There are Models, and then there are Models: Not-so-subtle Distinctions between Mathematical and Statistical Modeling

Commentary[1] by Richard W.J. Neufeld[2], James T. Townsend[3], and Matthew J. Shanahan[4] in response to Joseph L. Rodgers' article (2010), "The epistemology of mathematical and statistical modeling: A quiet methodological revolution", *American Psychologist*, *65*, 1-12.

A vote of thanks is owed to Joseph Lee Rodgers (2010) for delineating the current movement toward a model-testing approach to data treatment. We applaud Rodgers' erudite and pellucid chronicling of this quiet revolution. Yet, we would underscore and elaborate upon his distinction between mathematical and statistical modeling, in ways that implicate graduate-education curricula beyond those currently advocated. Second, we agree that attention given to modeling developments waned in the mid-1990's, arguably giving ground to the Null-Hypothesis-Significance-Testing (NHST) debate. Alongside, however, we would point to a formidable rearguard action specifically in clinical mathematical psychology, whose impetus precisely during and after that period has contributed to the preservation and bolstering of the "quiet-revolution".

As noted in Rodgers' article, mathematical and statistical modeling stand as alternatives to classical NHST practices. In addition, they overlap in their methods of empirical evaluation of

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[2] The University of Western Ontario, Department of Psychology, rneufeld@uwo.ca

[3] Indiana University, Department of Psychological and Brain Sciences.

[4] The University of Western Ontario, Department of Psychology.

a single model, or comparative tenability between competing models. Nevertheless, this common ground is where the intersection ends, and indeed serves as a point of departure of the two forms of modeling. Statistical modeling (e.g., structural equation modeling [SEM]) entails the application of specified computations to empirical data sets, whatever the latter's substantive origin. On the other hand, mathematical modeling is dedicated to specific subject matter, such as the organization of cognitive operations transacting a trial of visual information processing. A stochastic model, characterizing much if not most mathematical psychology, is "The mathematical abstraction of an empirical process, whose development is governed by probabilistic laws" (Doob, 1953, p. v). For example, in the field of cognition, a stochastic mathematical process model would incorporate an intrinsic element of indeterminacy in mathematically stipulating the theoretical interconnections between a cognitive process and task-performance. Measures of the latter would be compared to predictions, and possibly evaluated using appropriate statistical distributions. Statistical modeling, on the other hand, entails the organization and assessment of data deemed to express hypothesized inter-variable relations, whether or not the hypotheses owe their existence to explicit formulae delivered by precise derivations.

Mathematical modeling impels content-specific mathematical developments. And, cases where pre-existing models are recruited to new arenas of application are not exempt. In clinical cognitive science, for instance, currently tenable models of normal cognition can be put to the service of illuminating cognitive psychopathology. Extant models of normal cognitive performance are titrated to accommodate clinical-sample deviations in performance speed and/or accuracy. Aspects of the models that survive intact tentatively signify spared functions, and those necessarily altered are "triaged" as signifying functions that are disorder-affected. Here, rote importation of extant modeling is seldom an option. Rather, productive implementation necessitates mathematical adaptations and extensions (e.g., Johnson, Blaha, Houpt & Townsend, 2010). The critical responsibility for executing such quantitative elaborations rests squarely with the user. Statistical modeling, on the other hand, although demanding care in application, and fidelity of interpretation to the data-model infrastructure, typically does not demand analogous input by the user-- which in effect would correspond to mathematically customizing the infrastructure itself.

Often customized to a specific behavioral domain, particular mathematical modeling developments are most useful to researchers within that domain. Reasonable progress within the staked-out problem area, however, defensibly hinges on the very substantive exclusiveness that accompanies in-depth theoretical focus. Mathematical modeling may sacrifice breadth of coverage for penetration of analysis; statistical modeling, however, carries its own restrictions. Different problem areas engage a statistical model according to research design and general data format prescribed by the given model architecture (e.g, a demarcated covariance structure, in the case of SEM). Mathematical modeling stands in contrast to generic model application, inasmuch as part of model-construction payoff consists of unique empirical tests emanating from the formal developments themselves (e.g., Townsend & Wenger, 2004).

A case can be made, moreover, that theory beautification is proprietary of content-focussed formal-model elegance (Neufeld, 2007). As for utility, its necessitating of precision has been deemed essential to dislodging otherwise intractable logjams. Examples from clinical cognitive science are plentiful, ranging from the disentangling of sources of cognitive-capacity reduction in schizophrenia, to the decomposition of memory deficit in Developmental Dyslexia. Progress in clinical mathematical modeling since the mid-1990's increasingly has been represented in regular articles and journal special issues or sections, such as *Psychological Assessment*'s "Process Models in Psychological Assessment" (Neufeld, 1998), and "Contributions of Mathematical Psychology to Clinical Science and Assessment", in the *Journal of Mathematical Psychology* (Neufeld & Townsend, 2010).

Realizing the scientific benefits of formal modeling requires facility with tools equal to the task. Self-tutelage is often the most accessible means of bolstering one's background. Resources from other disciplines can be recruited to the service of such efforts (e.g., "(Advanced) Mathematics for Engineers and Scientists"; "Calculus with Applications"). Workshops such as the APA Advanced Institutes provide similar opportunities. Progressive change to existing curricula, would require from our students "behavior fit to design models designed to fit behavior". Calculus and probability theory should be a fundamental tenet of training. Every psychologist should take two 5-credit courses in calculus, and statistics courses incorporating elementary probability.

Despite an emphasis on mathematical training, more globally it is the versatility and power of psychological theory we wish to see bolstered. Such opportunities abound at the intersection of psychological and mathematical expertise. With regard to the need for deep understanding of the content of one's science: future Nobel Laureate John Nash shared his mathematical ideas about gravity, friction, and radiation with Albert Einstein, whereupon the response of this most creative and rigorous of mathematical scientists was "You had better study some more physics, young man." (Nasar, 1998, p.71). With reference to the value of serious mathematical preparation, fellow Nobel laureate and cognitive scientist *extraordinaire* Herbert Simon (1978) writes in his autobiographical statement: "The social sciences, I thought, needed the same kind of rigor and the same mathematical underpinnings that had made the "hard" sciences so brilliantly successful." Together, these comments urge a rigorous, creative mathematical approach with deep appreciation for the extant corpus of psychological knowledge.

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