PSY 9556B (Feb 26) Update on Measurement Invariance Methods, and Longitudinal Panel Models

Updates on Measurement Invariance

- Gordon Cheung's presentation
  - Instead of chi-square difference test we can directly calculate invariance of each parameter
  - The issue of the referent item
  - My modified procedure using Little's approach to setting the scale
- Kateryna Keefer's presentation
  - Interpreting the impact of partial invariance

Cheung, G. W. & Lau, R. S. (2012). A direct comparison approach for testing measurement invariance. Organizational Research Methods, 15(2), 167-198.

Keefer, K. V., Holden, R. R., & Parker, J. D. A. (2013). Longitudinal assessment of trait emotional intelligence: Measurement invariance and construct continuity from late childhood to adolescence.

Example – One Latent Variable (4 indicators) at Two Time Points

Often in research we simply create an aggregate score without assessing MI

```
USEVARIABLES ARE w1enjcr w1pleas w1well w1enjun
w2enjcr w2pleas w2well w2enjun uni_ew uni_ew2;
DEFINE:
uni_ew = mean(w1enjcr w1pleas w1well w1enjun);
uni_ew2 = mean (w2enjcr w2pleas w2well w2enjun);
ANALYSIS:
type=general;
```

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Means				
W1ENJCR	6.349	0.107	59.279	0.000
W1PLEAS	6.119	0.127	48.046	0.000
W1WELL	5.833	0.110	53.163	0.000
W1ENJUN	7.506	0.105	71.155	0.000
W2ENJCR	6.588	0.099	66.744	0.000
W2PLEAS	6.098	0.126	48.321	0.000
W2WELL	6.029	0.112	54.004	0.000
W2ENJUN	7.348	0.106	69.214	0.000
UNI_EW	6.457	0.089	72.778	0.000
UNI_EW2	6.516	0.088	73.642	0.000

Estimating Invariance of the Loadings

```
!two-time point using Mplus default;
MODEL:
univx1 by w1enjcr* (L1)
w1pleas (L2)
w1well (L3)
w1enjun (L4);
univx2 by w2enjcr* (L5)
w2pleas (L6)
w2well (L7)
w2enjun (L8);
w1enjcr with w2enjcr;
w1pleas with w2pleas;
w1well with w2well;
w1enjun with w2enjun;
[univx1 univx2];
[w1enjcr] (T1)
[w1pleas] (T2)
[w1well] (T3)
[w1enjun] (T4);
[w2enjcr] (T5)
[w2pleas] (T6)
[w2well] (T7)
[w2enjun] (T8);
MODEL CONSTRAINT: L1 = 4 - L2 - L3 - L4;
                T1 = 0 - T2 - T3 - T4;
                L5 = 4 - L6 - L7 - L8;
                T5 = 0 - T6 - T7 - T8;
                NEW(DiffL1 DiffL2 DiffL3 DiffL4);
                DiffL1 = L1 - L5;
                DiffL2 = L2 - L6;
                DiffL3 = L3 - L7;
                DiffL4 = L4 - L8;
ANALYSIS:
BOOTSTRAP = 1000;
output: sampstat residual stdyx tech4 cinterval(BCBOOTSTRAP);
```

Estimating Invariance of the Loadings

```
Chi-Square Test of Model Fit

          Value                              55.002
          Degrees of Freedom                     15
          P-Value                            0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.080
          90 Percent C.I.                     0.058  0.104
          Probability RMSEA <= .05            0.013

CFI/TLI

          CFI                                 0.976
          TLI                                 0.955
```

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
UNIVX1 BY				
W1ENJCR	0.677	0.063	10.671	0.000
W1PLEAS	1.266	0.066	19.242	0.000
W1WELL	1.270	0.054	23.602	0.000
W1ENJUN	0.787	0.051	15.455	0.000
UNIVX2 BY				
W2ENJCR	0.537	0.065	8.262	0.000
W2PLEAS	1.344	0.051	26.131	0.000
W2WELL	1.326	0.056	23.751	0.000
W2ENJUN	0.793	0.054	14.564	0.000
UNIVX2 WITH				
UNIVX1	1.605	0.180	8.896	0.000
W1ENJCR WITH				
W2ENJCR	1.603	0.197	8.139	0.000
W1PLEAS WITH				
W2PLEAS	0.353	0.170	2.071	0.038
W1WELL WITH				
W2WELL	0.151	0.133	1.142	0.253
W1ENJUN WITH				
W2ENJUN	1.725	0.243	7.110	0.000
Means				
UNIVX1	6.434	0.089	71.999	0.000
UNIVX2	6.524	0.089	73.056	0.000

Estimating Invariance of the Loadings

```
Intercepts
   W1ENJCR            1.984       0.427       4.643       0.000
   W1PLEAS           -2.055       0.462      -4.450       0.000
   W1WELL            -2.362       0.360      -6.553       0.000
   W1ENJUN            2.432       0.366       6.651       0.000
   W2ENJCR            3.094       0.457       6.768       0.000
   W2PLEAS           -2.666       0.363      -7.352       0.000
   W2WELL            -2.616       0.383      -6.828       0.000
   W2ENJUN            2.188       0.391       5.599       0.000

Variances
   UNIVX1             2.440       0.209      11.670       0.000
   UNIVX2             2.398       0.211      11.379       0.000

Residual Variances
   W1ENJCR            3.213       0.253      12.723       0.000
   W1PLEAS            2.398       0.339       7.076       0.000
   W1WELL             0.703       0.208       3.387       0.001
   W1ENJUN            2.830       0.321       8.813       0.000
   W2ENJCR            2.946       0.248      11.889       0.000
   W2PLEAS            1.656       0.215       7.694       0.000
   W2WELL             0.556       0.161       3.446       0.001
   W2ENJUN            2.658       0.252      10.534       0.000

New/Additional Parameters
   DIFFL1             0.140       0.070       1.995       0.046
   DIFFL2            -0.077       0.066      -1.165       0.244
   DIFFL3            -0.056       0.053      -1.063       0.288
   DIFFL4            -0.006       0.050      -0.123       0.902
```

Estimating Invariance of the Loadings

CONFIDENCE INTERVALS OF MODEL RESULTS

	Lower .5%	Lower 2.5%	Lower 5%	Estimate	Upper 5%	Upper 2.5%	Upper .5%
UNIVX1 BY							
W1ENJCR	0.488	0.539	0.559	0.677	0.768	0.787	0.827
W1PLEAS	1.097	1.140	1.166	1.266	1.378	1.395	1.451
W1WELL	1.146	1.168	1.186	1.270	1.361	1.377	1.417
W1ENJUN	0.654	0.689	0.705	0.787	0.870	0.886	0.916
UNIVX2 BY							
W2ENJCR	0.354	0.401	0.421	0.537	0.639	0.658	0.685
W2PLEAS	1.215	1.237	1.257	1.344	1.427	1.445	1.487
W2WELL	1.194	1.227	1.241	1.326	1.419	1.440	1.495
W2ENJUN	0.648	0.680	0.702	0.793	0.886	0.900	0.923

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New/Additional Parameters							
DIFFL1	-0.037	0.004	0.030	0.140	0.258	0.280	0.326
DIFFL2	-0.274	-0.217	-0.191	-0.077	0.024	0.050	0.087
DIFFL3	-0.201	-0.165	-0.153	-0.056	0.026	0.043	0.082
DIFFL4	-0.132	-0.109	-0.095	-0.006	0.071	0.084	0.116

Constraining Invariant Loadings and Evaluating Invariance of Intercepts

```
!first nested model with contraints of the loadings across time;
MODEL:
univx1 by w1enjcr* (L1)
w1pleas (L2)
w1well (L3)
w1enjun (L4);
univx2 by w2enjcr* (L1)
w2pleas (L2)
w2well (L3)
w2enjun (L4);
w1enjcr with w2enjcr;
w1pleas with w2pleas;
w1well with w2well;
w1enjun with w2enjun;
[univx1 univx2];
[w1enjcr] (T1)
[w1pleas] (T2)
[w1well] (T3)
[w1enjun] (T4);
[w2enjcr] (T5)
[w2pleas] (T6)
[w2well] (T7)
[w2enjun] (T8);
MODEL CONSTRAINT: L1 = 4 - L2 - L3 - L4;
                  T1 = 0 - T2 - T3 - T4;
                ! L5 = 4 - L6 - L7 - L8;
                  T5 = 0 - T6 - T7 - T8;
NEW(DiffI1 DiffI2 DiffI3 DiffI4);
                DiffI1 = T1 - T5;
                DiffI2 = T2 - T6;
                DiffI3 = T3 - T7;
                DiffI4 = T4 - T8;
ANALYSIS:
BOOTSTRAP = 1000;
output: sampstat residual stdyx tech4 cinterval(BCBOOTSTRAP);
```

```
Means
   UNIVX1              6.435       0.089      72.031       0.000
   UNIVX2              6.524       0.089      73.041       0.000
```

Evaluating Invariance of Intercepts

```
Chi-Square Test of Model Fit

          Value                              61.290
          Degrees of Freedom                     18
          P-Value                            0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.076
          90 Percent C.I.                     0.056  0.098
          Probability RMSEA <= .05            0.018

CFI/TLI

          CFI                                 0.974
          TLI                                 0.959
```

CONFIDENCE INTERVALS OF MODEL RESULTS

	Lower .5%	Lower 2.5%	Lower 5%	Estimate	Upper 5%	Upper 2.5%	Upper .5%
New/Additional Parameters							
DIFFI1	-0.422	-0.367	-0.350	-0.202	-0.085	-0.063	-0.010
DIFFI2	-0.110	-0.057	-0.029	0.110	0.233	0.253	0.299
DIFFI3	-0.288	-0.228	-0.210	-0.111	-0.015	0.006	0.041
DIFFI4	0.055	0.084	0.106	0.204	0.314	0.334	0.378

Constraining 2 Invariant Intercepts and Assessing Invariance of the Residuals

```
!constraining two invariant intercepts across time;
MODEL:
univx1 by w1enjcr* (L1)
w1pleas (L2)
w1well (L3)
w1enjun (L4);
univx2 by w2enjcr* (L1)
w2pleas (L2)
w2well (L3)
w2enjun (L4);
w1enjcr with w2enjcr;
w1pleas with w2pleas;
w1well with w2well;
w1enjun with w2enjun;
[univx1 univx2];
[w1enjcr] (T1)
[w1pleas] (T2)
[w1well] (T3)
[w1enjun] (T4);
[w2enjcr] (T5)
[w2pleas] (T2)
[w2well] (T3)
[w2enjun] (T8);
w1enjcr (r1)
w1pleas (r2)
w1well (r3)
w1enjun (r4);
w2enjcr (r5)
w2pleas (r6)
w2well (r7)
w2enjun (r8);
MODEL CONSTRAINT: L1 = 4 - L2 - L3 - L4;
                 T1 = 0 - T2 - T3 - T4;
NEW(Diffr1 Diffr2 Diffr3 Diffr4);
                Diffr1 = r1 - r5;
                Diffr2 = r2 - r6;
                Diffr3 = r3 - r7;
                Diffr4 = r4 - r8;
ANALYSIS:
BOOTSTRAP = 1000;
output: sampstat residual stdyx tech4 cinterval(BCBOOTSTRAP);
```

Constraining 2 Invariant Intercepts and Assessing Invariance of the Residuals

```
Chi-Square Test of Model Fit

          Value                              65.549
          Degrees of Freedom                     19
          P-Value                            0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.077
          90 Percent C.I.                     0.057  0.098
          Probability RMSEA <= .05            0.014

CFI/TLI

          CFI                                 0.972
          TLI                                 0.958
```

```
 Means
    UNIVX1             6.415      0.089     71.988      0.000
    UNIVX2             6.552      0.096     68.401      0.000
```

If I constrain all four intercepts:

```
 Means
    UNIVX1             6.418      0.088     72.558      0.000
    UNIVX2             6.534      0.091     72.187      0.000
```

```
CONFIDENCE INTERVALS OF MODEL RESULTS

                  Lower .5%  Lower 2.5%   Lower 5%    Estimate    Upper 5%  Upper 2.5%   Upper .5%

 New/Additional Parameters
    DIFFR1          -0.272      -0.164      -0.094       0.359       0.829       0.933       1.021
    DIFFR2          -0.120       0.114       0.229       0.749       1.293       1.424       1.635
    DIFFR3          -0.384      -0.269      -0.228       0.061       0.335       0.386       0.507
    DIFFR4          -0.473      -0.332      -0.230       0.203       0.672       0.805       0.905
```

Summary of “Latent” Mean Differences Across Time

Model	M Time 1	M Time 2	Diff
Equal weights (across loadings and across time)	6.457	6.516	0.059
CFA with unconstrained loadings and intercepts	6.434	6.524	0.090
All constrained loadings (across time)	6.435	6.524	0.089
Two constrained intercepts (across time)	6.415	6.552	0.137
Four constrained intercepts(across time)	6.418	6.534	0.116
Could remove item with invariant intercepts (if enough items left over)			

Panel Longitudinal Models

- Can be viewed as the structural part of a longitudinal SEM model
- Typical parts:
  - Auto-regressive regression paths
  - Cross-lagged regression paths
  - Correlated residuals
  - Can add time invariant covariates (a few different ways of modeling)
  - Can add time variant covariates
  - Could add a LGM structure (ALT models – next week)
- Basic simplex (autoregressive) change process
  - AR1, AR2,…
  - What is the meaning of AR2
  - Simplex change process vs. latent growth model - meaning

Example Using Mplus

In the following example I use single observed variables across 8 time points. A more powerful design would use latent variables and would test for measurement invariance first.

```
DEFINE:
if (gender eq 1) then ngender = 1;
if (gender eq 2) then ngender = 0;
analysis: estimator = mlr;
model: dep2 on dep1;
dep3 on dep2;
dep4 on dep3;
dep5 on dep4;
dep6 on dep5;
dep7 on dep6;
dep8 on dep7;
drink2 on drink1;
drink3 on drink2;
drink4 on drink3;
drink5 on drink4;
drink6 on drink5;
drink7 on drink6;
drink8 on drink7;
dep1 with drink1;
dep2 with drink2;
dep3 with drink3;
dep4 with drink4;
dep5 with drink5;
```

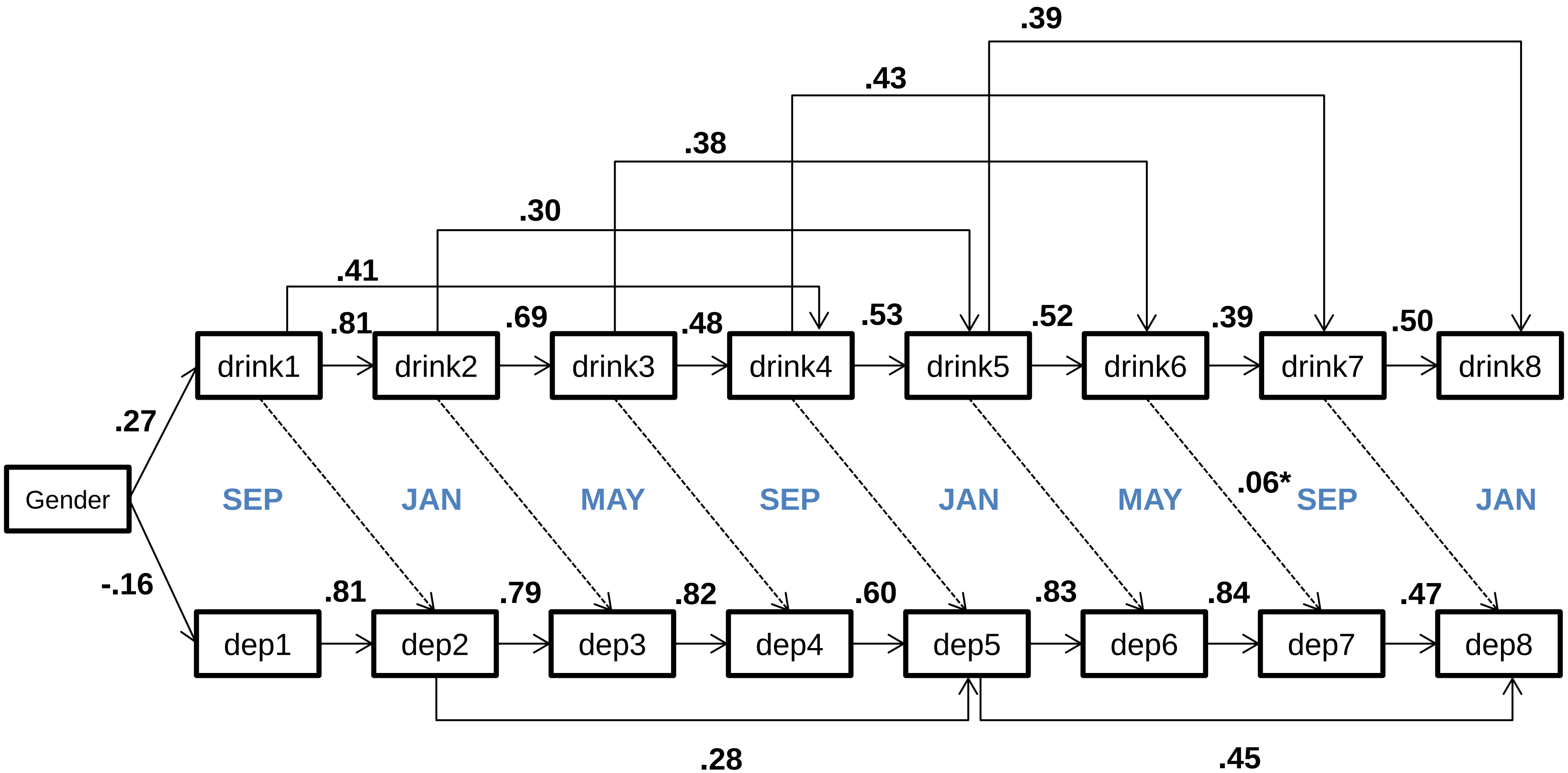
syntax continued

```
dep6 with drink6;
dep7 with drink7;
dep8 with drink8;
drink4 on drink1;
drink5 on drink2;
drink6 on drink3;
drink7 on drink4;
drink8 on drink5;
dep8 on dep5;
dep5 on dep2;
dep1 drink1 on ngender;
dep2 on drink1;
dep3 on drink2;
dep4 on drink3;
dep5 on drink4;
dep6 on drink5;
dep7 on drink6;
dep8 on drink7;
output: sampstat stdyx modindices;
```

Example Using Mplus

```
Chi-Square Test of Model Fit

          Value                              199.547*
          Degrees of Freedom                      98
          P-Value                             0.0000
          Scaling Correction Factor           1.4174
            for MLR

*   The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used
    for chi-square difference testing in the regular way.  MLM, MLR and WLSM
    chi-square difference testing is described on the Mplus website.  MLMV, WLSMV,
    and ULSMV difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.050
          90 Percent C.I.                     0.040  0.060
          Probability RMSEA <= .05            0.484

CFI/TLI

          CFI                                 0.965
          TLI                                 0.951

Chi-Square Test of Model Fit for the Baseline Model

          Value                             3020.090
          Degrees of Freedom                     136
          P-Value                             0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                               0.052
```

Example Using Mplus



Standardized coefficients all significant p < .001, except .06* significant p < .05. Note that drink-dep correlated residuals were included but not significant. An alternative model testing the effects of depression on drinking revealed no significant cross-lag associations.