PSY 9556B (Feb 5) Latent Growth Modeling

- “Fixed” and “random” word confusion
- Simplest LGM – knowing how to calculate dfs
- How many time points needed?
- Power, sample size
- Nonlinear growth – quadratic
- Nonlinear growth freeing loadings
- Piecewise models
- Linear growth (different ways of scaling time)
- Associative LGM
- Higher order LGM: curve-of-factors model
- Conditional models (time-invariant, time-variant)
- Multiple groups (group covariate or multiple-groups analysis)
- Similarity between LGM and MLM
- When to use LGM, when to use MLM

“Fixed” and “Random” word clarification

- Fixed and random effects in MLM and LGM
  - Fixed effect: single values that estimate of population values
    - (e.g., a regression coefficient, a mean intercept or slope in LGM, MLM)
  - Random effect: provide information about the variation in the regression coefficient or intercept parameters across the clustering units
    - (e.g., variance of intercepts and slopes in LGM or MLM)
- Fixed and random factors in ANOVA
  - Fixed factor levels chosen apriori
  - Random factor: no particular interest in the levels; chosen at random
    - Best example of a random factor: persons
    - Repeated measures design
    - At least two observations nested within persons
    - Persons as a random factor is also referred to as the clustering unit or in MLM
- Mixed models
  - One fixed factor crossed with one random factor (e.g., split plot ANOVA)

Number of Parameters and Degrees of Freedom
Example: 2 time points (linear)

Parameters and dfs

Elements:
(v (v+3))/2 = (2*5)/2 = 5

Parameters:
2 residuals (2 time points): left-over variance not explained by latent variables
1 mean intercept: the mean start-point of individual trajectories
1 mean slope: the mean slope (e.g., growth/learning/decrease) of individual trajectories
1 variance of the intercepts: variation in individual start-points
1 variance of the slopes: variation in individual slopes
1 correlation between intercept and slope
(note, indicator intercepts fixed at 0)

Total parameters = 7
dfs = to many parameters

Number of Parameters and Degrees of Freedom
Example: 3 time points (linear)

Parameters and dfs

Elements:
(v (v+3))/2 = (3*6)/2 = 9

Parameters:
3 residuals (3 time points): left-over variance not explained by latent variables
1 mean intercept: the mean start-point of individual trajectories
1 mean slope: the mean slope (e.g., growth/learning/decrease) of individual trajectories
1 variance of the intercepts: variation in individual start-points
1 variance of the slopes: variation in individual slopes
1 correlation between intercept and slope
(note, indicator intercepts fixed at 0)

Total parameters = 8
dfs = 9 - 8 = 1

Number of Parameters and Degrees of Freedom
Example: 4 time points (linear + quadratic)

Parameters and dfs

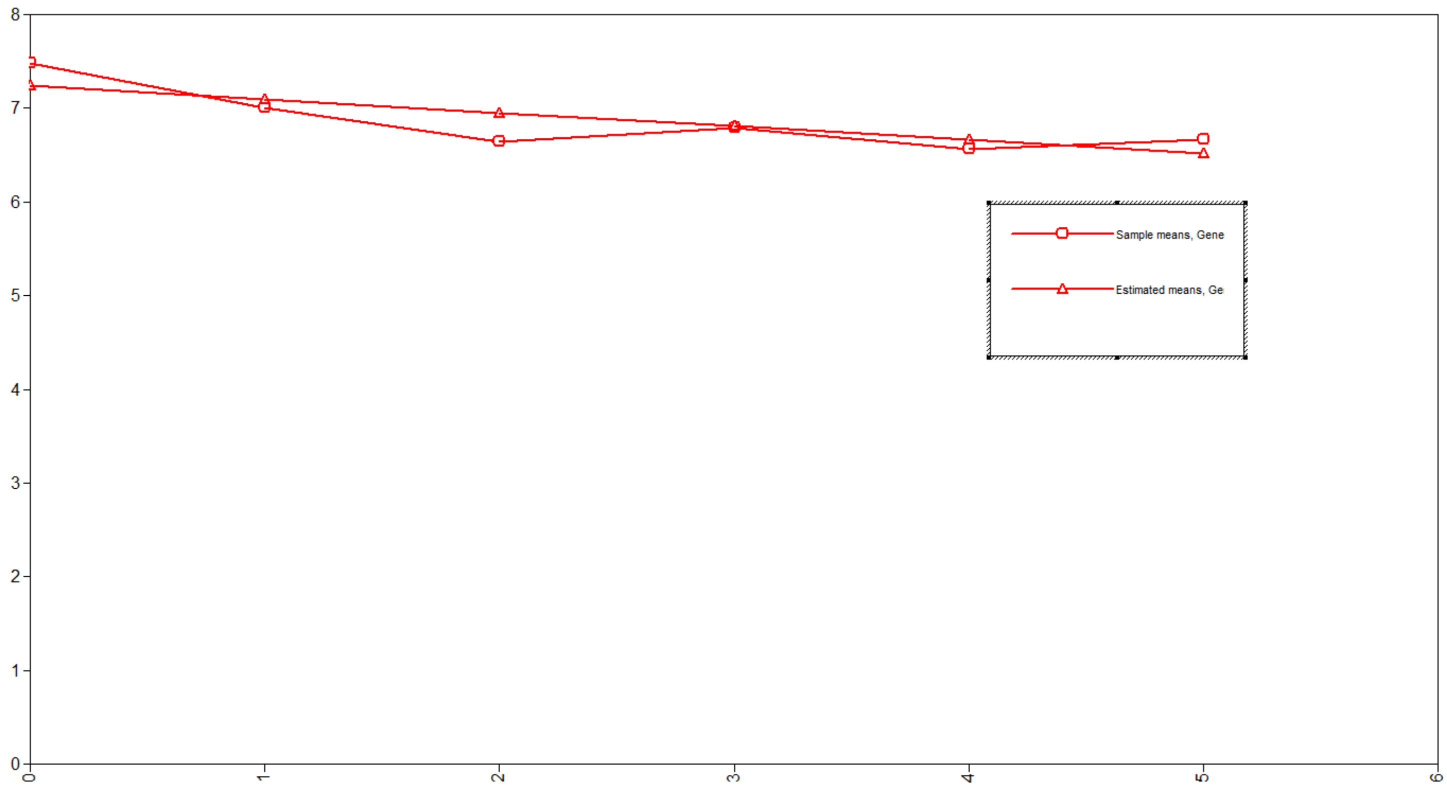
Elements:
(4 (4+3))/2 = (4*7)/2 = 14

Parameters:
4 residuals (4 time points): left-over variance not explained by latent variables
1 mean intercept: the mean start-point of individual trajectories
1 mean slope: the mean slope (e.g., growth/learning/decrease) of individual trajectories
1 mean quadratic component
1 variance of the intercepts: variation in individual start-points
1 variance of the slopes: variation in individual slopes
1 variance of quadratic component
3 correlations between intercept, slope, quadratic
(note, indicator intercepts fixed at 0)

Total parameters = 13
dfs = 14 -13 = 1

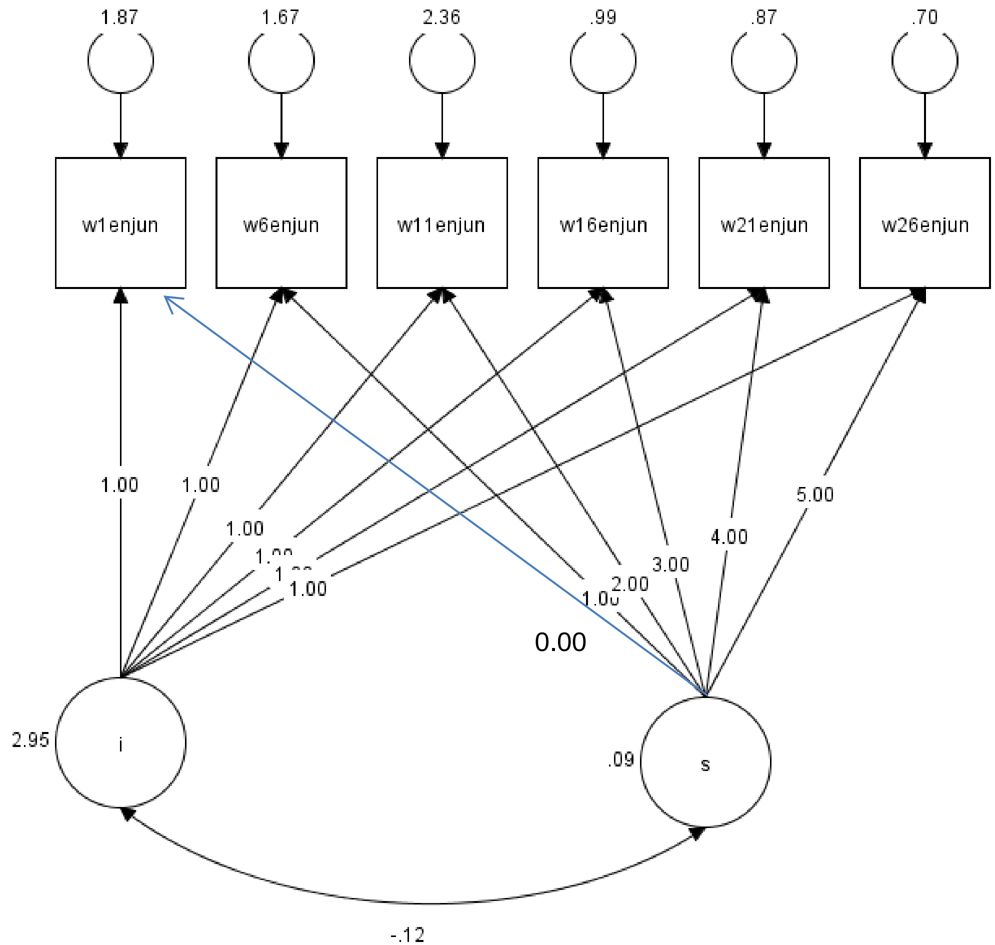
Example of a LGM with Five Time Points

```
USEVARIABLES ARE w1enjun w6enjun w11enjun w16enjun w21enjun w26enjun;
model:
I S | w1enjun@0 w6enjun@1 w11enjun@2 w16enjun@3 w21enjun@4 w26enjun@5;
plot:
type=plot3;
series = w1enjun(0) w6enjun(1) w11enjun(2) w16enjun(3) w21enjun(4) w26enjun(5);
output: sampstat residual stdyx tech4 modindices;
```



Example of a LGM with Five Time Points

In this figure you should also indicate the means of the latent variables: Intercept = 2.948 and Slope = 0.094



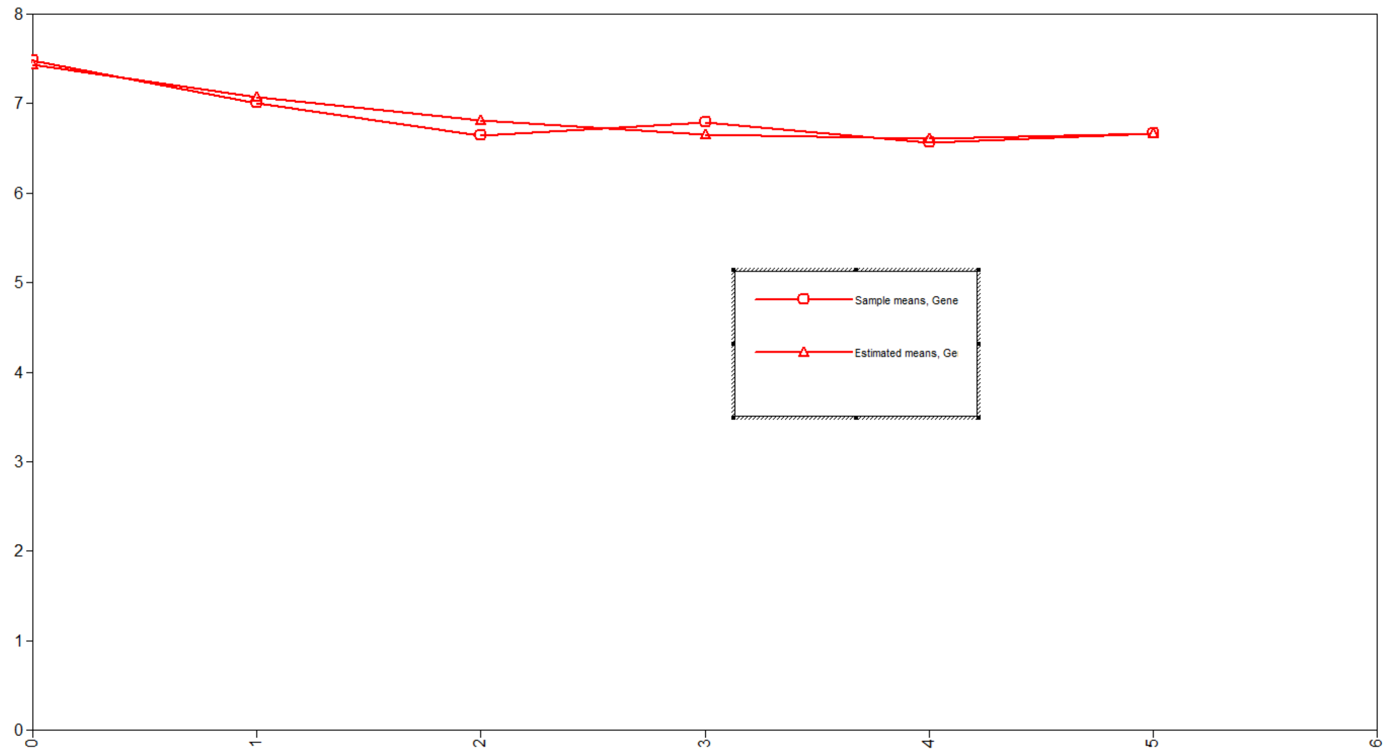
Example of a LGM with Five Time Points

```
Number of Free Parameters                        11

Loglikelihood

          H0 Value                        -3779.045
          H1 Value                        -3745.014

Information Criteria

          Akaike (AIC)                     7580.091
          Bayesian (BIC)                   7624.349
          Sample-Size Adjusted BIC         7589.443
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                              68.063
          Degrees of Freedom                     16
          P-Value                            0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.089
          90 Percent C.I.                     0.068  0.111
          Probability RMSEA <= .05            0.002

CFI/TLI

          CFI                                 0.957
          TLI                                 0.960

Chi-Square Test of Model Fit for the Baseline Model

          Value                            1239.482
          Degrees of Freedom                     15
          P-Value                            0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                               0.078
```

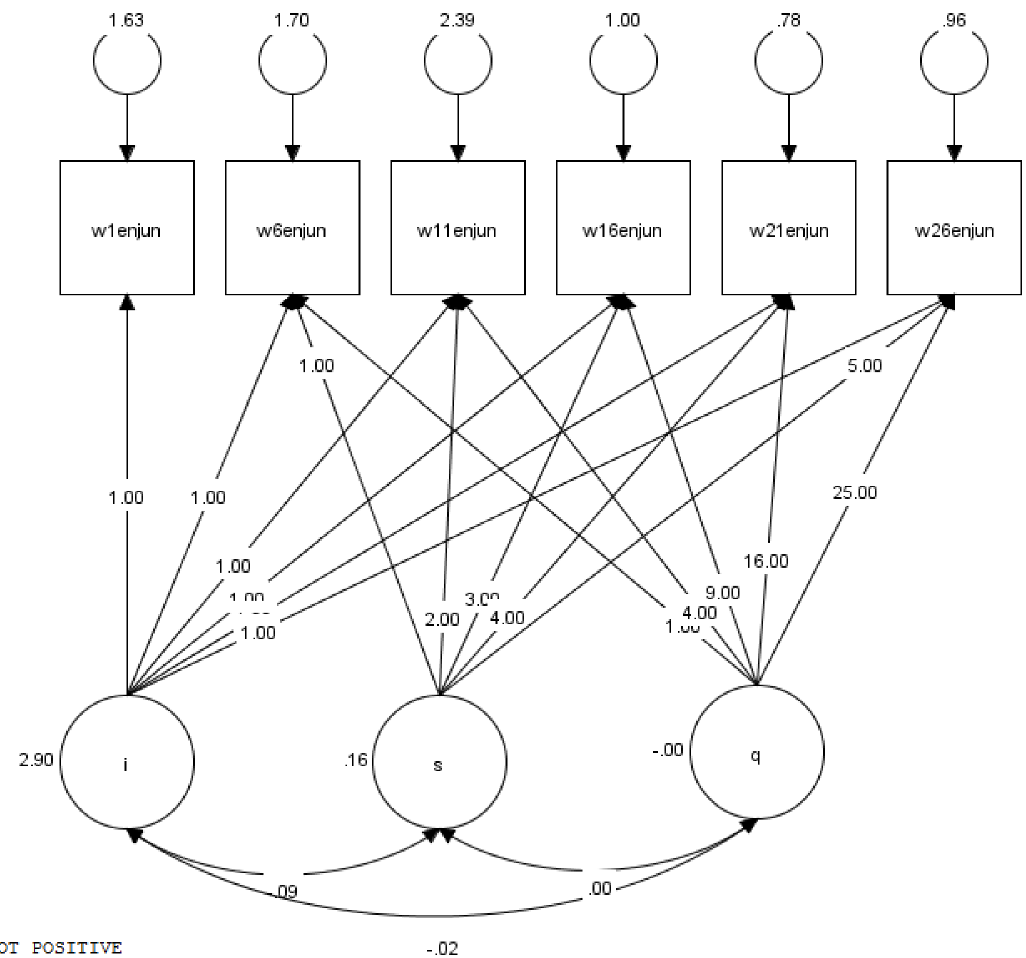
	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
I |				
W1ENJUN	1.000	0.000	999.000	999.000
W6ENJUN	1.000	0.000	999.000	999.000
W11ENJUN	1.000	0.000	999.000	999.000
W16ENJUN	1.000	0.000	999.000	999.000
W21ENJUN	1.000	0.000	999.000	999.000
W26ENJUN	1.000	0.000	999.000	999.000
S |				
W1ENJUN	0.000	0.000	999.000	999.000
W6ENJUN	1.000	0.000	999.000	999.000
W11ENJUN	2.000	0.000	999.000	999.000
W16ENJUN	3.000	0.000	999.000	999.000
W21ENJUN	4.000	0.000	999.000	999.000
W26ENJUN	5.000	0.000	999.000	999.000
S WITH				
I	-0.116	0.050	-2.300	0.021
Means				
I	7.241	0.100	72.384	0.000
S	-0.145	0.022	-6.442	0.000
Intercepts				
W1ENJUN	0.000	0.000	999.000	999.000
W6ENJUN	0.000	0.000	999.000	999.000
W11ENJUN	0.000	0.000	999.000	999.000
W16ENJUN	0.000	0.000	999.000	999.000
W21ENJUN	0.000	0.000	999.000	999.000
W26ENJUN	0.000	0.000	999.000	999.000
Variances				
I	2.948	0.289	10.192	0.000
S	0.094	0.014	6.718	0.000
Residual Variances				
W1ENJUN	1.872	0.208	9.019	0.000
W6ENJUN	1.666	0.166	10.026	0.000
W11ENJUN	2.363	0.216	10.949	0.000
W16ENJUN	0.992	0.099	10.068	0.000
W21ENJUN	0.871	0.105	8.273	0.000
W26ENJUN	0.704	0.128	5.504	0.000

Example of a LGM with Five Time Points
Adding a Quadratic Component

```
USEVARIABLES ARE w1enjun w6enjun w11enjun w16enjun w21enjun w26enjun;
model:
I S Q| w1enjun@0 w6enjun@1 w11enjun@2 w16enjun@3 w21enjun@4 w26enjun@5;
plot:
type=plot3;
series = w1enjun(0) w6enjun(1) w11enjun(2) w16enjun(3) w21enjun(4) w26enjun(5);
output: sampstat residual stdyx tech4 modindices;
```



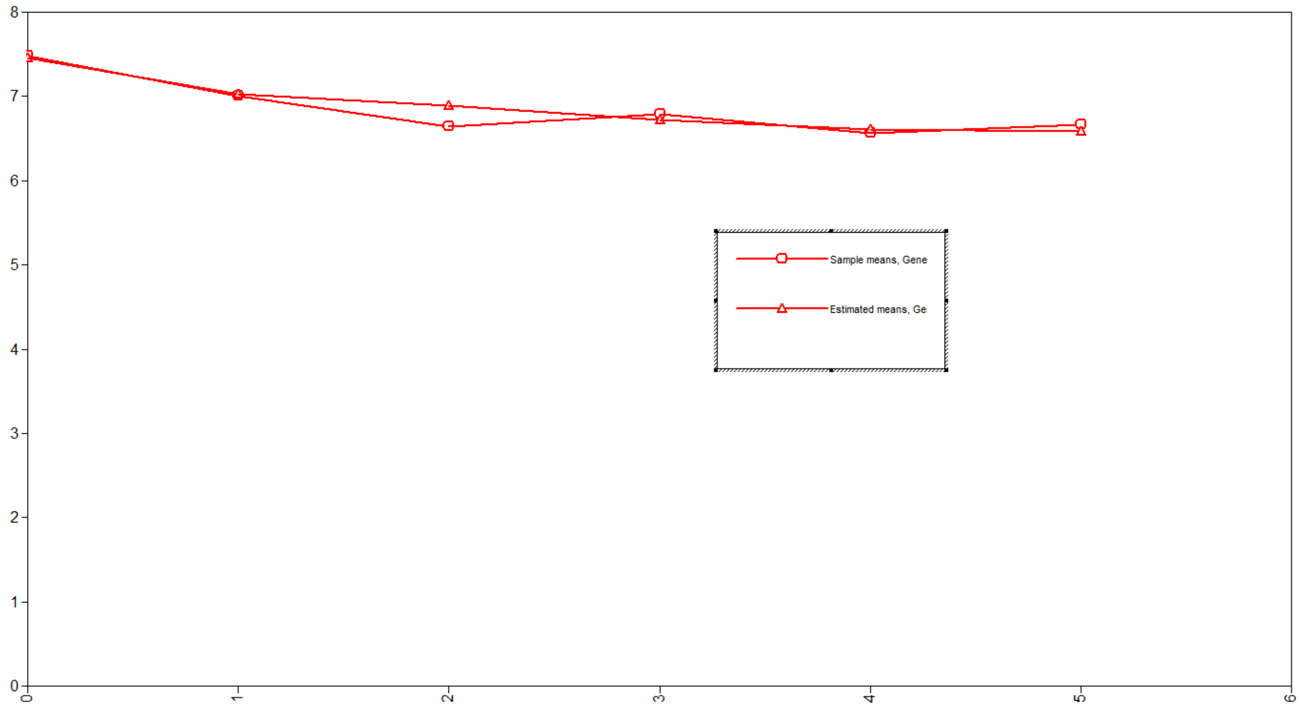
Example of a LGM with Five Time Points
Adding a Quadratic Component




```
THE MODEL ESTIMATION TERMINATED NORMALLY

     WARNING:  THE LATENT VARIABLE COVARIANCE MATRIX (PSI) IS NOT POSITIVE
     DEFINITE.  THIS COULD INDICATE A NEGATIVE VARIANCE/RESIDUAL VARIANCE FOR A
     LATENT VARIABLE, A CORRELATION GREATER OR EQUAL TO ONE BETWEEN TWO LATENT
     VARIABLES, OR A LINEAR DEPENDENCY AMONG MORE THAN TWO LATENT VARIABLES.
     CHECK THE TECH4 OUTPUT FOR MORE INFORMATION.
     PROBLEM INVOLVING VARIABLE Q.
```

Example of a LGM with Five Time Points
Time Points Free

```
USEVARIABLES ARE w1enjun w6enjun w11enjun w16enjun w21enjun w26enjun;
model:
I S | w1enjun@0 w6enjun@1 w11enjun* w16enjun* w21enjun* w26enjun*5;
plot:
type=plot3;
series = w1enjun(0) w6enjun(1) w11enjun(2) w16enjun(3) w21enjun(4) w26enjun(5);
output: sampstat residual stdyx tech4 modindices;
```



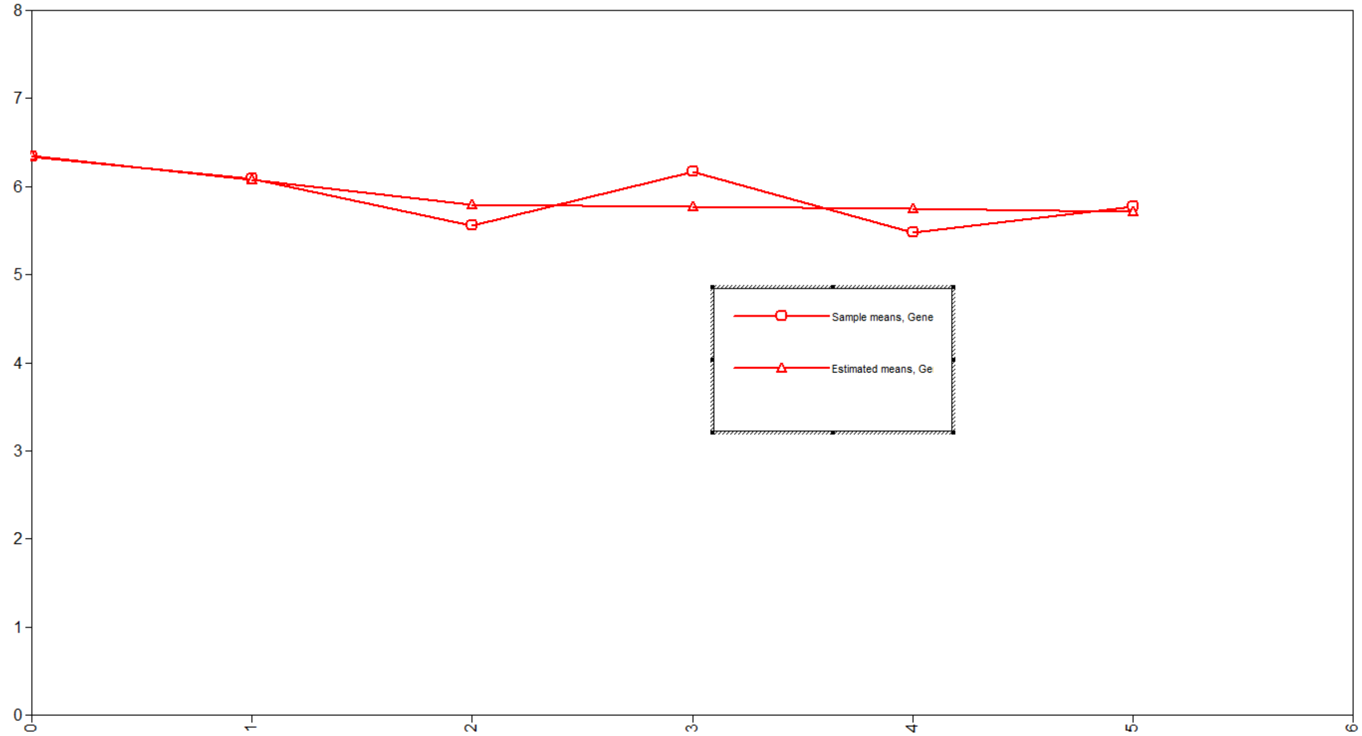
Example of a LGM with Five Time Points
Time Points Free

```
Number of Free Parameters                        15

Loglikelihood

          H0 Value                        -3764.199
          H1 Value                        -3745.014

Information Criteria

          Akaike (AIC)                     7558.398
          Bayesian (BIC)                   7618.749
          Sample-Size Adjusted BIC         7571.151
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                              38.370
          Degrees of Freedom                     12
          P-Value                            0.0001

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.073
          90 Percent C.I.                     0.048  0.099
          Probability RMSEA <= .05            0.065

CFI/TLI

          CFI                                 0.978
          TLI                                 0.973

Chi-Square Test of Model Fit for the Baseline Model

          Value                            1239.482
          Degrees of Freedom                     15
          P-Value                            0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                               0.030
```

```
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 I        |
    W1ENJUN            1.000      0.000    999.000    999.000
    W6ENJUN            1.000      0.000    999.000    999.000
    W11ENJUN           1.000      0.000    999.000    999.000
    W16ENJUN           1.000      0.000    999.000    999.000
    W21ENJUN           1.000      0.000    999.000    999.000
    W26ENJUN           1.000      0.000    999.000    999.000

 S        |
    W1ENJUN            0.000      0.000    999.000    999.000
    W6ENJUN            1.000      0.000    999.000    999.000
    W11ENJUN           1.304      0.173      7.525      0.000
    W16ENJUN           1.706      0.233      7.319      0.000
    W21ENJUN           1.954      0.287      6.818      0.000
    W26ENJUN           2.013      0.289      6.965      0.000

 S        WITH
    I                 -0.687      0.288     -2.383      0.017

 Means
    I                  7.452      0.108     69.287      0.000
    S                 -0.429      0.087     -4.911      0.000

 Intercepts
    W1ENJUN            0.000      0.000    999.000    999.000
    W6ENJUN            0.000      0.000    999.000    999.000
    W11ENJUN           0.000      0.000    999.000    999.000
    W16ENJUN           0.000      0.000    999.000    999.000
    W21ENJUN           0.000      0.000    999.000    999.000
    W26ENJUN           0.000      0.000    999.000    999.000

 Variances
    I                  3.523      0.444      7.928      0.000
    S                  0.723      0.291      2.481      0.013

 Residual Variances
    W1ENJUN            0.935      0.374      2.500      0.012
    W6ENJUN            1.869      0.177     10.554      0.000
    W11ENJUN           2.435      0.218     11.175      0.000
    W16ENJUN           0.939      0.101      9.314      0.000
    W21ENJUN           0.774      0.111      6.946      0.000
    W26ENJUN           0.910      0.124      7.330      0.000
```

Example of a LGM with Five Time Points Piecewise

```
USEVARIABLES ARE w1enjcr w6enjcr w11enjcr w16enjcr w21enjcr w26enjcr;
model:
I S1 | w1enjcr@0 w6enjcr@1 w11enjcr@2 w16enjcr@2 w21enjcr@2 w26enjcr@2;
I S2 | w1enjcr@0 w6enjcr@0 w11enjcr@0 w16enjcr@1 w21enjcr@2 w26enjcr@3;
plot:
type=plot3;
series = w1enjcr(0) w6enjcr(1) w11enjcr(2) w16enjcr(3) w21enjcr(4) w26enjcr(5);
output: sampstat residual stdyx tech4 modindices;
```



Example of a LGM with Five Time Points Piecewise

```
MODEL FIT INFORMATION

Number of Free Parameters                       15

Loglikelihood

          H0 Value                       -4038.467
          H1 Value                       -3997.736

Information Criteria

          Akaike (AIC)                    8106.934
          Bayesian (BIC)                  8167.249
          Sample-Size Adjusted BIC        8119.651
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             81.462
          Degrees of Freedom                    12
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.119
          90 Percent C.I.                    0.095  0.144
          Probability RMSEA <= .05           0.000

CFI/TLI

          CFI                                0.902
          TLI                                0.877

Chi-Square Test of Model Fit for the Baseline Model

          Value                            722.674
          Degrees of Freedom                    15
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.074
```

Poor model

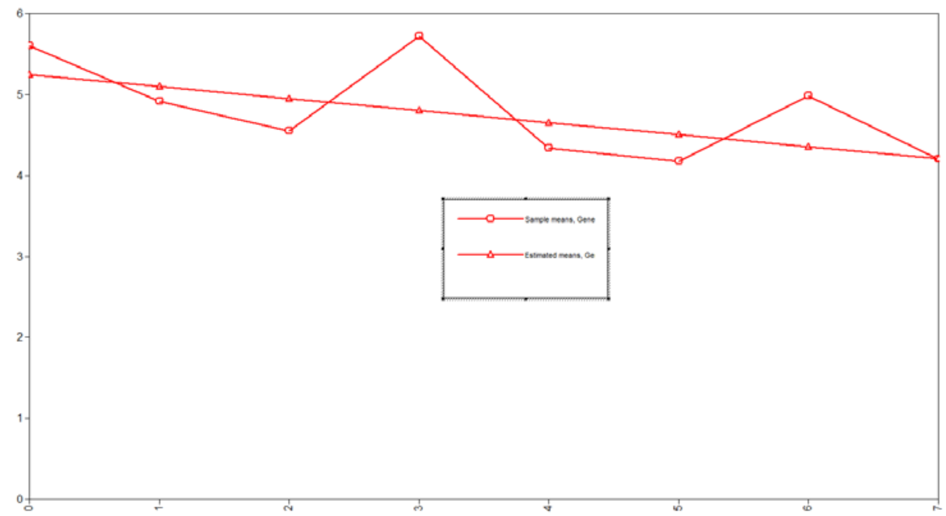
Power in LGM

- Fan (2003) showed that LGM consistently showed higher statistical power for detecting group differences in the linear growth slope than repeated measures ANOVA.
- Study done with five time points
- More research needed on the impact of number of time points

Fan, X. (2003). Power of latent growth modeling for detecting group differences in linear growth trajectory parameters. *Structural Equation Modeling: A Multidisciplinary Journal*, *10*, 380-400.

Alternative Ways of Scaling Slope: Drinking Example (8 time points)

```
MODEL FIT INFORMATION

Number of Free Parameters                       13

Loglikelihood

          H0 Value                       -7809.924
          H1 Value                       -7736.326

Information Criteria

          Akaike (AIC)                   15645.849
          Bayesian (BIC)                 15695.927
          Sample-Size Adjusted BIC       15654.687
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                            147.197
          Degrees of Freedom                    31
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.104
          90 Percent C.I.                    0.087  0.121
          Probability RMSEA <= .05           0.000

CFI/TLI

          CFI                                0.942
          TLI                                0.948

Chi-Square Test of Model Fit for the Baseline Model

          Value                           2027.634
          Degrees of Freedom                    28
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.059
```



Alternative Ways of Scaling Slope: Drinking Example (8 time points)

```
usevariables are d1 d2 d3 d4 d5 d6 d7 d8;
model: i s | d1@0 d2@0.143 d3@0.286 d4@0.429 d5@0.572 d6@0.715 d7@0.858 d8@1;
```

```
 S        WITH
    I              -13.709      2.624     -5.225      0.000

 Means
    I                5.246      0.351     14.941      0.000
    S               -1.038      0.348     -2.978      0.003

 Variances
    I               36.513      3.251     11.230      0.000
    S               24.601      3.386      7.266      0.000
```

```
usevariables are d1 d2 d3 d4 d5 d6 d7 d8;
model: i s | d1@0 d2@1 d3@2 d4@3 d5@4 d6@5 d7@6 d8@7;
```

```
 S        WITH
    I               -1.959      0.375     -5.225      0.000

 Means
    I                5.245      0.351     14.941      0.000
    S               -0.148      0.050     -2.978      0.003

 Variances
    I               36.511      3.251     11.230      0.000
    S                0.502      0.069      7.266      0.000
```

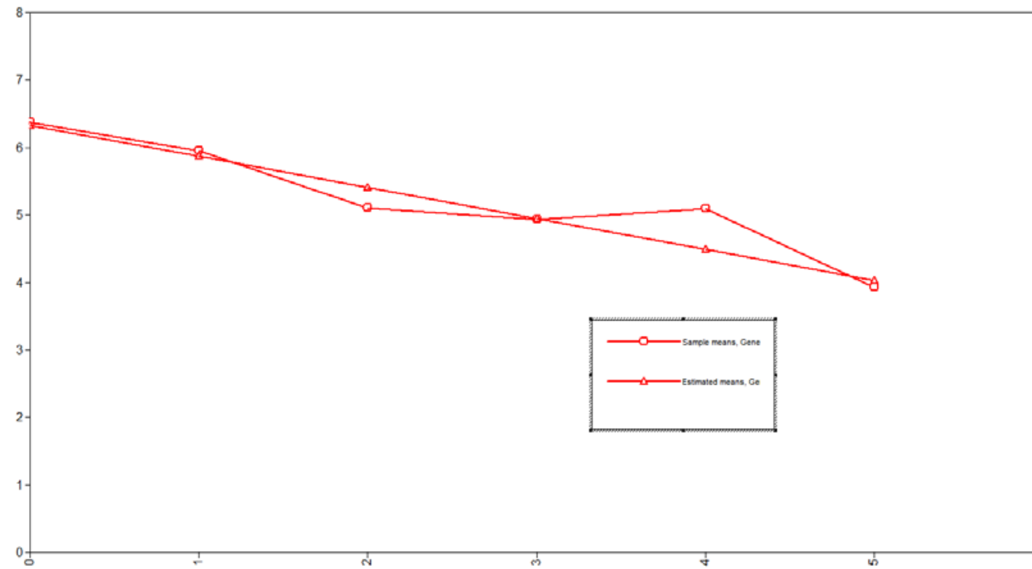
```
usevariables are d1 d2 d3 d4 d5 d6 d7 d8;
model: i s | d1@-7 d2@-6 d3@-5 d4@-4 d5@-3 d6@-2 d7@-1 d8@0;
```

```
 S        WITH
    I                1.557      0.359      4.332      0.000

 Means
    I                4.208      0.340     12.385      0.000
    S               -0.148      0.050     -2.978      0.003

 Variances
    I               33.698      3.047     11.060      0.000
    S                0.502      0.069      7.266      0.000
```

Associative LGM (i.e., Trajectory of Two Different Variables)

I will investigate whether the slope in enjoying courses is associated with the slope in drinking. Here is a LGM of drinking only:

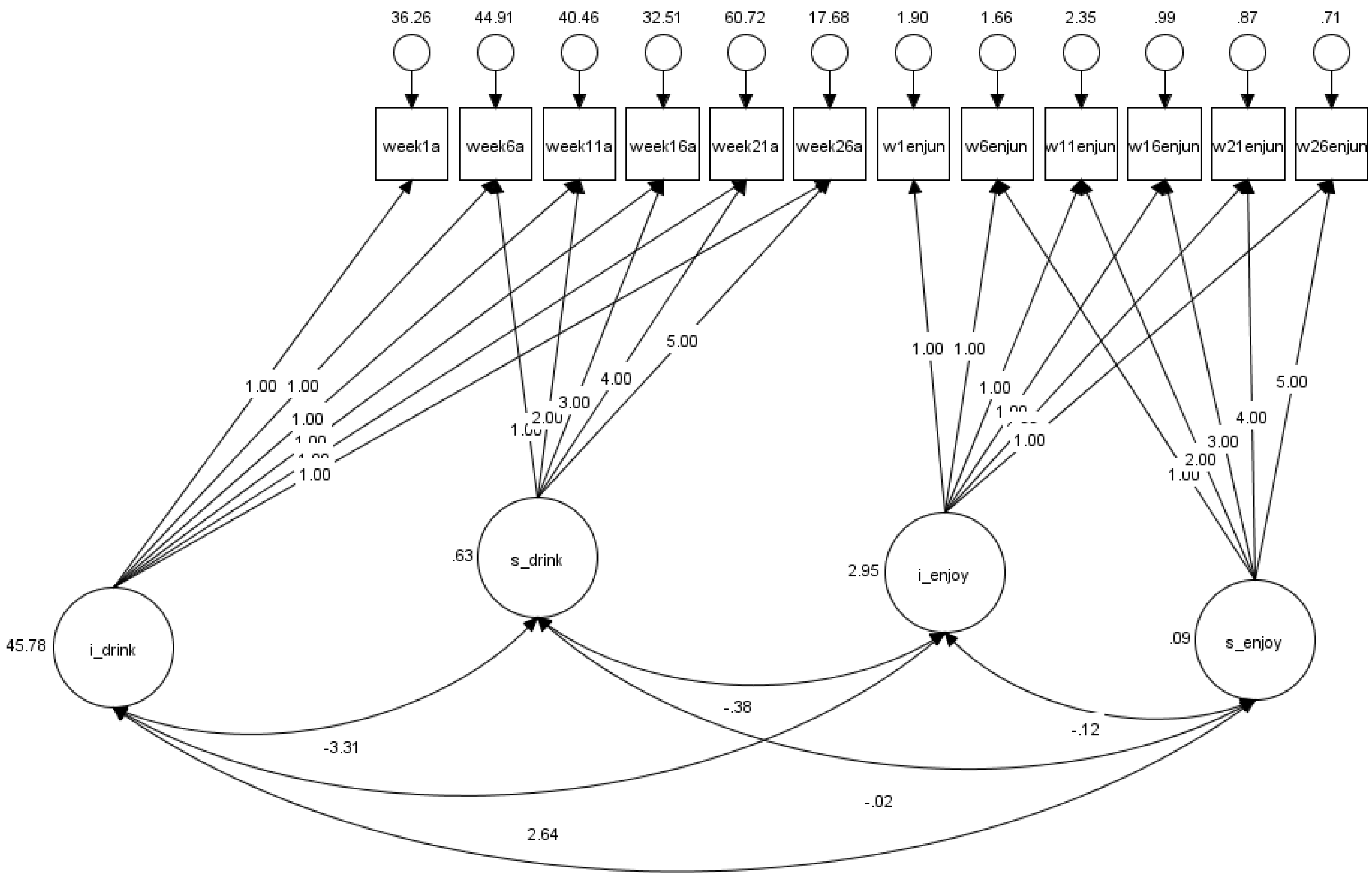
```
Number of Free Parameters                        11

Loglikelihood

          H0 Value                        -6995.646
          H1 Value                        -6952.667

Information Criteria

          Akaike (AIC)                    14013.292
          Bayesian (BIC)                  14057.550
          Sample-Size Adjusted BIC        14022.645
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                              85.958
          Degrees of Freedom                     16
          P-Value                            0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.103
          90 Percent C.I.                     0.082  0.125
          Probability RMSEA <= .05            0.000

CFI/TLI

          CFI                                 0.888
          TLI                                 0.895

Chi-Square Test of Model Fit for the Baseline Model

          Value                             642.245
          Degrees of Freedom                     15
          P-Value                            0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                               0.066
```




```
 S        WITH
    I                 -3.351      0.969     -3.459      0.001

 Means
    I                  6.328      0.406     15.568      0.000
    S                 -0.459      0.083     -5.551      0.000

 Variances
    I                 45.801      5.084      9.009      0.000
    S                  0.650      0.288      2.260      0.024
```

Associative LGM (i.e., Trajectory of Two Different Variables)

```
USEVARIABLES ARE week1a week6a week11a week16a week21a week26a
w1enjun w6enjun w11enjun w16enjun w21enjun w26enjun;
model:
I_drink S_drink | week1a@0 week6a@1 week11a@2 week16a@3 week21a@4 week26a@5;
I_enjoy S_enjoy | w1enjun@0 w6enjun@1 w11enjun@2 w16enjun@3 w21enjun@4 w26enjun@5;
output: sampstat residual stdyx tech4 modindices;
```



Associative LGM (i.e., Trajectory of Two Different Variables)

```
Loglikelihood

          H0 Value                       -10767.889
          H1 Value                       -10665.836

Information Criteria

          Akaike (AIC)                    21587.778
          Bayesian (BIC)                  21692.388
          Sample-Size Adjusted BIC        21609.884
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             204.105
          Degrees of Freedom                     64
          P-Value                            0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.073
          90 Percent C.I.                     0.062  0.084
          Probability RMSEA <= .05            0.000

CFI/TLI

          CFI                                 0.925
          TLI                                 0.923

Chi-Square Test of Model Fit for the Baseline Model

          Value                            1945.417
          Degrees of Freedom                     66
          P-Value                            0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                               0.066
```

```
 Means
    I_DRINK            6.309      0.407     15.514      0.000
    S_DRINK           -0.454      0.083     -5.485      0.000
    I_ENJOY            7.240      0.100     72.342      0.000
    S_ENJOY           -0.144      0.022     -6.408      0.000

 Variances
    I_DRINK           45.779      5.085      9.003      0.000
    S_DRINK            0.631      0.287      2.201      0.028
    I_ENJOY            2.949      0.290     10.181      0.000
    S_ENJOY            0.093      0.014      6.705      0.000

 S_DRINK  WITH
    I_DRINK           -0.616      0.090     -6.833      0.000

 I_ENJOY  WITH
    I_DRINK            0.228      0.069      3.307      0.001
    S_DRINK           -0.277      0.133     -2.078      0.038

 S_ENJOY  WITH
    I_DRINK           -0.013      0.089     -0.145      0.884
    S_DRINK           -0.087      0.146     -0.599      0.549
    I_ENJOY           -0.220      0.081     -2.701      0.007
```

One Trajectory with Time Varying Covariates

I will investigate the previous trajectory of drinking and add time varying covariates of Enjoyed University. Recall that in the previous example, Enjoyed University was modelled as a separate trajectory. In the present example, I ignore the growth factor of Enjoyed University.

```
model:
I_drink S_drink | week1a@0 week6a@1 week11a@2 week16a@3 week21a@4 week26a@5;
week1a on w1enjun;
week6a on w6enjun;
week11a on w11enjun;
week16a on w16enjun;
week21a on w21enjun;
week26a on w26enjun;
I_drink S_drink with w1enjun w6enjun w11enjun w16enjun w21enjun w26enjun;
output: sampstat residual stdyx tech4 modindices;
```

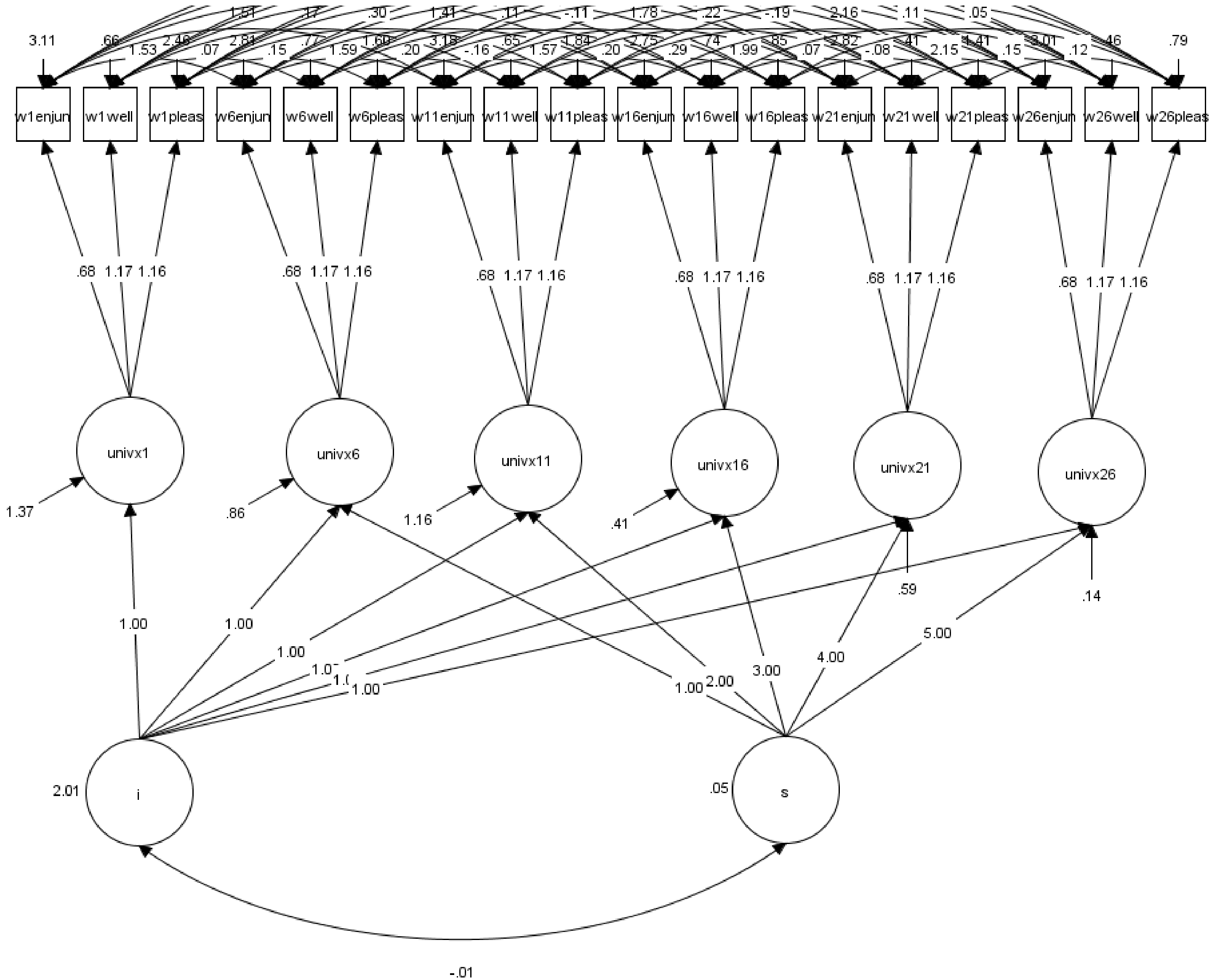
One Trajectory with Time Varying Covariates

```
Number of Free Parameters                        56

Loglikelihood

          H0 Value                       -10728.518
          H1 Value                       -10665.836

Information Criteria

          Akaike (AIC)                    21569.035
          Bayesian (BIC)                  21794.348
          Sample-Size Adjusted BIC        21616.648
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             125.363
          Degrees of Freedom                     34
          P-Value                            0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.081
          90 Percent C.I.                     0.066  0.096
          Probability RMSEA <= .05            0.000

CFI/TLI

          CFI                                 0.861
          TLI                                 0.791

Chi-Square Test of Model Fit for the Baseline Model

          Value                             705.935
          Degrees of Freedom                     51
          P-Value                            0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                               0.045
```

Not a good model;
covariate do not add much

```
 WEEK1A   ON
    W1ENJUN            0.151      0.225      0.671      0.502

 WEEK6A   ON
    W6ENJUN            0.228      0.179      1.274      0.203

 WEEK11A  ON
    W11ENJUN           0.145      0.159      0.914      0.361

 WEEK16A  ON
    W16ENJUN           0.265      0.182      1.457      0.145

 WEEK21A  ON
    W21ENJUN           0.336      0.255      1.320      0.187

 WEEK26A  ON
    W26ENJUN           0.285      0.327      0.874      0.382

 I_DRINK  WITH
    W1ENJUN            1.101      1.223      0.900      0.368
    W6ENJUN            2.469      1.193      2.069      0.039
    W11ENJUN           2.848      1.265      2.251      0.024
    W16ENJUN           2.331      1.122      2.079      0.038
    W21ENJUN           2.411      1.083      2.227      0.026
    W26ENJUN           1.830      1.154      1.587      0.113

 S_DRINK  WITH
    W1ENJUN           -0.282      0.324     -0.872      0.383
    W6ENJUN           -0.540      0.337     -1.602      0.109
    W11ENJUN          -0.541      0.333     -1.624      0.104
    W16ENJUN          -0.599      0.364     -1.648      0.099
    W21ENJUN          -0.654      0.358     -1.829      0.067
    W26ENJUN          -0.666      0.412     -1.619      0.106
    I_DRINK           -3.332      1.000     -3.332      0.001
```

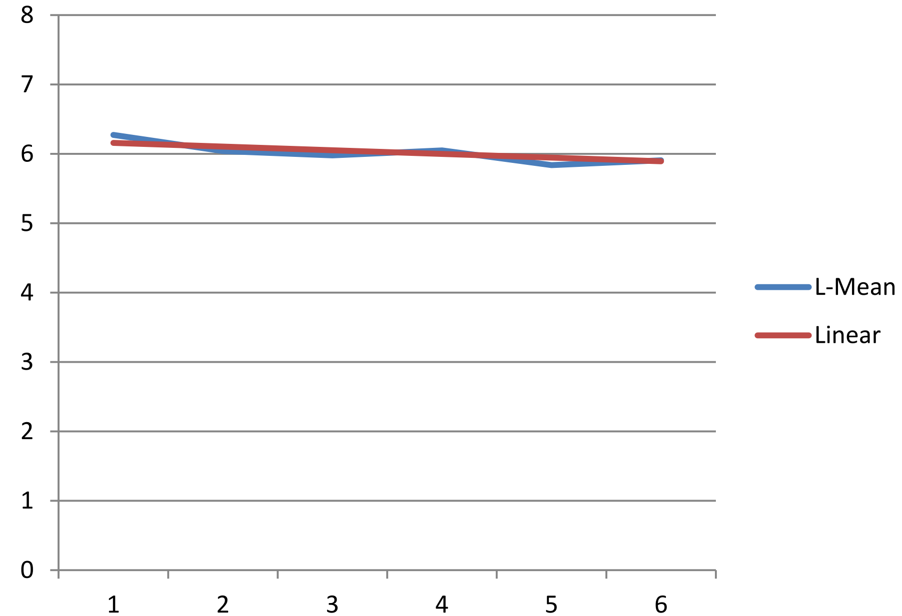
Higher Order LGM: Curve of Factor Model



Higher Order LGM: Curve of Factor Model

Steps:

- Allow residuals of indicators to correlate across time
- Test for strong invariance (loadings and intercepts across time points)
- Constrain loadings and intercepts to equality across time points
- Fix intercepts of the level-1 factors (i.e., univx1 univx2…in this example) at 0
- In Mplus, the intercept growth factor is fixed at 0; free this parameter



Higher Order LGM: Curve of Factor Model

```
MODEL:
univx1 by w1enjun* (L1)
w1well (L2)
w1pleas (L3);
univx6 by w6enjun* (L1)
w6well (L2)
w6pleas (L3);
univx11 by w11enjun* (L1)
w11well (L2)
w11pleas (L3);
univx16 by w16enjun* (L1)
w16well (L2)
w16pleas (L3);
univx21 by w21enjun* (L1)
w21well (L2)
w21pleas (L3);
univx26 by w26enjun* (L1)
w26well (L2)
w26pleas (L3);
[w1enjun] (T1);
[w1well] (T2);
[w1pleas] (T3);
[w6enjun] (T1);
[w6well] (T2);
[w6pleas] (T3);
[w11enjun] (T1);
[w11well] (T2);
[w11pleas] (T3);
[w16enjun] (T1);
[w16well] (T2);
[w16pleas] (T3);
[w21enjun] (T1);
[w21well] (T2);
[w21pleas] (T3);
[w26enjun] (T1);
[w26well] (T2);
[w26pleas] (T3);
```

```
w1pleas with w6pleas w11pleas w16pleas w21pleas w26pleas;
w6pleas with w11pleas w16pleas w21pleas w26pleas;
w11pleas with w16pleas w21pleas w26pleas;
w16pleas with w21pleas w26pleas;
w21pleas with w26pleas;
w1well with w6well w11well w16well w21well w26well;
w6well with w11well w16well w21well w26well;
w11well with w16well w21well w26well;
w16well with w21well w26well;
w21well with w26well;
w1enjun with w6enjun w11enjun w16enjun w21enjun w26enjun;
w6enjun with w11enjun w16enjun w21enjun w26enjun;
w11enjun with w16enjun w21enjun w26enjun;
w16enjun with w21enjun w26enjun;
w21enjun with w26enjun;
![univx1 univx6 univx11 univx16 univx21 univx26];
I S | univx1@0 univx6@1 univx11@2 univx16@3 univx21@4 univx26@5;
[I];
MODEL CONSTRAINT: L1 = 3 - L2 - L3;
T1 = 0 - T2 - T3;
output: sampstat residual stdyx tech4 modindices (5);
```

Higher Order LGM: Curve of Factor Model

```
Number of Free Parameters                        78

Loglikelihood

          H0 Value                       -10821.321
          H1 Value                       -10686.827

Information Criteria

          Akaike (AIC)                    21798.643
          Bayesian (BIC)                  22112.472
          Sample-Size Adjusted BIC        21864.960
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             268.989
          Degrees of Freedom                    111
          P-Value                            0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.059
          90 Percent C.I.                     0.050  0.068
          Probability RMSEA <= .05            0.054

CFI/TLI

          CFI                                 0.970
          TLI                                 0.959

Chi-Square Test of Model Fit for the Baseline Model

          Value                            5487.539
          Degrees of Freedom                    153
          P-Value                            0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                               0.076
```

```
 S        WITH
    I                 -0.003      0.033     -0.077      0.939

 Means
    I                  6.158      0.087     71.160      0.000
    S                 -0.053      0.018     -2.998      0.003

 Variances
    I                  1.986      0.210      9.454      0.000
    S                  0.053      0.010      5.249      0.000

 Residual Variances

    UNIVX1             1.444      0.183      7.909      0.000
    UNIVX6             0.878      0.120      7.297      0.000
    UNIVX11            1.171      0.138      8.512      0.000
    UNIVX16            0.421      0.070      5.994      0.000
    UNIVX21            0.644      0.089      7.209      0.000
    UNIVX26            0.148      0.080      1.846      0.065
```

LGM with Intercept and Slope Regressed on Gender (Time Invariant Covariate)

Mplus - [linear_semesterdrinking8.inp]

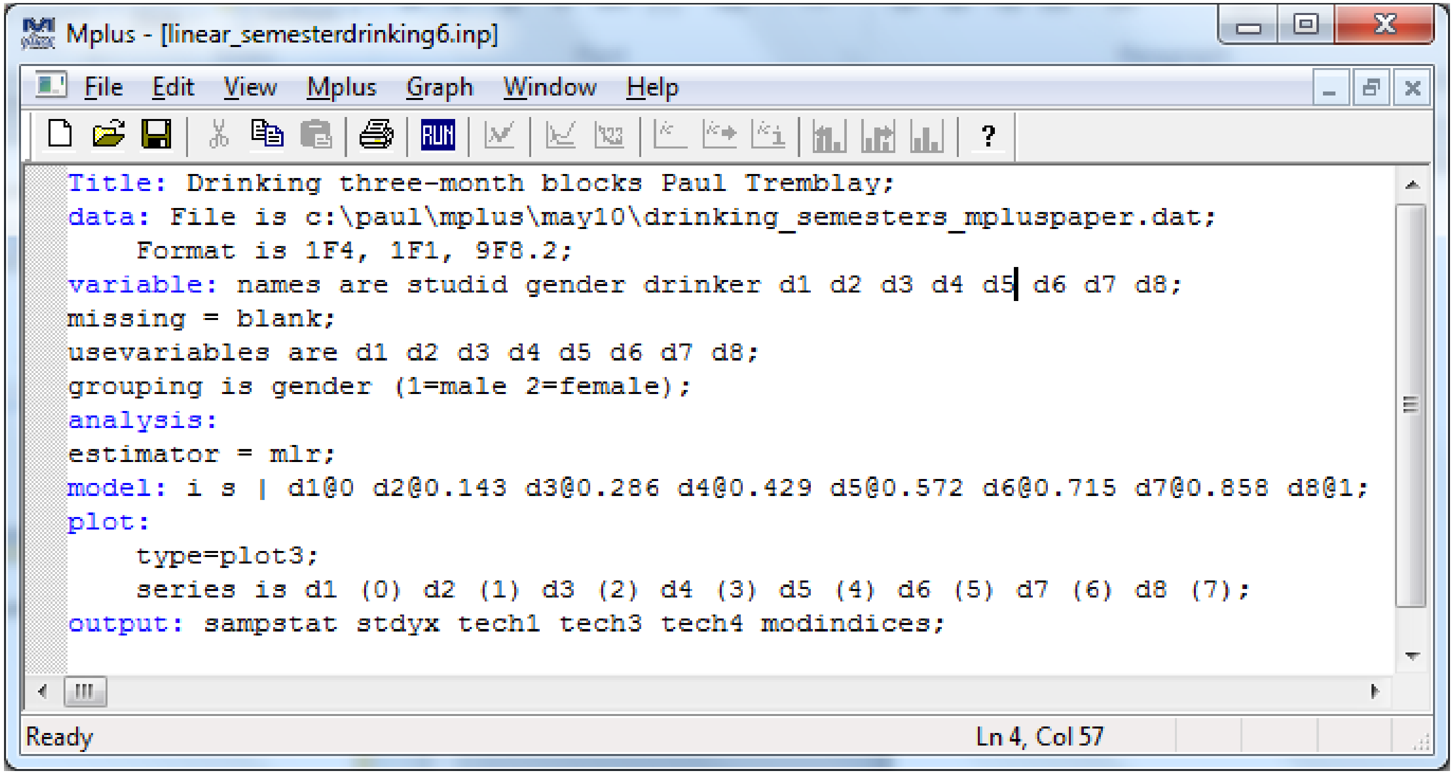
```
Title: LGM Drinking three-month blocks - Paul Tremblay;
data: File is drinking_semesters_mpluspaper.dat;
    Format is 1F4, 1F1, 9F8.2;
variable: names are studid gender drinker d1 d2 d3 d4 d5 d6 d7 d8;
missing = blank;
usevariables are d1 d2 d3 d4 d5 d6 d7 d8 gender_d;
define:
if (gender eq 1) then gender_d = 1;
if (gender eq 2) then gender_d = 0;
analysis: estimator=mlr;
model: i s | d1@0 d2@0.143 d3@0.286 d4@0.429 d5@0.572 d6@0.715 d7@0.858 d8@1;
i s on gender_d;
plot:
    type=plot3;
    series is d1 (0) d2 (1) d3 (2) d4 (3) d5 (4) d6 (5) d7 (6) d8 (7);
output: sampstat standardized tech1 tech3 tech4;
```

Mplus - [linear_semesterdrinking8.out]

```
 I        ON
    GENDER_D          3.884      0.870      4.462      0.000

 S        ON
    GENDER_D         -1.587      0.952     -1.667      0.095

 S        WITH
    I               -12.411      6.048     -2.052      0.040
```

Ready   Ln 265, Col 31

LGM Multiple groups (Gender)



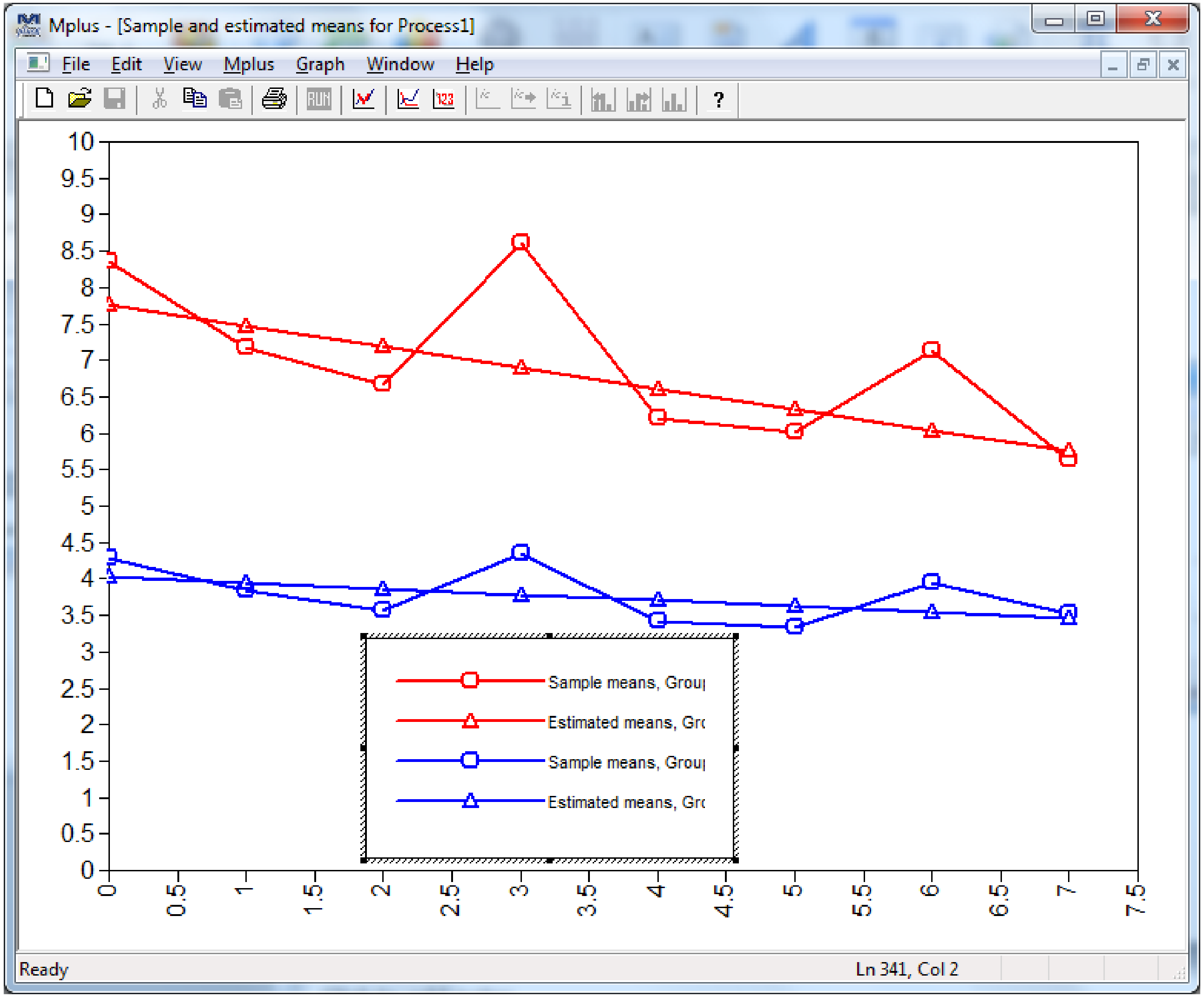
LGM Multiple groups (Gender)

Mplus - [linear_semesterdrinking6.out]

```
MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Group MALE

 Means
    I                  7.764      0.805      9.647      0.000
    S                 -2.011      0.888     -2.265      0.024

 Variances
    I                 61.958     16.424      3.772      0.000
    S                 52.528     19.285      2.724      0.006


Group FEMALE



 Means
    I                  4.029      0.312     12.909      0.000
    S                 -0.569      0.290     -1.961      0.050

 Variances
    I                 18.580      2.549      7.288      0.000
    S                 10.387      2.640      3.935      0.000
```

Ready   Ln 341, Col 2

LGM Multiple groups (Gender)



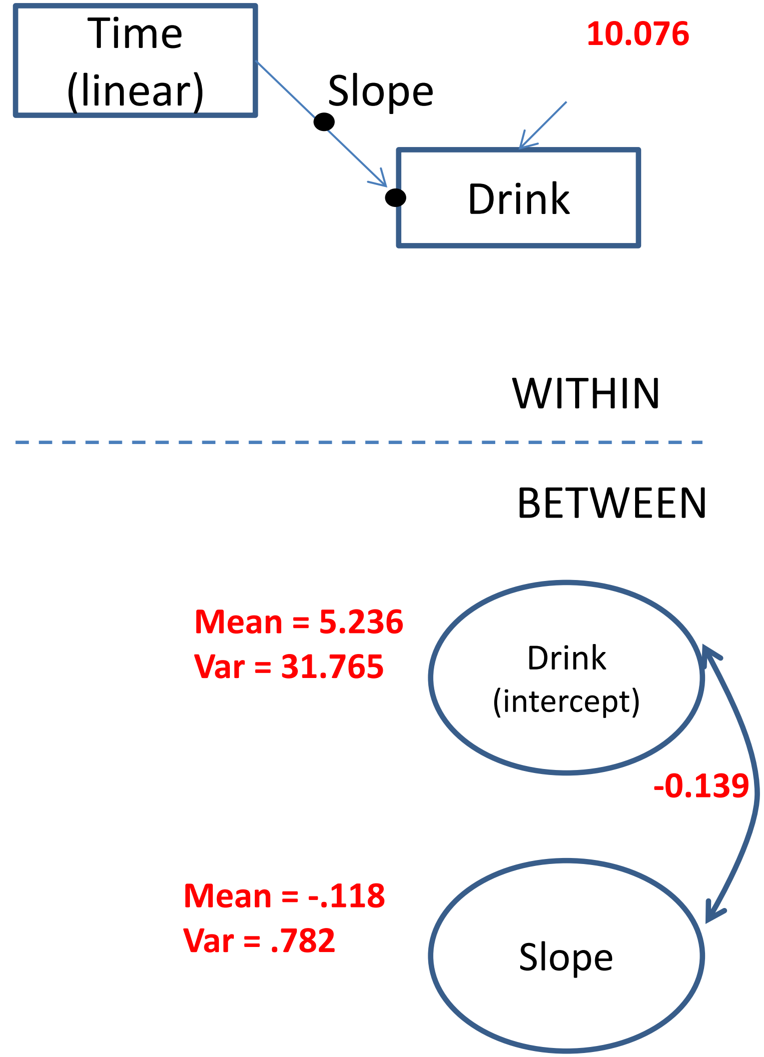
Measures within Persons

```
Title: PSY9555 Regression examples;
!note that two outliers of 0 were removed in average;
data: File is sem_mplus2.dat;
    Format is 1F4, 1F1, 1F2, 23F8.2, 1F11.3, 72F8.2;
data widetolong:
wide = drink1 drink2 drink3 drink4;
long = drink;
idvariable = person;
repetition = time;
variable: names are studid gender age
bppa bpv bpa bph bptot
sq1 sq2 sq3 sq4 sq5 sq6 sq7 sq8 sq9 sq10 sq11 sq12 sq13 sq14 sq15
es es_pt es_fin grade
drink1 drink2 drink3 drink4 epis1 epis2 epis3 epis4
stress1 stress2 stress3 stress4 pleased1 pleased2 pleased3 pleased4
enjoyc1 enjoyc2 enjoyc3 enjoyc4 enjoyu1 enjoyu2 enjoyu3 enjoyu4
effort1 effort2 effort3 effort4 harm1 harm2 harm3 harm4 dep1 dep2 dep3 dep4
drink1b drink2b drink3b drink4b epis1b epis2b epis3b epis4b
stress1b stress2b stress3b stress4b please1b please2b please3b please4b
enjoyc1b enjoyc2b enjoyc3b enjoyc4b enjoyu1b enjoyu2b enjoyu3b enjoyu4b
effort1b effort2b effort3b effort4b harm1b harm2b harm3b harm4b
dep1b dep2b dep3b dep4b;
missing = blank;
usevariables are drink person time;
cluster = person;
within = time;
analysis:
type = twolevel random;
model:
%within%
s | drink on time;
%between%
s with drink;
output: sampstat tech1;
```

When your data file is structured in the conventional one line per subject with repeated measures on the same line

Measures within Persons

```
Loglikelihood

          H0 Value                         -4756.126
          H0 Scaling Correction Factor        3.5791
            for MLR

Information Criteria

          Akaike (AIC)                      9524.253
          Bayesian (BIC)                    9556.568
          Sample-Size Adjusted BIC          9537.507
            (n* = (n + 2) / 24)


MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Within Level

 Residual Variances
    DRINK             10.076      1.428      7.056      0.000

Between Level

 S        WITH
    DRINK             -0.139      1.168     -0.119      0.905

 Means
    DRINK              5.236      0.306     17.090      0.000
    S                 -0.118      0.085     -1.396      0.163

 Variances
    DRINK             31.765      4.088      7.769      0.000
    S                  0.782      0.478      1.636      0.102
```



Measures within Persons (Previous Model specified as LGM)

```
Title: PSY9555 Regression examples;
!note that two outliers of 0 were removed in average;
data: File is sem_mplus2.dat;
    Format is 1F4, 1F1, 1F2, 23F8.2, 1F11.3, 72F8.2;
!LISTWISE = ON;
variable: names are studid gender age
bppa bpv bpa bph bptot
sq1 sq2 sq3 sq4 sq5 sq6 sq7 sq8 sq9 sq10 sq11 sq12 sq13 sq14 sq15
es es_pt es_fin grade
drink1 drink2 drink3 drink4 epis1 epis2 epis3 epis4
stress1 stress2 stress3 stress4 pleased1 pleased2 pleased3 pleased4
enjoyc1 enjoyc2 enjoyc3 enjoyc4 enjoyu1 enjoyu2 enjoyu3 enjoyu4
effort1 effort2 effort3 effort4 harm1 harm2 harm3 harm4 dep1 dep2 dep3 dep4
drink1b drink2b drink3b drink4b epis1b epis2b epis3b epis4b
stress1b stress2b stress3b stress4b please1b please2b please3b please4b
enjoyc1b enjoyc2b enjoyc3b enjoyc4b enjoyu1b enjoyu2b enjoyu3b enjoyu4b
effort1b effort2b effort3b effort4b harm1b harm2b harm3b harm4b
dep1b dep2b dep3b dep4b;
missing = blank;
usevariables are drink1 drink2 drink3 drink4;
analysis:
type = general;
estimator = mlr;
model:
I S | drink1@0 drink2@1 drink3@2 drink4@3;
drink1-drink4 (1);
plot:
type is plot3;
series = drink1 (0) drink2 (1) drink3 (2) drink4 (3);
output: sampstat residual stdyx tech4 modindices;
```

Back to the conventional data structure specification

IN LGM these residuals are usually not constrained to equality but they are in MLM. I constrained them here.

Example: Measures within Persons (LGM and MLM)

LGM

Mplus - [lgm.out]

```
MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 S        WITH
    I                 -0.142      1.167     -0.122      0.903

 Means
    I                  5.238      0.307     17.088      0.000
    S                 -0.119      0.085     -1.404      0.160

 Variances
    I                 31.784      4.093      7.765      0.000
    S                  0.776      0.477      1.626      0.104

 Residual Variances
    DRINK1            10.082      1.430      7.052      0.000
    DRINK2            10.082      1.430      7.052      0.000
    DRINK3            10.082      1.430      7.052      0.000
    DRINK4            10.082      1.430      7.052      0.000
```

Ready   Ln 219, Col 2

MLM

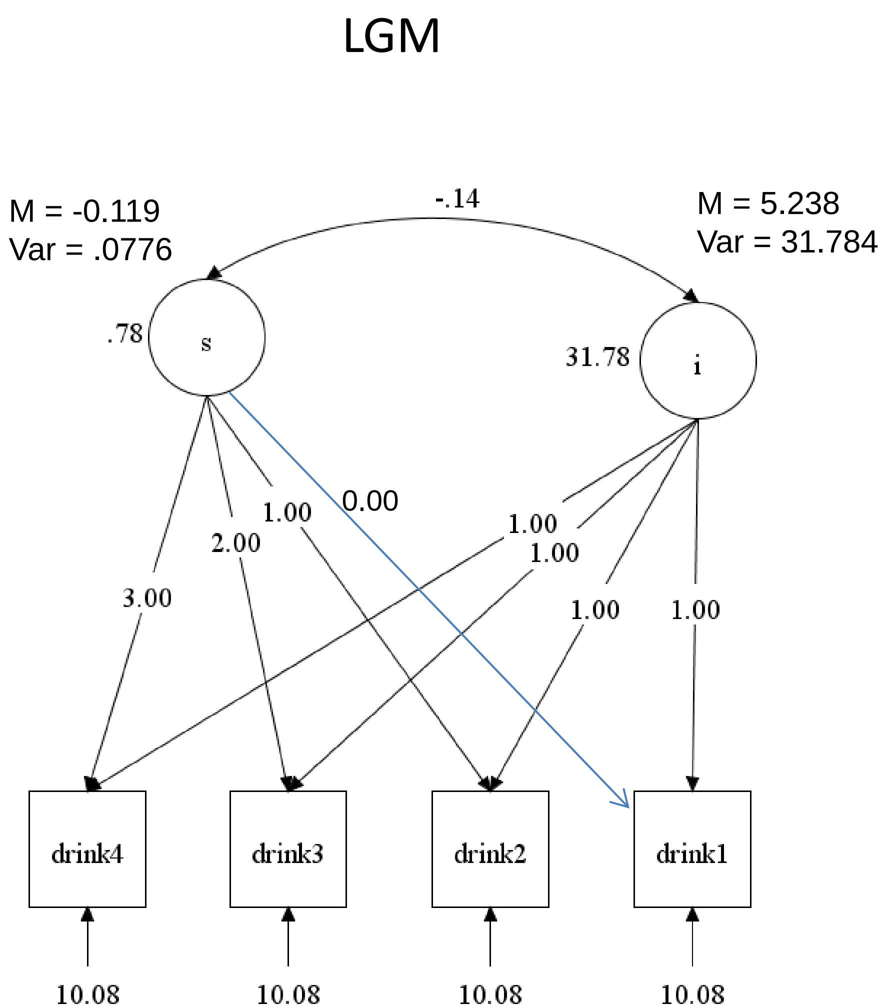
Mplus - [lgm_mlm.out]

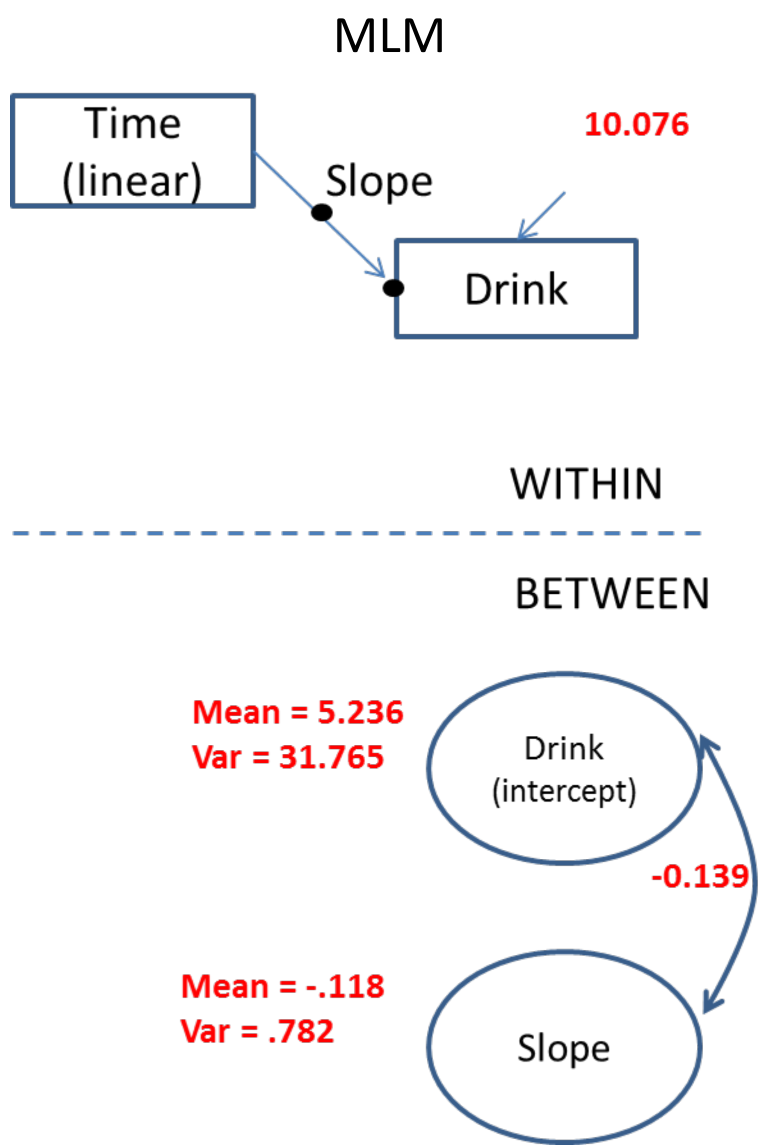
```
MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Within Level

 Residual Variances
    DRINK             10.076      1.428      7.056      0.000

Between Level

 S        WITH
    DRINK             -0.139      1.168     -0.119      0.905

 Means
    DRINK              5.236      0.306     17.090      0.000
    S                 -0.118      0.085     -1.396      0.163

 Variances
    DRINK             31.765      4.088      7.769      0.000
    S                  0.782      0.478      1.636      0.102
```

Ready   Ln 1, Col 1

Same as in slide 13

Example: Measures within Persons (LGM and MLM)





Example: Measures within Persons (LGM and MLM)

LGM

```
MODEL FIT INFORMATION

Number of Free Parameters                        6

Loglikelihood

          H0 Value                       -4756.126
          H0 Scaling Correction Factor      3.5794
            for MLR
          H1 Value                       -4731.180
          H1 Scaling Correction Factor      3.6974
            for MLR

Information Criteria

          Akaike (AIC)                    9524.252
          Bayesian (BIC)                  9548.407
          Sample-Size Adjusted BIC        9529.368
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             13.178*
          Degrees of Freedom                     8
          P-Value                           0.1059
          Scaling Correction Factor         3.7860
            for MLR
```

MLM

```
MODEL FIT INFORMATION

Number of Free Parameters                        6

Loglikelihood

          H0 Value                       -4756.126
          H0 Scaling Correction Factor      3.5791
            for MLR

Information Criteria

          Akaike (AIC)                    9524.253
          Bayesian (BIC)                  9556.568
          Sample-Size Adjusted BIC        9537.507
            (n* = (n + 2) / 24)
```