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Nonlinear Bifurcations of Psychological Stress Negotiation: New Properties of a Formal Dynamical Model

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Abstract: *Dynamical systems analysis is applied to a nonlinear model of stress and coping (Neufeld, 1999). The model is composed of 6 order parameters and 11 control parameters, and integrates core constructs of the topic domain, including variants of cognitive appraisal, differential stress susceptibility, stress activation, and coping propensity. In part owing to recent advances in Competitive Modes Theory (Yao, Yu & Essex, 2002), previously intractable but substantively significant dynamical properties of the 6-dimensional model are identified. They include stable and unstable fixed-point equilibria (higher-dimensional saddle-node bifurcation), oscillatory patterns attending fixed-point de-stabilization, and chaotic behaviors. Examination of the nature of system fixed-point de-stabilization, in relation to its control parameters, unveils mechanisms of re-stabilization, and dynamic stability control. All identified dynamics emerge naturally from a system whose construction guideposts are lodged in the addressed content domain. Dynamical complexities therefore may be intrinsic to the present content domain, possibly no less so than in other disciplines where the presence of such attributes has been established.*

Key Words: stress, coping, dynamical systems, decisional control

INTRODUCTION

Negotiation of psychological stress is deemed to comprise progressive transactions among environmental demands and threats to well-being (stressors), coping activity aimed at their resolution, and subjective evaluation of coping effectiveness (see, e.g., Monroe, 2008). Stress and coping variously have been

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considered to entail dynamical interactions among constituent variables; inter-variable interconnectedness and interdependence; self-regulation; reciprocal determination; process-like properties; mutual adaptation of implicated variables; and recursion (reviewed in Neufeld, 1999). This characterization retains currency in contemporary accounts (e.g., Gunnar & Quevedo, 2007; Monroe, 2008).

Such descriptors have beckoned the implementation of a quantitative systems approach to expressing the dynamics of stress-coping interactions (cf. Staddon, 1984). Accordingly, a six-variable (six-dimensional) nonlinear dynamical systems model, with eleven parameters, has been constructed to express the above tenets of these interactions (Neufeld, 1999). The model incorporates prominent theoretical constructs such as “primary appraisal of threat”, and “secondary appraisal of potential coping effectiveness” (Arnold, 1960; Lazarus, 1966; Lazarus & Folkman, 1984). It has, moreover been tendered as a theoretical lens through which to view stress-negotiation deficit in selected psychopathology (Neufeld, Boksman, George & Carter, 2010), thereby bearing on related issues in the stress-coping literature (Coyne & Racioppo, 2000).

Numerical analyses have shown the model to be capable of expressing self-regulation and the effects of varying levels of dimension interdependence. Self regulation, for example, occurs as follows. After a sudden increase in exogenous stressor level, system dimensions undergo a transitory damped oscillation, eventuating in their return to pre-dislodgement, stable-equilibrium values. Elevating dimensional interdependence, in turn, has taken the form of increasing feedback to the system of information comprising “secondary appraisal of coping efficacy” (thus instantiating increased system coupling; see, e.g., Enns & McGuire, 2000). Effects of greater coupling have included heightened system ability to withstand elevated exogenous-stressor levels without suffering a catastrophic breakdown (i.e., without encountering a mathematical singularity, or sudden infinitely positive, or negative dimension values). The latter may signify a collapsing of coping activity (Neufeld, 1982), not unlike that modeled in terms of catastrophe theory (Neufeld, 1989).

The identification of other potential linkages between dynamical systems and stress and coping, however, has awaited the advent of selected mathematical developments for dynamical systems analysis called competitive modes (Yao, Yu & Essex, 2002; Yao, Yu, Essex & Davison, 2006). These methods analytically assess the capability for extant systems to express rich, and arguably substantively significant dynamical behaviors. They become especially important when, as with the current model, system complexity resists such assessment through strictly numerical computer simulation techniques.

Furthermore, to the motivation for using competitive modes, Nicolis and Prigogine (1977) originally suggested that species competition within an ecological system would generate survival-enhancing bio-diversity. Haken (1983), in turn averred that such competition may result in complex system behavior such as chaos, and arguably transcends the biological domain. Rather, the complexity could attend generalized modes of any system’s interacting dimensions (order parameters), essentially entailing competition for limited resources, such as system energy.

Observe that construction of the present six-dimensional system (Neufeld, 1999) has been rooted in substantive theory of the addressed domain of study, without recourse to quantitative methods known to generate targeted dynamics, or architectures from other disciplines such as nonlinear physics or mathematical ecology (cf. Neufeld & Nicholson, 1991; Nicholson & Neufeld, 1992; Yao, et al, 2006; Yu, 2006). Because capturing important constructs of the addressed content domain has driven the system design, resulting dynamical properties should be substantively significant in their own right (May, 2004).

Here, we present newly identified, theoretically salient properties of the six-dimensional model. We begin by complementing the detailed exposition available in Neufeld (1999) with specifics required to study the model's previously unknown dynamical features.

MODEL SPECIFICS

In this section, we delineate the form of coping addressed by the six-dimensional theoretical system (Neufeld, 1999), followed by enumeration of its constituent dimensions. The differential equation comprising this dynamical model then is set forth; it consists of six time derivatives, each expressing the structure of change at time *t* of a component dimension. Exposition of model composition then introduces the presentation of its uncovered higher-level dynamics.

The equation addresses a prominent form of coping labelled "decisional control". This form of coping is defined and illustrated as follows. Originally described by Averill (1973) as corresponding with the number of alternatives available to an individual for negotiating stressful situations (cf. Thompson, 1981), decisional control has been defined in formal developments (Morrison, Neufeld & Lefebvre, 1988) as "positioning oneself in a multifaceted stressing situation so as to minimize the probability of an untoward physical or social event" (Lees & Neufeld, 1999). Decisional control is thus cognition intensive, in terms of marshalling of predictive judgments concerning degrees of threat identified with alternatives embedded in one's presenting stressor environment. Under socially tense circumstances, for instance, interpersonal encounters may be selectively undertaken so as to minimize the judged likelihood of a gauche interchange. In a setting of physical threat (e.g., risky work setting), decisional control would prescribe engagement of the alternative (e.g., occupational task) of estimated minimal danger or discomfort. Extensive mathematical dissection (Morrison, et al, 1988; Shanahan & Neufeld, 2010a; 2010b) and psychophysiological and behavioural investigation of this form of coping have supported the above characterization (e.g., Morrison, et al, 1988; Kukde & Neufeld, 1994; Paterson & Neufeld, 1995).

The system's first dimension $Y_1(t)$ conveys the prevailing level of exogenous stressor properties potentially subject to decisional control (a simplifying assumption). Implicated environments, for example may involve certain academic settings, those of evaluative social interaction, contexts of executive decision making, and arguably selected industrial or military circumstances.

The system's second dimension $Y_2(t)$ represents stress arousal levels. Stress arousal has subjective, behavioral and psychophysiological constituents (Blascovich, 2008; Ice & James, 2007; Monroe & Kelley, 1995; Neufeld, 1989), and $Y_2(t)$ serves as a summary expression of stress activation at time t (e.g., Neufeld & Davidson, 1974). The third dimension $Y_3(t)$ is one of cognitive efficiency, specifically as it impinges on predictive judgments of threat, pivotal to the exercise of decisional control (detailed in Shanahan & Neufeld, 2010a; 2010b). Dimension $Y_4(t)$ in turn, refers to the degree of decisional-control coping outlay, entailing the generation and appropriation of control-mediating predictive judgments.

Dimension $Y_5(t)$ is a dynamical weighting factor expressing the degree to which the stress-arousal apparatus is responsive to its sources of activation (elaborated upon, below). Last, $Y_6(t)$, again a dynamical weighting factor, corresponds to reactivity of coping-engagement $Y_4(t)$ to any shift in external stressor-property level $Y_1(t)$.

The system's time derivatives $dY_i(t)/dt$, $i = 1,2, \ldots, 6$, now are presented, followed by an exposition of each one in turn. They appear as follows:

$$
\begin{aligned}
&dY_1(t)/dt = a - bY_3(t)\, Y_4(t) - cY_1(t),\\
&dY_2(t)/dt = Y_5(t)\, [dY_1(t)/dt]\, [1 + Y_6(t)] - eY_2(t) - \{f\, Y_3(t)\, Y_4(t)\, Y_1(t) - g\},\\
&dY_3(t)/dt = h - iY_2(t), \qquad (1)\\
&dY_4(t)/dt = Y_6(t)\, [dY_1(t)/dt] - jY_4(t) + k\, \{f\, Y_3(t)\, Y_4(t)\, Y_1(t) - g\} + d,\\
&dY_5(t)/dt = 1.0 - Y_5(t)\, [Y_1(t) + Y_4(t) + 1],\\
&dY_6(t)/dt = 1.0 - Y_6(t)\, [Y_1(t) + 1].
\end{aligned}
$$

Proceeding term by term, stipulations of each time derivative's composition, and accompanying theoretical rationale; the non-negative control parameters a through k; and assumptions, are available in Eq. 1's inaugural presentation (Neufeld, 1999). The following abridgement comprises additional nuances of the system, with an eye toward explicating its currently identified formal dynamical properties, and their substantive significance.

In $dY_1(t)/dt$, the parameter a expresses the tendency for environmental stressor properties to accumulate when unchecked by countering influences (e.g., Staddon, 1984); b denotes the degree to which coping activity $Y_4(t)$, scaled by its efficiency $Y_3(t)$, reduces these properties; and c depicts the tendency for extrinsic environmental factors to attenuate an increase in the level of stressor properties when it is relatively higher, and the opposite when it is lower.

Turning to $dY_2(t)/d$, recall that $Y_5(t)$ regulates the impact of agents effecting stress activation. Note that these agents are deemed to include not only movement in the level of external stressor properties $Y_1(t)$, but also that in the amount of “cognitive work” expended to produce decisional-control-mediating predictive judgments $Y_4(t)$ (e.g., Shanahan & Neufeld, 2010a; 2010b; Solomon, Holmes & McCaul, 1980; Wenger & Townsend, 2000). The insertion of the dynamical weighting factor $Y_6(t)$ into $dY_2(t)/dt$ selectively brings into play the $Y_4(t)$-related sources of stress activation. It does so inasmuch as stress-altering attributes of $Y_4(t)$ are ascribable mainly to that portion of its change yoked to stressor-property level $Y_1(t)$ (Neufeld, 1990; 1999); and, second, because coping activity $Y_4(t)$ is responsive to fluctuation in $Y_1(t)$ specifically as determined by the dynamical weighting factor $Y_6(t)$ [as seen in Eq. 1's $dY_4(t)/dt$, above].

Overall, then, the structure of this dimension's time derivative is such that $Y_2(t)$ tracks changes in $Y_1(t)$; it also tracks changes in $Y_4(t)$, expressly as the latter are forced by those of $Y_1(t)$. Altogether, this contribution of alteration in $Y_1(t)$ to that of $Y_2(t)$ is implemented by importing $dY_1(t)/dt$ directly into $dY_2(t)/dt$, and apropos of $Y_4(t)$, additionally weighting $dY_1(t)/dt$ by $Y_6(t)$.

The concept of cognitive appraisal (Arnold, 1960; Lazarus, 1966; Paterson & Neufeld, 1987) enters Eq. 1 as the expression in braces $fY_3(t)Y_4(t)Y_1(t) - g$ (qualified below). The subjective salience of coping effectiveness, quantified as the efficiency of cognitive operations bearing on decisional control $Y_3(t)$, is enhanced according to existing coping outlay $Y_4(t)$. This product, in turn, is thrown into relief based on the level of stressor properties $Y_1(t)$ subjected to $Y_3(t)Y_4(t)$. The multiplicative term $Y_3(t)Y_4(t)Y_1(t)$ parallels certain operations in classical mathematical ecology, where prey-consumption *rate* directly depends on the density of prey to which ongoing predation is applied (e.g., Roughgarden, 1979). The impact of this ternary product on the system is filtered through the appraisal mechanism, brought into play here by the parameter f. The assessed return on coping expenditure $fY_3(t)Y_4(t)Y_1(t)$ is contrasted to a subjective standard g, altogether reducing stress level $Y_2(t)$ when the appraisal is more favorable, and the opposite.

Remaining momentarily with this time derivative, we draw out connections with the concepts, primary appraisal of threat, and secondary appraisal of coping resources (Lazarus, 1966; Lazarus & Folkman, 1984). Primary appraisal is positioned in $dY_2(t)/dt$'s right-hand side according to $dY_1(t)/dt$. Secondary appraisal is brought into play as the expression in braces, whose $Y(t)$ terms implicitly bespeak provision for plasticity of dynamical updating (so-called reappraisal). The dynamical interplay of variables making for ongoing cognitive appraisal is in keeping with the declared process nature of this construct, and the network in which it operates.

Cognitive appraisal transparently permeates this highly-coupled system. Through $Y_2(t)$, there are concurrent effects on cognitive efficiency (i.e., the right-hand side of $dY_3(t)/dt$, below), which then ramify via $Y_3(t)$ to $dY_1(t)/dt$ and $dY_4(t)/dt$. In turn, $Y_1(t)$ is implemented in the time derivatives of four other dimensions in Eq. 1, and $Y_4(t)$ is integrated into those of three other dimensions. The equation thereby rigorously stipulates reverberations of covert appraisal, intractable by verbal reasoning alone, by embedding them within a decidedly formal system (e.g., Braithwaite, 1968; Staddon, 1984).

The third time derivative $dY_3(t)/dt$ states that cognitive efficiency impinging on decisional-control is adversely affected by stress arousal $Y_2(t)$ (e.g., Fisher, 1986; Neufeld, 1994). In this way, stress stands to undermine its own resolution (Neufeld & McCarty, 1994; Shanahan & Neufeld, 2010a). The parameter i reflects individual susceptibility to stress effects on decisional-control related cognitive functioning (e.g., Eysenck, 1989); the parameter h, on the other hand, indicates the counter-tendency toward steady improvement, possibly through practice (e.g., Neufeld, 2007). Whatever the source, h expresses individual immunity to the toll on functioning exacted by $Y_2(t)$.

It is possible that under selected conditions, stress activation can improve cognitive performance (e.g., Anderson, 1990; Neiss, 1990), and moreover has been shown to do so (e.g., Neufeld & McCarty, 1994; Neufeld, Townsend & Jetté, 2007). As with restriction to decisional-control amenable stressor properties $Y_1(t)$, above, the present developments are restricted to the realm of stress-induced performance decline (e.g., Fisher, 1986; Neufeld 1994).

Time derivative dY_4/dt defines current alterations in the level of decisional-control coping activity. Responsiveness to changes in stressor level $dY_1(t)/dt$ depends on the dynamical weighting factor $Y_6(t)$. The product $jY_4(t)$ is a self-regulating mechanism analogous to $cY_1(t)$ of the first time derivative. The expression in braces, weighted by k, invokes coping-(dis)incentive properties of cognitive appraisal of coping efficacy. Relatively higher values spur coping activity. Finally, the parameter d represents the infusion of exogenous factors exerting upward movement in coping outlay (elaborated upon, below).

The parameter k in effect governs feedback to the system of the latter's self-generated informational ensemble concerning its own functioning. The parameter d puts into place steady exogenous influences facilitating coping outlay. Such influences, for example, can take the form of social support (Field & Schuldberg, 2011; Levy, 2005).

The fifth time derivative $d_5(t)/dt$, found in $dY_2(t)/dt$ (above), institutes time-varying sensitivity of the stress-activation mechanism to variation in exogenous stressor level along with stressing aspects of coping expenditure. The first value of 1.0 represents a time-derivative constant conveying a certain resistance to sensitivity-reducing input. Here, $Y_5(t)$ decreases if and only if the adjacent negatively-valenced right-hand expression exceeds 1.0. The negatively-valenced expression includes the extant value of $Y_5(t)$, multiplied against stress-activation sources $Y_1(t)$, and $Y_4(t)$. The value of 1.0 inside the square brackets instantiates a self-regulation component, resembling $cY_1(t)$ and $jY_4(t)$, above.

Recall that the representation of $Y_4(t)$ in $dY_2(t)/dt$ was restricted to that portion of the former driven by exogenous stressor level. The dimension $Y_5(t)$, however, is affected by elevation in $Y_4(t)$ *in toto*. In this way, sensitivity of the stress-activation mechanism is "cognizant" that the degree to which elevation in $Y_1(t)$ can recruit additional decisional-control resources is limited by the overall level of existing commitment, regardless of origins.

Note that the 1.0's appearing in this time derivative are arbitrary, reducing model parameterization (see Bamber & van Santen, 2000). These values, however, could be replaced with adjustable parameters; $d_5(t)/dt$ can be viewed as entailing such parameters, currently set to 1.0 (Neufeld, 1999).

The composition of $dY_6(t)/dt$ resembles that of $dY_5(t)/dt$, and like considerations apply. In a certain sense, this time derivative embellishes the "memory" aspect of the proposed system: the dimension adjusts its own variation selectively, with respect to the domain of $dY_4(t)/dt$ on which it bears, specifically that of $Y_1(t)$.

Through the structure of its time differentials, this six-dimensional model integrates prominent constructs from the topic domain. Its architecture provides for person-environment interchange, and includes order parameters of stress arousal and coping propensity (Lees & Neufeld, 1999), which together incorporate the involvement of theoretically prominent forms of cognitive appraisal. The system allows for a certain amount of self-awareness of its own workings. It also allows for a structural element of "memory" (in addition to the usual element of memory intrinsic to a system whose time derivatives are functions of their own history). Provision is made for interaction between stressor negotiation and cognitive efficiency, including the undermining by stress of its own cognitively-mediated resolution. The system also admits input from potentially stabilizing agents such as social facilitation of coping activity.

The model arguably is comprehensive, if necessarily complex. Such complexity, however, additionally may be requisite to the emergence of substantively significant dynamical properties, an issue deferred pending detailed consideration of the properties themselves.

FIXED POINT EQUILIBRIA

An important ingredient to deciphering dynamics of the present system is that of pinpointing the nature of its fixed-point equilibria. The system's fixed-point equilibria occur when the values for terms on the right-hand side of the time derivatives sum to zero, thereby expressing a stationary state of the system. Such a state, however, may not qualify as a genuine homeostasis, if it is demonstrably unstable. An unstable fixed-point configuration is one susceptible to dislodgement by slight perturbation from exogenous influences, possibly leading to either a more stable fixed point, non-stationary behavior, or unbounded values. The latter is described above with respect to variation in system coupling.

We now identify the fixed-points of the Eq. 1 system, and examine the sustainability, from a substantive perspective, of positions taken up by the respective dimensions. The substantive inferences then are tested analytically, through characteristic-polynomial-and eigenvalue, single-point bifurcation and stability analysis (Borelli & Coleman, 1998; Guckenheimer & Holmes, 1983; see also Appendix of Field & Schuldberg, 2011).

Despite the complexity of the Eq. 1 system, its structure nevertheless renders tractability of its fixed points, identified in terms of the specified control parameters. Setting the time derivatives of Eq. 1 to 0, and solving simultaneously, the fixed-point solutions, denoted Y^*_i, are found to be:

$$Y_1^* = \frac{a}{2c}\left[1 \pm \sqrt{1 - \frac{4bc(ig - eh)}{a^2 if}}\right],$$

$$Y_2^* = \frac{h}{i},$$

$$Y_3^* = \frac{j(ig - eh)}{f\frac{a}{2c}\left[1 \pm \sqrt{1 - \frac{4bc(ig - eh)}{a^2 if}}\right](id - keh)}, \quad (2)$$

$$Y_4^* = \frac{id - keh}{ij},$$

$$Y_5^* = \frac{1}{1 + \frac{a}{2c}\left[1 \pm \sqrt{1 - \frac{4bc(ig - eh)}{a^2 if}}\right] + \frac{id - keh}{ij}},$$

$$Y_6^* = \frac{1}{1 + \frac{a}{2c}\left[1 \pm \sqrt{1 - \frac{4bc(ig - eh)}{a^2 if}}\right]}.$$

Equation 2 discloses fixed-point existence if and only if its radicand equals or exceeds 0.0. If greater than 0.0, there are two sets of fixed points. It is apparent, in turn, that for any possibility of a prevailing low stressor level, denoted by a negative region for Y_1*, the radicand must be greater than 1.0, implying $eh > ig$. That is, the product between the parameter *e*, which acts to diminish Eq. 1's stress activation, and the parameter *h*, which acts to increase individual cognitive efficiency, must surpass the product between the parameter *i*, involved in decreasing cognitive efficiency, and *g* whose higher values in Eq. 1's expression in braces adversely affects cognitive appraisal of coping efficacy.

For stressor level $Y^*_1 < 0$ now to be accompanied by concurrently active, efficient coping Y_4*, $Y_3^* > 0$ (altogether a seemingly sustainable configuration), *keh* must be less than *id*. This necessity is readily apparent in Y_4*. In Y_3*, the square-bracketed term multiplied by $a/(2c)$ equals Y_1*. Overall, with $ig < eh$ and $Y_1^* < 0$, $Y_3^* > 0$ requires $keh < id$. In other words degree of feedback *k* of the organism's functioning with respect to resolution of environmental stressors – the expression in braces in Eq. 1's $dY_4(t)/d(t)$ – must be modulated both according to exogenous or environmental (exemplified under System Re-stabilization, and Model Evaluation**,** below) input *d* (as scaled by *i*), and *eh*. From the standpoint of apparent steady-state robustness, *k* must be less than *id*/(*eh*).

As systemic stress vulnerability increases, as expressed by elevation in *i*,which denotes susceptibility of decisional-control related cognitive functioning to the effects of stress arousal $Y_2(t)$, reduction in *e* (identified with self-regulation of stress arousal, in part attenuating higher levels of $Y_2(t)$) , reduction in *h* (expressing individual resistance to the toll on cognitive functioning taken by stress arousal), or some combination of the above, so evidently does tolerance of elevation in *k*. The non-destabilizing elevation in *k* indicates that in the face of (compromised) coping efficacy, the system accommodates increased monitoring of returns on coping engagement. Latitude for feedback on itself of the system's internally-generated information also increases if met with strengthened exogenous influences *d*.

The second set of fixed points occurring to $eh > ig$ and $k < id/(eh)$ retains $Y_4^* > 0$, but addition of the radical now renders $Y^*_1 > 0$ and $Y_3^* < 0$. The substantive picture becomes one of more tenuous stability, inasmuch as a relatively higher level of prevailing stress is accompanied by relatively less efficiency of extant coping outlay. Whereas the first fixed-point constellation indicates dynamic stability, the second indicates instability. Such a configuration would be expressed analytically as a *higher-dimensional saddle node bifurcation* (e.g., Guckenheimer & Holmes, 1983).

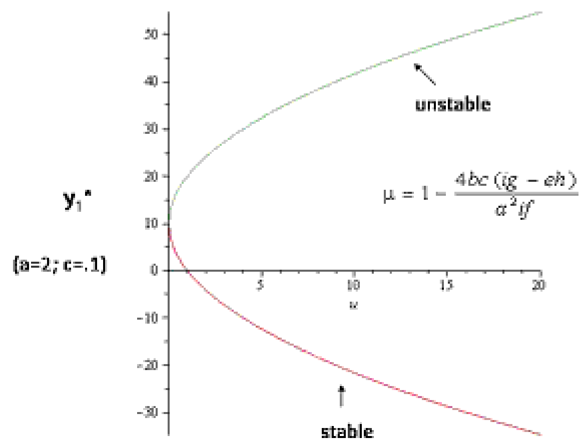

Fig. 1. Saddle-node bifurcation expressed on Y_1*, as the radicand of Eq. 2 increases from 0.

Accordingly, the two equilibria of Eq. 2 were studied within parameter-value constraints $k < id/(eh)$. Characteristic-polynomial-and-eigenvalue, fixed-point bifurcation and stability analysis demonstrated the following stability properties. In keeping with their contrasting predicted tenability, the first set of fixed points was observed to be stable, and the second, unstable. To illustrate, for parameter values $a = 2$, $b = c = .1$, $d = e = f = h = i = j = 1.0$, $g = 0.15$,and $k = 0.5$ (Neufeld, 1999), the pair of fixed-point sets are $Y_1^* = -0.042410$, 20.042;

Y_2* = 1, 1; Y_3* = 40.085, -0.084820; Y_4* = 0.5, 0.5; Y_5* = 0.68606, 0.046420; and Y_6* = 1.0443, 0.047523.[1] Eigenvalues for the first set are -1.4576, -1.0000, -0.95759, -.025010, and -3.0555 ± 5.5513*I*. By accepted criteria, as all real parts are negative, this is a stable-equilibrium set of fixed-points. Eigenvalues for the second set are -21.542, -21.042, -4.2230, -1.0000, -.10019 and 2.3736. As one of the eigenvalues is positive, the second fixed-point set is unstable. Taken together, these observations bear out analytically the qualitatively inferred differences in tenability of the system's pair of fixed-point equilibria for for $k < id/(eh)$. The higher-dimensional saddle-node bifurcation is depicted in Fig. 1.

Stability properties are further illustrated for an additional set of parameter values, with $k<id/(eh)$, in Fig. 2a. The figure presents time-series (integral) curves for $Y_1(t)$ and $Y_5(t)$. Initial conditions are located on, or close to the unstable fixed points, with the dimensions eventually veering toward their stable-equilibrium positions.

Figure 2a illustrates system behavior in a saddle-node parameter region, $k < id/(eh)$, illustrated for $Y_1(t)$ [prevailing stressor level], and Y_5(t) [responsiveness of stress-arousal system to its activation sources]. Integral curves to the right of t = 10,000 take on their stable fixed-point values. Initial conditions for the system's six dimensions approximate their unstable fixed-point values. Parameter values: $a = b = c =d = e =g$ =.10000, f = .20000, h = 2.1000, i = 1.1000, j = 2.4850, k = 0 .5; initial conditions for $Y_1(t)$ through $Y_6(t)$ are 1.3394, 1.9091, -185.00, 0.0018000, 0.42700, and 0.42746. Figure 2b illustrates limit cycle behavior illustrated for $Y_1(t)$ and $Y_4(t)$ [decisional-control coping activity], for t = 1,000 to 2,000, following de-stabilization, $k > id/(eh)$. Parameter values as above, except k = 1.1440; initial conditions for $Y_1(t)$ through $Y_6(t)$ are 1.0000, 2.0000, 0, -4.0000, 0, and 0.

System Destabilization

Inspection of Eq. 2 reveals dubious sustainability of both fixed-point sets where $k > id/(eh)$; ig remaining less than eh. Accompanied by Y_4 * < 0, subtraction of the radical yields Y_1*, Y_3* < 0, and its addition yields Y_1*, Y_3* > 0. Relatively lower Y_1* in the first instance takes place amidst comparatively inefficient coping Y_3*. In the second instance, an elevated Y_1* co-occurs with increased coping efficiency Y_3*, but again in the company of "conservative" coping investment Y_4*.

The expected instability of these fixed-point departures from the stable equilibrium configuration Y_1* < 0< Y_3*, Y_4*, above, was obtained for both fixed points, according to the stated eigenvalue criteria. With the loss of fixed-point stability, the system gives way to limit-cycle oscillations, as instantiated for parameter values of Fig. 2b, and illustrated for dimensions $Y_1(t)$ and $Y_4(t)$.

The limit-cycle pattern conveys a prototypical interplay between stressor level and amount of stressor-directed coping, not unlike selected predator-prey dynamics in mathematical ecology (e.g., Neufeld & Nicholson, 1991). Intercepting the loop at the upper right portion, and proceeding counter-

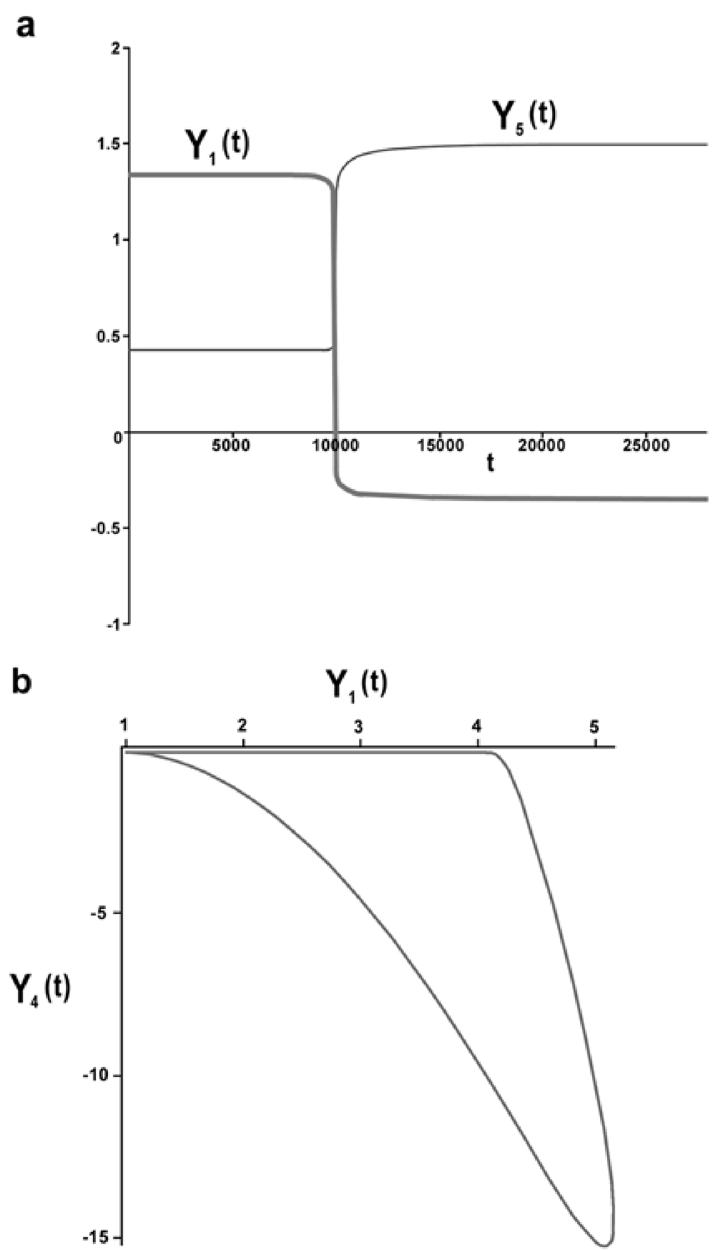

Fig. 2. De-stabilization of Eq. 1 system**.**

clockwise, relatively heightened coping expenditure issues in reduced stressor level (upper left), whereupon coping subsides, with consequential stressor accumulation, to the point of activating a rise in coping deployment (lower right), followed by a return to the upper right portion.

Amidst the higher-dimensional workings of the present system, with $k > id/(eh)$, this distinctive reciprocality between the present pair of dimensions nevertheless comes through. It is intrinsic, moreover, to a system that springs expressly from the invoked stress-coping subject matter, unaided by insertion of previously known periodicity-inducing mechanisms, or external forcing (cf. Field & Schuldberg, 2011; Neufeld & Nicholson, 1991; Savic & Jelic, 2005; Sprott, 2005).

Chaotic Behavior

Discerning the existence of chaotic and periodic (limit cycle, quasi-periodic and multiple-limit-cycle) behavior of the Eq. 1 system has challenged numerical analysis. The potential for chaotic behavior, however, is disclosed through recent advances in dynamical systems theory and methodology, known as *competitive modes* (Yao et al, 2002; 2006). Competitive modes theory attributes complex behavior of a dynamical system to the competition among some system dimensions in dominating system behavior ("competitive modes," stipulated formally in Yao, 2002; 2006; Yu, 2006). It has been shown that a dynamic system exhibits a form of complexity such as chaos only if the system exists at a minimum of two competitive modes. Currently, for any system described by ordinary differential equations, the forms of competitive modes can be analytically derived, and further expressed by the system parameters. The potential chaotic parameter regions are thus determined by letting at least one pair of competitive modes be equal. We determined that competitive modes were present in the Eq. 1 system (entailing dimensions 3, 4, and 5), and identified their parameter regions.

Competitive modes nevertheless can exploit other analyses in charting system attributes, as follows. Configurations of chaos-compatible system parameters disclosed by competitive-modes technology can be scrutinized in conjunct-tion with those prescribing fixed-point stability, thereby disclosing candidate control-parameters, governing system bifurcations. In turn, constellations of order parameters evincing stable and unstable fixed-point values can be consulted to decipher system structure issuing in fixed-point homeostasis.

Using this system-assessment technology, Eq. 1 has been shown to produce requisite competitive-modes for many parameter groups (Yao, et al, 2006; Yu, 2006). One such group is that for the dimension trajectories shown in Fig. 3, but with k (the degree to which the system feeds back to itself its appraisal of coping efficacy at time t) now being increased to the neighborhood of 5.0000.

Figure 3 illustrates the chaotic behavior of the Eq. 1 System. In Fig. 3a, chaotic behavior is expressed on $Y_1(t)$ (prevailing stressor level), $Y_2(t)$ (stress-arousal level], and $Y_3(t)$ [decisional-control related cognitive efficiency]. In Fig.

3b, chaotic behavior is expressed on $Y_4(t)$ (decisional-control coping activity), $Y_5(t)$ [responsiveness of stress-arousal system to its activation sources], and $Y_6(t)$ (propensity to mobilize coping activity in response to changes in $Y_1(t)$). Parameter values a through j are identical to those of Fig. 2; k =5.0000. Initial conditions are identical to those of Fig. 2b. Figure 4, is a power spectrum graph showing relative signal strength vs. signal frequency.

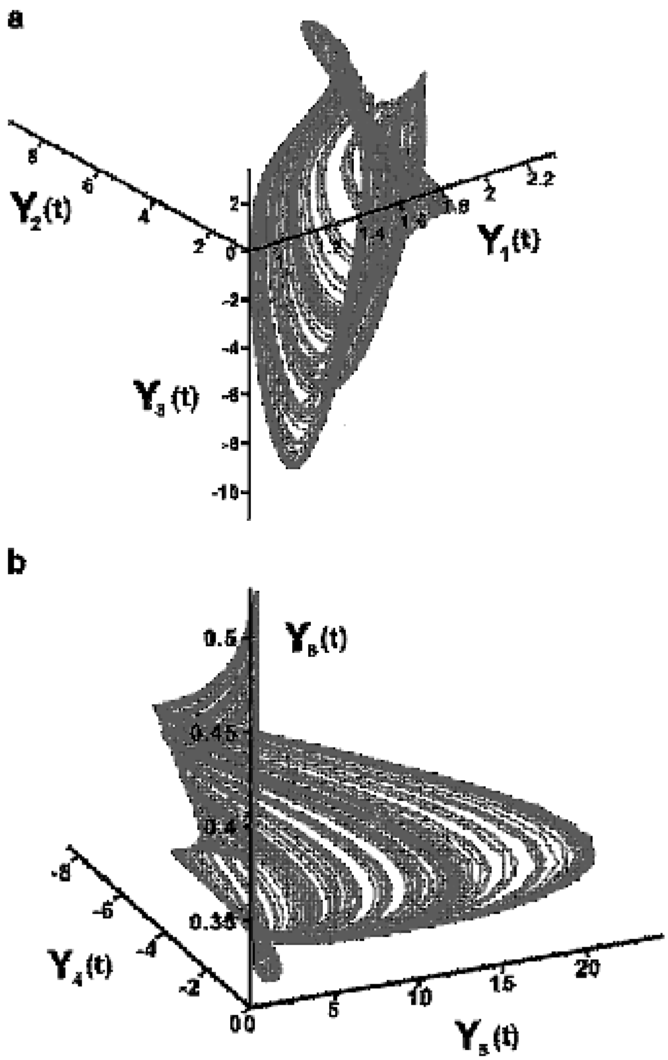

Fig. 3. Chaotic behavior of Eq. 1 system**.**

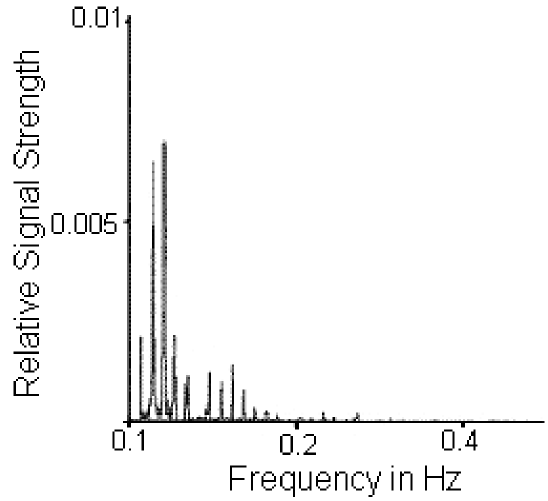

Fig. 4. Power spectrum graph showing relative signal strength vs. signal frequency.

System behavior progresses to chaos as *k* (system sensitivity to its self-generated signals surrounding coping efficacy) rises beyond 0.52381 = *id*/(*eh*), there being a series of single, quasi-periodic and multiple-limit-cycle bifurcations along the way. A Feigenbaum period doubling (see, e.g., Enns & McGuire, 2000) eventuates in chaos at $k \cong 4.55$. The system's chaotic attractor at $k = 5.0000$ is displayed for its complement of 6 dimensions in Fig. 3a and 3b. The associated broad-band power-spectrum graph, attendant to chaos (e.g., Sprott & Rowlands, 1995), is presented in Fig. 4.[2]

One characteristic of chaos is its continuous power spectrum. Periodic systems have only finite peaks in the spectrum. Weak chaotic systems have high finite peaks and also low continuous regions in their spectra. The chaos found in the model defensibly belongs to the domain of weak chaos, potentially more typical of stress-and-coping phenomena, as individuals' behavior in this realm arguably should not be totally chaotic.

Different from their limit-cycle behavior, system trajectories now only approximate, rather than duplicate their previous values. Moreover, dimensional workings are such that outlay of cognition-intensive coping $Y_4(t)$ never aligns with coping-potentiating cognitive efficiency $Y_3(t)$ so as to fully subdue stressor level $Y_1(t)$ to a point of mutual stability, or even predictability. This outcome evinces an ongoing but non-repetitive stress-coping kinesis, incorporating chaotically changing effects of the organism's responses on environmental properties themselves. As with oscillatory behavior, following a substantively meaningful shift in its control parameters, chaotic behavior now spontaneously arises from the system whose formal structure arises expressly from stress-coping relationships.

All in all, an individual with parameter configuration $k < id/(eh)$ can be expected to manifest stable, fixed-point equilibrium stress-coping behavior. An individual whose value of k exceeds $id/(eh)$ can be expected to show cyclical, waxing and waning of stress-coping patterns, depending on the degree of this inequality. If k sufficiently exceeds $id/(eh)$, chaotic stress-coping behavior should ensue. System behavior can be conjectured as being deflected away from environmental transactions, in favor of internally-generated signals; harnessing $Y_3(t)$ and $Y_4(t)$ so as to contain $Y_1(t)$ ostensibly becomes elusive, rendering the defined chaotic trajectory.

System Re-stabilization

Distinguishing the configuration of system parameters that produce equilibrium stability can potentiate a certain control over system behavior (Yu, 2006). Reinstating a stability-producing parameter configuration arguably can reverse undesired consequences of destabilization, including those occurring to chaotic behaviors (Yu, 2006). One such consequence entails an untoward behavioral-affective condition, known as “maladaptive over-determinism” (Heiby, Pagano, Blaine, Nelson & Heath, 2003). This condition comprises an elevated influence by internally-generated signals, over and against environmental signals, on patterns of affective experience. The over-determination by endogenous sources is deemed to undermine adaptive spontaneity in responding emotionally to environmental events.

Maladaptive over-determinism has been inferred from selected properties of extensive mood-diary time-series data, obtained from an individual with unipolar affective disorder, and from a control participant. Time-series analysis accommodating ordinal-data properties, known as Monotonic Multiscale Entropy (MMSE; Costa, Goldberger & Peng, 2002; Costa, Peng, Goldberger, & Hausdorff, 2003; Heath, Heiby & Pagano, 2007), was applied to each set of data. Relative to those obtained from control data, MMSE values obtained from patient data were found to be closer to MMSE values generated by a determine-istic-chaos benchmark (chaotic Henon System), and further from values generated by a stochastic-signal benchmark (Gaussian-distribution generated). The Eq.1 system presents a parametric parallel to maladaptive over-determinism, as follows. Self-generated signals in Eq. 1 are identified with its cognitive-appraisal term, in braces. Elevated influence of these signals occurs as k goes up. Chaotic behavior, in turn, ensues as k increasingly exceeds $id/(eh)$.

Turning to system-property control, forestalling fixed-point destabilization appears achievable by increasing the range or variability of k where $k < id/(eh)$. An instance of doing so takes the form of increasing the magnitude of d , the steadying environmental input to $dY_4(t)/d(t)$ of Eq. 1.

Where extant $k > id/(eh)$, restoring stability can assume one or a combination of three routes: increasing systemic stress-vulnerability specifically by increasing Eq. 1’s parameter i, and/or decreasing either or both of its parameters *e* and *h* (see this section, above); decreasing *k*; and, third, increasing *d*. Attenuation of *k* may be achieved through psychological intervention, such as cognitive behavior therapy targeting re-deployment of attentional resources to happenings in the environment. Elevating *d* as a stabilizing agent, possibly through raising perceived social support for actively engaging stressor level, nevertheless may be feasible. However accomplished, reinstating a stable stress-coping equilibrium, by returning the system to a region of fixed-point stability, is analogous to eliminating vibration (limit cycle) or turbulence (chaos) of a physical mechanism (Yu, 2006).

MODEL EVALUATION

The importance to their tenability of grounding dynamical models in real-world observations at the outset, has been underscored by May (2004). Doing so appears fundamental to ascertaining potentially meaningful system attractors (fixed-point, periodic, and chaotic). Although the production of chaos, for example, with randomly generated networks increases with system dimensionality (possibly because of an escalating chance of competitive-mode criteria among two or more dimensions; Yao et al., 2006), it nevertheless remains below 2% even with dimensionality up to 100 (Bagley & Glass, 1996).

The Eq. 1 system appears internally consistent in terms of tenability of attractors, parametric structure of fixed-point stability, and coherence between imputed meaning of parameters, and their effects on system behavior. Empirical testing, in turn, can adopt two main strategies in the present case. One entails model predictions of empirical trajectories of system dimensions, as they are repeatedly measured across time. This strategy assesses correspondence of empirical paths of system dimensions with their theoretical trajectories (their numerically solved integral curves, following parameter estimation). The second strategy examines numerical diagnostics (e.g., correlation dimension; estimated entropy) computed from empirically monitored model dimensions, as set against those emanating from the model's numerically solved integral curves.

Pursuant to the first strategy, Jette (2003) studied diary data obtained from 17 female and 3 male students, enrolled in Psychology graduate studies at the University of Western Ontario (mean age of females = 24.82, *S. D.* = 2.24; mean age of males = 29.33, *S. D.* = 3.05).[3] Graduate students were recruited because they were considered more likely than others to remain fastidious in completing the daily diaries over the course of the study.

Participant responses took place at approximately the same time daily (before retiring) for 60 days. A four-part structured inventory was designed to track the status of the six respective system dimensions, each as subjectively summarized over the preceding twenty-four hours (see, e.g., Ekenrode & Bolger, 1995). For example, the first dimension $Y_1(t)$, exogenous stressor level, was measured using the Daily Stress Inventory (DSI; Brantley, Waggoner, Jones & Rappaport, 1987). This measure covers a wide range of stressing events, has a format suitable for daily measurement, and possesses desirable psychometric properties (e.g., coefficients alpha for subscales entering into a composite estimate of the first dimension are .83 and .87 -- for "Event Frequency" and "Summed Event Impact", respectively). The second dimension $Y_2(t)$, stress arousal, was measured using the short version of the Stress Arousal Checklist (SACL; King, Burrows & Stanley, 1983), with instructions adapted to the current application. The Canadian-normed version of the SACL consists of 18 adjectives, rated on a 4-point scale, describing various states of activation (e.g., uneasy, energetic). Empirical evidence supporting the SACL's construct validity has been obtained from diverse sources, such a settings of physical activity and competitive sports (e.g., Kerr & Van DenWollenberg, 1997), and those involving groups deemed to diverge in levels of "stress and arousal" (King et al., 1983). At the outset, participants were familiarized with their responsibilities should they choose to participate. Each was paid one hundred dollars Cdn. after taking part.

Model assessment included the following three methods. The first consisted of significance testing of correlations computed across time, between diary-monitored values of system dimensions and their model predictions ("cross correlations"; e.g., Holtzman, 1963). The second comprised dissection of departures between profiles of modeled and empirical trajectories, with respect to profile elevation, or mean; scatter, or across-time data variability; and shape, or across-time contour (Casdagli, 1989; Levine & Lodwick, 1992; Pindyck & Rubinfeld, 1981; Sterman, 1984). And the third evaluated model predictions of empirical observations, as set against those emanating from the observations themselves, but after they had been randomly scrambled (i.e., creating "surrogate data"; see, e.g., Schiff & Chang, 1994; Schuldberg & Gottleib, 2002; Theiler, Galdrikian, Longtin, Eubank, Eubank & Farmer, 1992).

Before testing statistical significance of cross correlations, degrees of freedom were adjusted for missing observations, which amounted to 1.9% of the 7,200 measurement points; replacement estimates comprised the means of the pair of data flanking a missing value (see, e.g., Kreindler & Lumsden, 2007). Adjustment also was made for auto-correlation (Hotzman, 1963), and for estimation of system parameters. The cost function to be minimized for purposes of parameter estimation was composed of the squared deviations of model predictions from empirical observations, summed across measurement times for all 6 system dimensions (Levine & Lodwick, 1999). The numerical search was implemented via the MATLAB Constrained Optimization Algorithm (Mathworks, Inc., 1984-1999). Parameter values were constrained to predictions of bounded (finite) dimension trajectories over the measurement time course (e.g., Boyce & DiPrima, 1977), and were not restricted to the positive domain.

Significant correspondence between model and empirical time series occurred for 16 of the 20 participants (binomial probability of 16 or more significant results at per-test α of .05 approaches 0). The average cross-correlation was .235 across participants for the first 4 dimensions, and .158 with inclusion of the more subjectively opaque, dynamical weighting dimensions $Y_5(t)$ and $Y_6(t)$. The modest if significant magnitude of these cross correlations should be considered with in light of the specifics of their computation. The empirical data bore a full representation of variance sources; to provide a currently balanced picture of model performance, model-exogenous noise was bona-fidely registered in empirical data, as opposed to being controlled through initial aggregation, or preliminary filtering (see, e.g., Carter, Neufeld & Benn, 1998; Neufeld & Gardner, 1990). Initial values were fixed at 1.0 (Neufeld, 1999), and model comprehensiveness was not sacrificed to computational convenience through arbitrary simplification. Results for 10 participants with the higher significantly fitting values (averaging .266, all dimensions considered) were further evaluated for possible inflationary artifact (e.g., spurious enhancement of model fit, such as that due to data-based parameter estimation, degrees of freedom penalization, above, notwithstanding). Model predictions were found to significantly outperform those of surrogate data in 9 of the 10 cases (binomial probability for test-wise α of .05 again approaches 0). As expected when throwing into relief the intact-data and model-prediction profile properties of shape, similar findings for surrogate-data comparisons were obtained.

Pursuant to the second strategy, above, Levy (2005) monitored variation in system dimensions among an additional 5 male and 14 female students enrolled in Psychology graduate studies at the University of Western Ontario (mean age = 24.5, *S.D.* = 2.0; and mean age = 24.0, *S.D.*. = 3.0, respectively). Aided by the use of personal digital assistants (Bolger, Davis & Rafaeli, 2003), recording entailed electronic Ecological Momentary Assessment (ESM), directed to the “current status” of the six respective system dimensions. For example, measurement of the first dimension $Y_1(t)$ again implemented the DSI (Brantley et al., 1987), adapted to the ESM methodology. The second dimension $Y_2(t)$ was monitored with the the SACL (King et al., 1983), also adapted to ESM recording methodology. Based on developments concerning field-assessment of cognitive-efficiency (Broadbent, Cooper, Fitzgerald, & Parkes, 1982; Klumb, 1995; Reason, 1988), Dimension three $Y_3(t)$ was assessed by asking participants to indicate on a scale ranging from 1 (extremely low) to 5 (extremely high) how effectively they were able to use decisional control to manage external stress. Participants first attended an information and training session, in which they were familiarized with the ESM-posed questions, and the personal digital assistant instrumentation. Once more, each was paid 100 dollars Cdn. for taking part.

Missing observations amounted to .168% if the 13,680 measurement points; replacement estimates were obtained as in Jette (2003), above. The correlation dimension, an estimate of nonlinear system complexity (Farmer, Ott & Yorke, 1983; Molnar, Gacs, Ujvari, Skinner & Karmos, 1997) was computed for the Eq. 1 system, using a simulative chaos benchmark (Levy, 2000). The computed value, averaged across the 6 dimensions, was 1.407. Similarly indicative of nonlinear system complexity, the value computed from the empirical time series, averaged over the 19 participants, was 0.8000.

Like findings were obtained when quantifying system entropy, or the degree to which extant system information is limited in its prediction of subsequent system behavior (Krantz & Schreiber, 1997; Prigogine, 1980). The Kolmogorov Entropy value computed from the simulative chaos benchmark was

0.175, as averaged across the 6 dimensions. Monotonic Multiscale Entropy, an estimate providing for ordinal properties of empirical data (Costa, et al., 2002; 2003; Heath, et al, 2007), was computed dimension wise for each dimension and each participant. As with Kolmogorov Entropy, above, the obtained value of 0.264, averaged over the 6 dimensions and across the 19 participants, was coherent with the presence of nonlinear complexity.

Observe that the present empirical evaluation was stringently demanding throughout. Rather than remaining non-committal as to specifics of system-architecture, and probing for numerical signatures general to nonlinear complexity, these assessments were driven expressly by Eq. 1.

Furthermore, to empirical predictions, the model forecasts that increased social support, or other exogenous agents of elevated coping outlay (d of Eqs. 1 and 2), will have favorable effects on system stability. Such stability should be expressed as sustained equilibrium of system dimensions. Other empirical implications comprise associations of system time-series properties (Schuldberg & Gottlieb, 2002) and parameter values (Jetté, 2003) with psychometrically-measured personality and other individual-difference variables (Sprott, 2005). Such would be the case for parameter d and perceived social support (Procidano & Heller, 1983), and for parameter h of Eq. 1's $dY_3(t)/dt$, and stress-susceptibility (Endler, Edwards & Vitelli, 1991).

Evaluation of psychological nonlinear dynamical systems models comprises an ongoing enterprise with progressive model refinement (Guastello, Koopmans & Pincus, 2009). Current challenges facing the study of nonlinear dynamics in psychology, psychological stress and coping included, resemble those met elsewhere: continuing development of empirical-assessment strategies conjoint with the construction of principled models to be tested (Heathcote, 2002). Part and parcel of the latter entails mathematical analysis disclosing substantively-significant combinations of system dynamics, and explanatory configurations of control parameters.

DISCUSSION

Comprehensiveness in modeling nonlinear dynamics of psychological stress and coping demands higher than the usual system dimensionality and parameterization. As with simpler and lower-dimensional physical and biological systems or large generic networks (Bagley & Glass, 1996; Enns & McGuire, 2000; Funahashi & Nakamura, 1993; Sompolinsky, Cristanti & Sommers, 1988; Sprott, 2008), a rich array of complex dynamical behaviors nevertheless are identified.

Numerical explorations indicate that the latter hinge on Eq. 1's comprehensive, if necessarily complex structure. Forfeiture of the current dynamical-complexity arsenal (i.e., one, or a combination of chaotic, oscillatory and even bounded behaviors) included the following deviations from Eq. 1: elimination of its parameter g; elimination of the integer 1 from the square-bracketed expression in $dY_5(t)/dt$, elimination of 1 from the square-bracketed expression in $dY_6(t)/dt$, and both; adding[4] $k\{fY_3(t)\ Y_4(t)\ Y_1(t) - g\} + d$, as found in

$dY_4(t)/dt$, to $dY_6(t)/dt$; and paring down the product $Y_3(t)Y_4(t)Y_1(t)$, appearing in $dY_2(t)/dt$ and in $dY_4(t)/dt$, to each of its 3 unique pairs of $Y(t)$ terms, and each of its individual $Y(t)$ terms. Other variations involve principled explorations attending $eh < ig$.

Simplifying the present 6-dimensional model to a 4-dimensional model retained dynamical complexity, including the production of chaos, but in a comparatively restricted parameter region (Levy, 2005). To the degree the present dynamics are general to stress and decisional-control coping, they should be robust to changes in control parameters, as the latter vary across individuals or settings. In this sense, consistency potentially lies in the dimensional architecture that hosts the control parameters (cf Mischel & Shoda, 1998), and the dynamics to which they give rise. On balance, the 6-dimensional system architecture fares well in analytical generalization analysis of its essential dynamics, in their surmounting tight parameter boundaries (Yao, et al, 2006; Yu, 2006).

As with many dynamically complex nonlinear systems models, the architecture of the present six-dimensional model is deterministic. A stochastic perturbation can occasionally cause such a system to move in and out of a deterministic attractor (chaotic, limit cycle, fixed point, etc.)[5]. Such a stochastic element is not woven into the present system composition (cf. Brown & Holmes, 2001).

Rather, stochastic sources of variation are deemed to enter into the system's empirical expression. Here, the system is considered to operate amidst model-exogenous noise variance. An apt designation of this confluence is one of *percolation* (Frisch & Hammersley, 1963), comprising a "process of deterministic flow through a stochastic medium" (Winsor, 1995, p. 181). Deterministic predictions thus are regarded as perturbed by stochastic sources that are analogous to error constituents of data theory, such as that of the general linear model.

In his enumeration of issues facing mathematical psychology, Luce (1997) noted that a challenge to implementation of nonlinear dynamical systems theory in psychology was a shortage of principled models in the field. As later stated by Heathcote, "The ultimate success of nonlinear dynamics in behavioral and other areas of psychology will rest on the further development of specific dynamical models and the adaptation of NDA (*nonlinear dynamical analysis*) techniques to test them (2002, p. 626; italics added)." The above concerns with model development appear to remain cogent, at least in the field of stress and coping. Exceptions include formulations of Savic, Knezevic & Opacic (2000), Field & Schuldberg (2011), and Smith and Stevens (1997).

The 7-dimensional model of Savic, et al (2000) addresses dynamical relations entailing the hypothalamic-pituitary-adrenocortical axis, and the "memory-system" (including past-episode, memory-trace mediated cognitive appraisal). Incorporated is provision for personality-related differences in stress vulnerability. This model emphasizes restoration of equilibrium states following stressor incidents[6].

Field and Schuldberg (2011) have extended to the domain of stress-related psychological symptoms, the "Oregonator model" (Field & Noyes, 1974), a dynamical account of a cyclic chemical reaction (the Belousov-Zhabotinsky reaction; Field, Käräs & Noyes, 1972). Emphasized are the buffering effects of social-support. This 3-dimensional model is shown to exhibit chaotic behavior upon the introduction of a time-driven, oscillatory forcing expression (cf. Sprott, 2005), which conveys periodicity of stress induction. Social support is brought into play as one of the system's three order parameters. Field and Schuldberg (2011) have provided an instructive comparison of their model to that of Eq. 1.

Smith and Stevens (1997) have studied arousal modulation from the perspective of separation-attachment affiliative tendencies. Their work has entailed the adaptation of Lotka-Volterra mathematical-ecology modeling and associated dynamics, including apparently chaotic behaviors. Six difference equations together translate Lotka-Volterra "species" variables into noradrenergic- and opioid-neurotransmitter population mechanisms, and Lotka-Volterra "resources" into separation and attachment behaviors. Altogether, with their focus on other aspects of the stress-coping domain, the above models may be viewed as complementary to the present 6-dimensional model, and vice versa.

CONCLUSION

The 6-dimensional model dissected here is substantively principled, in the sense of springing exclusively from what is known about stress-coping phenomena. Its fixed-point, oscillatory and chaotic attractors are properties naturally emerging from a dynamical system whose construction is designed to express domain-specific subject matter. Intricate dynamics therefore may typify the present field of study, akin to that of other fields where their presence has been established.

On balance, the current developments endorse nonlinear dynamical systems theory as a platform for integrating prominent stress-coping constructs. As occurs in other disciplines, the study of dynamics here itself is dynamical, with the challenges of empirical evaluation interacting with those of principled model development. It seems clear, nevertheless, that genuine progress in discerning the subject matter's complexities continues to lie in a rigorous synthesis of its core concepts with contemporary quantitative advances from the broader field of dynamical systems theory.

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ENDNOTES

[1]Note that negative values are taken to imply the opposite of their positive-valence status. In the case of Y^*_1, a negative value is interpreted as signifying relative well-being, or comparative buffering of prevailing stressor level, over and against corresponding positive values. Comparative disruption of decisional-control-related cognitive functioning is inferred from negative $Y^*_{3.}$ Interestingly, many, if not most systems, in a chaotic behavioral regime (discussed for the present system, below) exhibit both positive and negative order-parameter values (Lotka-Volterra and Mackey-Glass systems, for examples, being exceptions; e.g., Sprott, 2010; Yu, 2006).

[2]Numerical integration also disclosed the Feigenbaum cascade accompanying the onset of chaos, preceded by very high periodicity, the latter suggesting the imminence of chaos.

[3]Full reports of empirical studies (Jette, 2003; Levy, 2005) are available upon request through the corresponding author.

[4]Increased system coupling can be "desirable" or "undesirable", depending on its context (e.g., Perrow, 1999; Tenner, 1996). In the present case, the "context" entails system parameters, as follows. With reduced extant system feedback on itself $k < id/(eh)$, increased coupling produced system resistance to a singularity upon progressive elevation of $Y_1(0)$ (Neufeld, 1999). In contrast, with more system feedback on itself $k > id/(eh)$, increased coupling led to apparently chaotic behavior eventuating in a previously absent singularity (parameter values and initial conditions being those of Fig. 3). In these ways, effects of more versus less coupling were dependent on the configuration of parameter values, with associated stability properties – $k <,> id/(eh)$ – in which it took place.

[5]Also possible is periodic or aperiodic exogenous forcing (see, e.g., Field & Schuldberg, 2011; Sprott, 2005), for instance associated with diurnal variation in affect. (We thank an anonymous reviewer for pointing out this possibility.)

[6]Secondary analyses with competitive modes methodology and Hopf-bifurcation analysis indicates that this system's equilibrium state appears not to give way to oscillatory or chaotic attractors (however, cf. the 3-dimensional model of Savic & Jelic, 2005).

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