 | 0.000 |
| W1BDI7 | 0.796 | 0.028 | 28.314 | 0.000 |
| W1BDI8 | 0.667 | 0.037 | 18.170 | 0.000 |
| W1BDI9 | 0.647 | 0.057 | 11.381 | 0.000 |
| W1BDI10 | 0.558 | 0.052 | 10.813 | 0.000 |
| W1BDI11 | 0.499 | 0.048 | 10.440 | 0.000 |
| W1BDI12 | 0.644 | 0.042 | 15.479 | 0.000 |
| W1BDI13 | 0.689 | 0.040 | 17.280 | 0.000 |
| W1BDI14 | 0.922 | 0.027 | 33.553 | 0.000 |
| W1BDI15 | 0.766 | 0.026 | 29.624 | 0.000 |
| W1BD16R | 0.499 | 0.046 | 10.949 | 0.000 |
| W1BDI17 | 0.710 | 0.042 | 16.919 | 0.000 |
| W1BD18R | 0.587 | 0.043 | 13.718 | 0.000 |
| W1BDI19 | 0.687 | 0.035 | 19.366 | 0.000 |
| W1BDI20 | 0.660 | 0.034 | 19.670 | 0.000 |
| W1BDI21 | 0.598 | 0.053 | 11.384 | 0.000 |

# BDI-II: Items Specified as Continuous (ML) vs. Categorical (WLSMV)



# BDI-II: Items Specified as Continuous (ML) vs. Categorical (WLSMV)



# BDI-II: A Third Approach - Categorical (WLSMV) Items Dichotomized*

MODEL FIT INFORMATION

Number of Free Parameters 42

Chi-Square Test of Model Fit

| | |
|---|---|
| Value | 410.622* |
| Degrees of Freedom | 189 |
| P-Value | 0.0000 |

* The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM a
for chi-square difference testing in the regular way.
chi-square difference testing is described on the Mplu
and ULSMV difference testing is done using the DIFFTES

RMSEA (Root Mean Square Error Of Approximation)

| | | |
|---|---|---|
| Estimate | 0.055 | |
| 90 Percent C.I. | 0.048 | 0.062 |
| Probability RMSEA <= .05 | 0.124 | |

CFI/TLI

| | |
|---|---|
| CFI | 0.930 |
| TLI | 0.922 |

Chi-Square Test of Model Fit for the Baseline Model

| | |
|---|---|
| Value | 3375.787 |
| Degrees of Freedom | 210 |
| P-Value | 0.0000 |

WRMR (Weighted Root Mean Square Residual)

| | |
|---|---|
| Value | 1.222 |

STDYX Standardization

| | | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|---|---|---|---|---|---|
| BDI1 | BY | | | | |
| W1B1 | | 0.754 | 0.040 | 18.934 | 0.000 |
| W1B2 | | 0.658 | 0.045 | 14.731 | 0.000 |
| W1B3 | | 0.700 | 0.044 | 15.781 | 0.000 |
| W1B4 | | 0.600 | 0.051 | 11.726 | 0.000 |
| W1B5 | | 0.614 | 0.049 | 12.457 | 0.000 |
| W1B6 | | 0.652 | 0.060 | 10.924 | 0.000 |
| W1B7 | | 0.781 | 0.036 | 21.665 | 0.000 |
| W1B8 | | 0.614 | 0.047 | 13.206 | 0.000 |
| W1B9 | | 0.662 | 0.067 | 9.852 | 0.000 |
| W1B10 | | 0.598 | 0.055 | 10.776 | 0.000 |
| W1B11 | | 0.531 | 0.056 | 9.464 | 0.000 |
| W1B12 | | 0.638 | 0.048 | 13.196 | 0.000 |
| W1B13 | | 0.639 | 0.052 | 12.371 | 0.000 |
| W1B14 | | 0.897 | 0.039 | 22.763 | 0.000 |
| W1B15 | | 0.656 | 0.044 | 15.044 | 0.000 |
| W1B16 | | 0.448 | 0.067 | 6.678 | 0.000 |
| W1B17 | | 0.727 | 0.044 | 16.708 | 0.000 |
| W1B18 | | 0.546 | 0.053 | 10.222 | 0.000 |
| W1B19 | | 0.657 | 0.044 | 15.055 | 0.000 |
| W1B20 | | 0.543 | 0.052 | 10.414 | 0.000 |
| W1B21 | | 0.552 | 0.069 | 8.043 | 0.000 |

*In some of the later waves of data, respondents didn't use all responses…further explanation in class

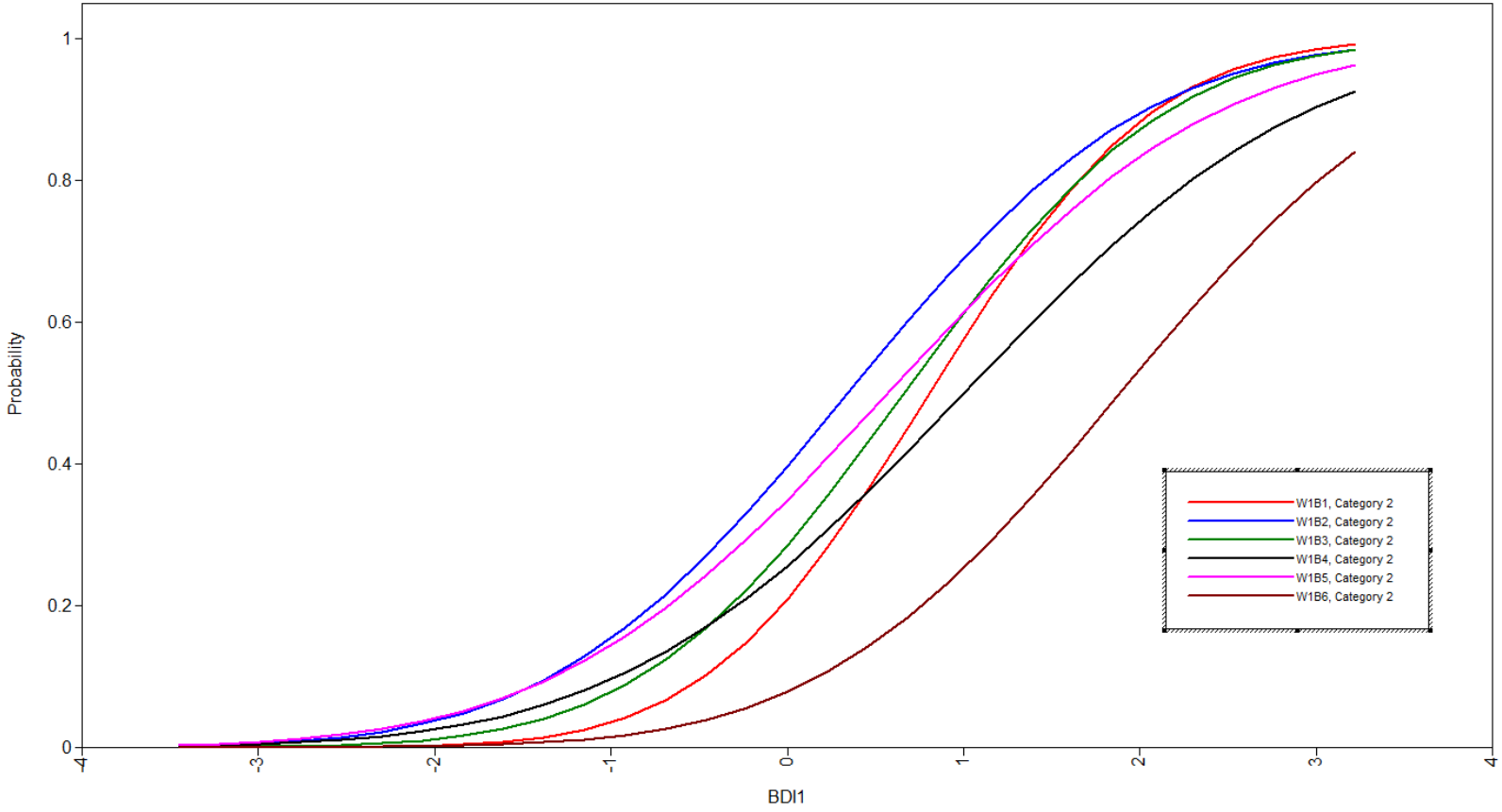
# BDI-II: A Third Approach - Categorical (WLSMV) Items Dichotomized*

IRT PARAMETERIZATION IN TWO-PARAMETER PROBIT METRIC
WHERE THE PROBIT IS DISCRIMINATION*(THETA - DIFFICULTY)

Item Discriminations

| BDI1 | BY | | | | | Item Difficulties | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | W1B1 | 1.149 | 0.141 | 8.157 | 0.000 | W1B1$1 | 0.703 | 0.098 | 7.192 | 0.000 |
| | W1B2 | 0.873 | 0.104 | 8.360 | 0.000 | W1B2$1 | 0.300 | 0.100 | 2.992 | 0.003 |
| | W1B3 | 0.980 | 0.122 | 8.049 | 0.000 | W1B3$1 | 0.578 | 0.102 | 5.643 | 0.000 |
| | W1B4 | 0.750 | 0.100 | 7.503 | 0.000 | W1B4$1 | 0.871 | 0.138 | 6.307 | 0.000 |
| | W1B5 | 0.778 | 0.100 | 7.762 | 0.000 | W1B5$1 | 0.497 | 0.113 | 4.384 | 0.000 |
| | W1B6 | 0.860 | 0.137 | 6.277 | 0.000 | W1B6$1 | 1.642 | 0.205 | 8.014 | 0.000 |
| | W1B7 | 1.251 | 0.148 | 8.449 | 0.000 | W1B7$1 | 0.415 | 0.086 | 4.832 | 0.000 |
| | W1B8 | 0.778 | 0.095 | 8.224 | 0.000 | W1B8$1 | 0.345 | 0.108 | 3.201 | 0.001 |
| | W1B9 | 0.884 | 0.160 | 5.528 | 0.000 | W1B9$1 | 1.973 | 0.256 | 7.710 | 0.000 |
| | W1B10 | 0.746 | 0.108 | 6.924 | 0.000 | W1B10$1 | 1.170 | 0.165 | 7.070 | 0.000 |
| | W1B11 | 0.627 | 0.092 | 6.795 | 0.000 | W1B11$1 | 0.123 | 0.122 | 1.012 | 0.311 |
| | W1B12 | 0.829 | 0.106 | 7.821 | 0.000 | W1B12$1 | 0.731 | 0.119 | 6.118 | 0.000 |
| | W1B13 | 0.832 | 0.114 | 7.313 | 0.000 | W1B13$1 | 0.982 | 0.137 | 7.151 | 0.000 |
| | W1B14 | 2.032 | 0.458 | 4.439 | 0.000 | W1B14$1 | 1.179 | 0.105 | 11.238 | 0.000 |
| | W1B15 | 0.870 | 0.102 | 8.560 | 0.000 | W1B15$1 | 0.035 | 0.097 | 0.356 | 0.722 |
| | W1B16 | 0.501 | 0.094 | 5.339 | 0.000 | W1B16$1 | -1.589 | 0.293 | -5.422 | 0.000 |
| | W1B17 | 1.058 | 0.134 | 7.881 | 0.000 | W1B17$1 | 0.785 | 0.107 | 7.329 | 0.000 |
| | W1B18 | 0.652 | 0.091 | 7.173 | 0.000 | W1B18$1 | -0.119 | 0.118 | -1.011 | 0.312 |
| | W1B19 | 0.871 | 0.102 | 8.557 | 0.000 | W1B19$1 | -0.020 | 0.097 | -0.204 | 0.839 |
| | W1B20 | 0.646 | 0.088 | 7.345 | 0.000 | W1B20$1 | -0.279 | 0.122 | -2.280 | 0.023 |
| | W1B21 | 0.662 | 0.118 | 5.593 | 0.000 | W1B21$1 | 1.850 | 0.281 | 6.588 | 0.000 |

# BDI-II: A Third Approach - Categorical (WLSMV) Items Dichotomized*



# BDI-II: Tests of Measurement Invariance – wk1 wk13 wk26 (see Mplus Manual 7 p. 486)

## Model 1 (Configural MI)

```
ANALYSIS:
parameterization = theta;
MODEL:
bdi1 by w1b1-w1b21;
bdi13 by w13b1-w13b21;
bdi26 by w26b1-w26b21;

!correlated residuals acrorss time
w1b1 with w13b1 w26b1;
w1b2 with w13b2 w26b2;
w1b3 with w13b3 w26b3;
w1b4 with w13b4 w26b4;
```

**Not all correlated residuals shown**

```
w13b19 with w26b19;
w13b20 with w26b20;
w13b21 with w26b21;
!set residual variances to 1;
w1b1-w1b21@1;
w13b1-w13b21@1;
w26b1-w26b21@1;
!set factor means at 0;
[bdi1@0 bdi13@0 bdi26@0];
OUTPUT: sampstat stdyx modindices;
SAVEDATA: difftest=deriv.dat;
```

## Model 2 (Loadings & Thresholds MI)

```
ANALYSIS:
parameterization = theta;
difftest = deriv.dat;
MODEL:
bdi1 by
w1b1-w1b21 (L1-L21);
bdi13 by
w13b1-w13b21 (L1-L21);
bdi26 by
w26b1-w26b21 (L1-L21);
[w1b1$1-w1b21$1] (T1-T21);
[w13b1$1-w13b21$1] (T1-T21);
[w26b1$1-w26b21$1] (T1-T21);
w1b1 with w13b1 w26b1;
w1b2 with w13b2 w26b2;
w1b3 with w13b3 w26b3;
w1b4 with w13b4 w26b4;
```

**Not all correlated residuals shown**

```
w13b19 with w26b19;
w13b20 with w26b20;
w13b21 with w26b21;
!set residual variances to 1;
w1b1-w1b21@1;
w13b1-w13b21;
w26b1-w26b21;
!set factor means at 0;
[bdi1@0];
[bdi13 bdi26];
OUTPUT: sampstat stdyx modindices;
!SAVEDATA: difftest=deriv.dat;
```

# BDI-II: Tests of Measurement Invariance – wk1 wk13 wk26 (see Mplus Manual 7 p. 486)

## Model 1 (Configural MI)

```
MODEL FIT INFORMATION

Number of Free Parameters                      192

Chi-Square Test of Model Fit

          Value                           2174.229*
          Degrees of Freedom                 1824
          P-Value                          0.0000

*   The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM a
    for chi-square difference testing in the regular way.
    chi-square difference testing is described on the Mplu
    and ULSMV difference testing is done using the DIFFTES

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                         0.022
          90 Percent C.I.                  0.018  0.025
          Probability RMSEA <= .05         1.000

CFI/TLI

          CFI                              0.956
          TLI                              0.953

Chi-Square Test of Model Fit for the Baseline Model

          Value                        10003.731
          Degrees of Freedom                1953
          P-Value                          0.0000

WRMR (Weighted Root Mean Square Residual)

          Value                            1.105
```

## Model 2 (Loadings & Thresholds MI)

```
Number of Free Parameters                      154

Chi-Square Test of Model Fit

          Value                           2234.787*
          Degrees of Freedom                 1862
          P-Value                          0.0000

Chi-Square Test for Difference Testing

          Value                             96.938
          Degrees of Freedom                    38
          P-Value                          0.0000

*   The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM a
    for chi-square difference testing in the regular way.
    chi-square difference testing is described on the Mplu
    and ULSMV difference testing is done using the DIFFTES

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                         0.022
          90 Percent C.I.                  0.018  0.026
          Probability RMSEA <= .05         1.000

CFI/TLI

          CFI                              0.954
          TLI                              0.951

Chi-Square Test of Model Fit for the Baseline Model

          Value                        10003.731
          Degrees of Freedom                1953
          P-Value                          0.0000

WRMR (Weighted Root Mean Square Residual)

          Value                            1.135
```